Numerical and Symbolic Inverse Laplace Transform

Maxima Laplace

Initial value problem: linear ODE with constants coefficients.

$m := -0.5$ $n := 1$ $q(t) := \sqrt{t}$

$$ode := \left[\frac{d^2}{dt^2} y(t) + m \cdot \frac{d}{dt} y(t) + n \cdot y(t) = q(t) \right]$$

$ic := [0\ 0]$ $[x(0)\ x'(0)]$

$fnc := [y\ Y]$ $[t\text{-domain}\ s\text{-domain}]$

Symbolic Solver call $ODE := LapOde(ode, ic, fnc, t, s)$

Laplace transform of the ode (mixing time and freq domain)

$$ODE_1 = \left[\frac{(2 + s \cdot (-1 + 2 \cdot s)) \cdot Y(s)}{2} = \frac{\sqrt{\pi}}{2 \cdot 2\sqrt{s}^3} \right]$$

Freq-domain solution

$$ODE_2 = \left[Y(s) = \frac{\sqrt{\pi}}{2\sqrt{s}^3 \cdot (2 + s \cdot (-1 + 2 \cdot s))} \right]$$

Time-domain symbolic solution: Maxima can't give a closed expression

$$ODE_3 = \left[y(t) = ilt\left(\frac{\sqrt{\pi}}{2\sqrt{s}^3 \cdot (2 + s \cdot (-1 + 2 \cdot s))}, s, t \right) \right]$$

Numerical ode solver solution

$ode = y'' + m \cdot y' + n \cdot y - q(t)$ $U = \begin{bmatrix} y \\ y' \end{bmatrix}$ $\frac{dU}{dt} = \begin{bmatrix} y' \\ y'' \end{bmatrix}$

$$D(t, U) := \begin{bmatrix} U_2 \\ -m \cdot U_2 - n \cdot U_1 + q(t) \end{bmatrix}$$

Plots: numerical solver and numerical inverse laplace solutions $[ti\ te\ N] := [0\ 12\ 20]$

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Assign(ODE_2)
ty := ILap(Y(s), ti, te, N)
y(t) := cinterp(ty, t)
ty2 := dn_GearsBDF(ic^T, ti, te, 100, D(t, U))
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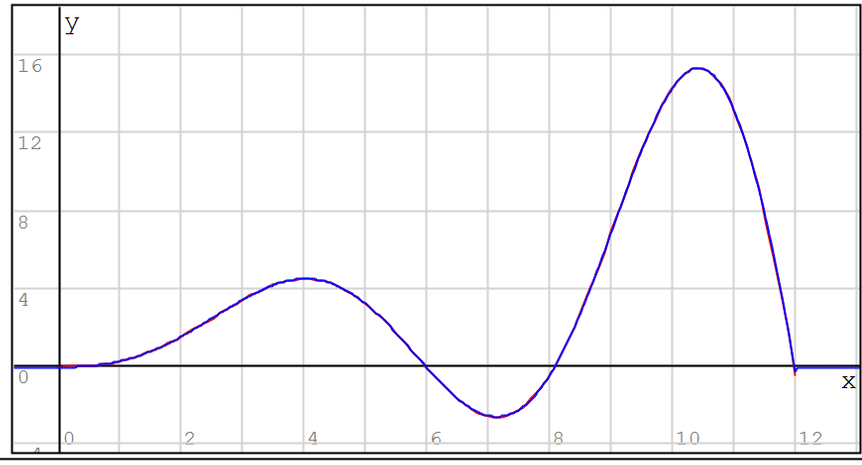

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