Lagrangians

Lagrangian Dynamics

$isol\left(\frac{1}{\partial t}\cdot d\left(\frac{\partial(L, q')}{\partial q'}\right) - \frac{\partial(L, q)}{\partial q}, q''\right)$ $id(x) := x$

$too := time(0)$

$frames := 99$

$NN := 2\cdot[5\ 5]$ Clear(D) = 1 $0 := 10^{-33}$

[m kg s K A cd] := [1 1 1 1 1 1]

dTime := "t"   dSymbol := "∂"   sign := 1

$$E = \sum_{i=1}^{n} \dot{q}_i \frac{\partial L}{\partial \dot{q}_i} - L.$$

```
L_vf(D(2), U#, a#, b#, N#, sf#) := [[m kg s K A cd] := [1 1 1 1 1 1] [α# β#] := [a# b#] + 1]
  if (N# = 0) ∨ (sf# = 0)
    eval({augment(col(U#, α#), col(U#, β#))
          augment(U#_{1 α#}, U#_{1 β#}, "o")})
  else
    [Clear(xx#, xx'#)  u# := U#_{1[2..cols(U#)]}  u#_{a#} := xx#  u#_{b#} := xx'#
    [G# := D(U#_{1 1}, u#)_{[a# b#]}  G(xx#, xx'#) := G#]
    SF#(t) := str2num(sf#)
    eval({augment(col(U#, α#), col(U#, β#))
          augment(U#_{1 α#}, U#_{1 β#}, "o")
          pVField("G", pBox(col(U#, α#), col(U#, β#), 10 %, 10 %), N#
```

Mechanics Braintwister Problem

Mechanics Braintwister Problem

https://fr.maplesoft.com/applications/view.aspx?SID=131117&view=html

Gen coords, constants & position

str2num(∂Vars("gc", ["r" "θ"], ["a" "m.1" "m.2" "g"])) = [t r r' θ θ']

$x1 := r - 2\cdot a$ $x2 := r\cdot\sin(\theta)$ $y2 := -r\cdot\cos(\theta)$

$x1' := \frac{d(x1)}{\partial t}$ $x2' := \frac{d(x2)}{\partial t}$ $y2' := \frac{d(y2)}{\partial t}$

Lagrangian

$$L := \frac{1}{2}\cdot m_1\cdot x1'^2 + \frac{1}{2}\cdot m_2\cdot\left(x2'^2 + y2'^2\right) - m_2\cdot g\cdot y2$$

Equations of motion

$r''_{eq} := L_{eq}(L, r)$ $\theta''_{eq} := L_{eq}(L, \theta)$

$$L = \frac{m_1\cdot r'^2 + m_2\cdot\left((\sin(\theta)\cdot r' + r\cdot\cos(\theta)\cdot\theta')^2 + (-\cos(\theta)\cdot r' + r\cdot\sin(\theta)\cdot\theta')^2\right) + 2\cdot m_2\cdot g\cdot r\cdot\cos(\theta)}{2}$$

$$r''_{eq} = \frac{\left(\theta'\cdot(\cos(\theta)\cdot(\sin(\theta)\cdot r' + r\cdot\cos(\theta)\cdot\theta') + \sin(\theta)\cdot(-\cos(\theta)\cdot r' + r\cdot\sin(\theta)\cdot\theta')) + g\cdot\cos(\theta)\right)\cdot m_2}{m_1 + m_2}$$

$$\theta''_{eq} = -\frac{\left(\cos(\theta)\cdot(\sin(\theta)\cdot r' + r\cdot\cos(\theta)\cdot\theta') + \sin(\theta)\cdot(-\cos(\theta)\cdot r' + r\cdot\sin(\theta)\cdot\theta') + \theta'\cdot r\right)\cdot r' + r\cdot g\cdot\sin(\theta)}{r^2}$$

Solutions

$N := 200$ $te := 2\ \mathrm{s}$

$[\,a\ m_1\ m_2\ g\,] := [\,50\ \mathrm{cm}\ 50\ \mathrm{g}\ 50\ \mathrm{g}\ g_e\,]$

$ic := [\,a\ 0\ \frac{\mathrm{m}}{\mathrm{s}}\ 90\ °\ 0\ \mathrm{rpm}\,]$

$D(t, u) := \left| \begin{array}{l} [\,r\ r'\ \theta\ \theta'\,] := [\,u_1\ u_2\ u_3\ u_4\,] \\ \mathrm{stack}\left(r', r''_{eq}, \theta', \theta''_{eq}\right) \end{array} \right.$

$U := L_{ode}\left(D(t, u), [\,0\ te\,], ic, N, \text{"Rkadapt"}\right)$

Find when m1 hit the pulley

$[\,X1\,] := L_{At}\left([\,x1\,], gc, U\right)$ $n := \sum \mathrm{findrows}\left(\mathrm{eval}\left(\overrightarrow{X1 < 0}\right), 1, 1\right) = 52$

