Binary Distillation with the McCabe-Thiele Method

1. Introduction

A Toluene - Benzene feed at 1 atm is to be separated in a multistage distillation unit to a required purity of both components.

This worksheet will calculate the required number of theoretical stages required. Data describing propane-pentane equilibrium composition is supplied.

2. Equilibrium Data and Required Purity

The following data gives the mole fraction of propane in the liquid and vapour phases in a Toluene / Benzene mixture at equilibrium at 1 Atm.

$$xPropLiq := \begin{bmatrix} 0 \\ 0.067 \\ 0.139 \\ 0.217 \\ 0.301 \\ 0.393 \\ 0.493 \\ 0.603 \\ 0.723 \\ 0.855 \\ 1 \end{bmatrix} \qquad xPropVap := \begin{bmatrix} 0 \\ 0.145 \\ 0.278 \\ 0.401 \\ 0.513 \\ 0.615 \\ 0.708 \\ 0.792 \\ 0.869 \\ 0.938 \\ 1 \end{bmatrix}$$

Required purity of Benzene in distillate $x_D := 97\%$

Required purity of Benzene in bottoms $x_B := 2\%$

Fraction of Benzene in Feed $x_F := 40\%$

q value $q := 1.5$

Reflux ratio $R := 3.5$

3. The Equilibrium Line

This is a continuous function which describes the equilibrium line

$$g(x) := \text{interp}\left(\text{cspline}\left(xPropLiq, xPropVap\right), xPropLiq, xPropVap, x\right)$$

4. The Top and Bottom Operating Lines and the q-Line

q-line $$qline(x) := x \cdot \frac{q}{q-1} - \frac{x_F}{q-1}$$

Top operating line $$top\_op(x) := x \cdot \frac{R}{1+R} + \frac{x_D}{1+R}$$

Bottom operating line $$bottom\_op(x) := x \cdot \frac{R \cdot (x_F - x_B) + q \cdot (x_D - x_B)}{q \cdot (x_D - x_B) + R \cdot (x_F - x_B) - x_D + x_F} + \frac{x_B \cdot (x_F - x_D)}{q \cdot (x_D - x_B) + R \cdot (x_F - x_B) - x_D + x_F}$$

Hence a function can be defined that described the combined operating line

$$operating\_line(x) := \begin{cases} bottom\_op(x) & \text{if } x < \frac{x_D \cdot (q-1) + x_F \cdot (R+1)}{q+R} \\ top\_op(x) & \text{else} \end{cases}$$

5. Required Theoretical Stages

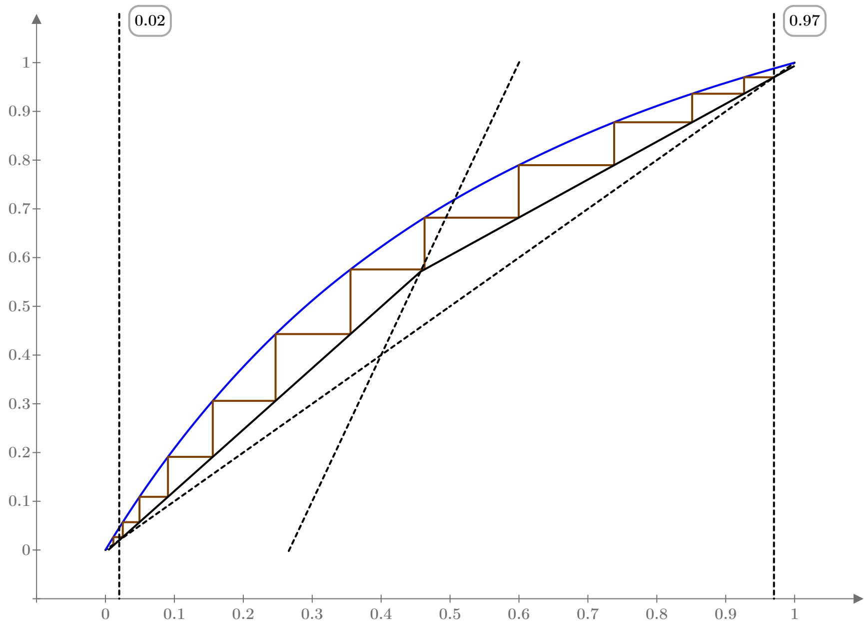
The following function steps off the required number of stages given a function f that describes the operating line and a function g that describes the equilibrium line

```
MC(f, g) := | xguess ← 0
            | X_1 ← x_D
            | Y_1 ← x_D
            | X_2 ← root(g(xguess) − Y_1, xguess)
            | Y_2 ← Y_1
            | i ← 3
            | while X_{i−2} > x_B
            |     | X_{i−1} ← X_{i−2}
            |     | Y_{i−1} ← f(X_{i−1})
            |     | X_i ← root(g(xguess) − Y_{i−1}, xguess)
            |     | Y_i ← Y_{i−1}
            |     | i ← i + 2
            | X_{i−1} ← X_{i−2}
            | Y_{i−1} ← X_{i−2}
            | return augment(X, Y)
```

6. Results

$x := 0, 0.001 .. 1$

Toluene - Benzene Equilibrium @ 1 atm



The required number of stages is:

$$stages := \text{floor}\left(\frac{\text{rows}\left(MC(operating\_line, g)^{\langle 1 \rangle}\right)}{2}\right) - 1$$

$$stages = 12$$

The efficiency of real stages will be about 0.5-0.7. Hence the actual required number of stages is about $\frac{stages}{0.5} = 24$.

$$MC(operating\_line, g) = \begin{bmatrix} 0.97 & 0.97 \\ 0.97 & 0.97 \\ 0.927 & 0.97 \\ 0.927 & 0.936 \\ 0.851 & 0.936 \\ 0.851 & 0.878 \\ 0.738 & 0.878 \\ 0.738 & 0.79 \\ 0.6 & 0.79 \\ 0.6 & 0.682 \\ 0.463 & 0.682 \\ 0.463 & 0.576 \\ 0.356 & 0.576 \\ 0.356 & 0.443 \\ 0.247 & 0.443 \\ 0.247 & 0.306 \\ 0.156 & 0.306 \\ 0.156 & 0.191 \\ 0.091 & 0.191 \\ 0.091 & 0.109 \\ 0.049 & 0.109 \\ 0.049 & 0.057 \\ 0.025 & 0.057 \\ 0.025 & 0.026 \\ 0.011 & 0.026 \\ 0.011 & 0.011 \end{bmatrix}$$