Solve for a simple pendulum, keeping r constant

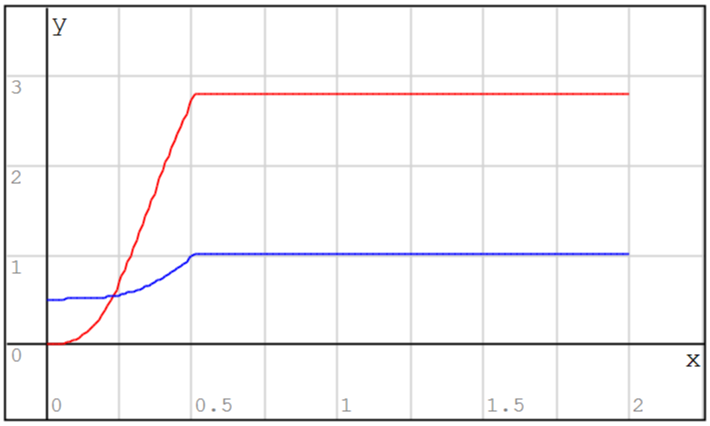
$D2(t, u) := \left| \begin{array}{l} [\,r\ r'\ \theta\ \theta'\,] := [\,u_1\ u_2\ u_3\ u_4\,] \\ \mathrm{stack}\left(0, 0, \theta', -\frac{g \cdot \sin(\theta)}{a}\right) \end{array} \right.$

Use U for the start time and IC's

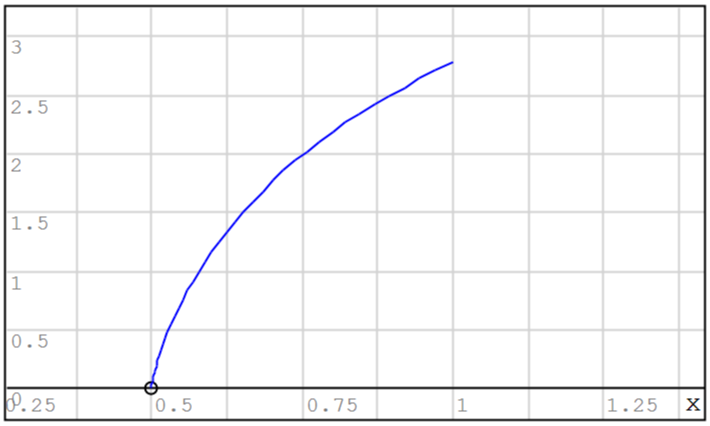
$U2 := L_{ode}\left(D2(t, u), [\,U_{n\,1}\ te\,], U_{n\,[2..\mathrm{cols}(U)]}, N - n, \text{"Rkadapt"}\right)$

Merge into one solution

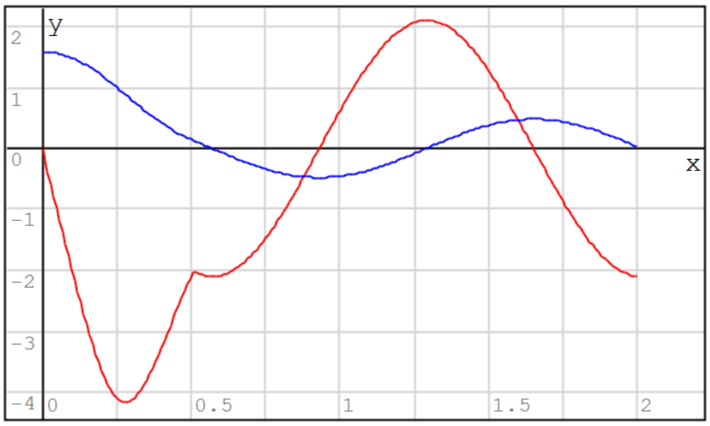
$U := \mathrm{stack}\left(U_{[1..n][1..\mathrm{cols}(U)]}, U2\right)$ $\mathrm{rows}(U) = 201$



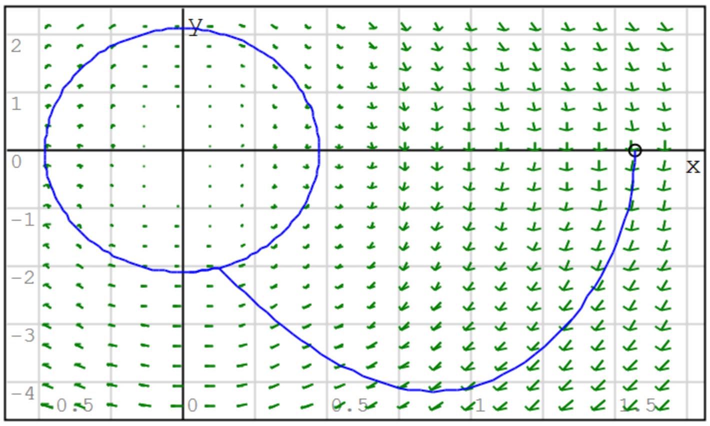
$\left\{ \begin{array}{l} \mathrm{augment}(\mathrm{col}(U, 1), 1.0 \cdot \mathrm{col}(U, 2)) \\ \mathrm{augment}(\mathrm{col}(U, 1), 1.0 \cdot \mathrm{col}(U, 3)) \end{array} \right.$



$L_{vf}(D(t, u), U, 1, 2, NN, 0)$

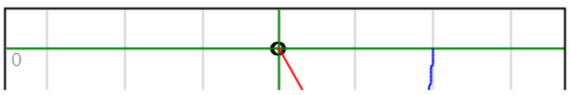


$\left\{ \begin{array}{l} \mathrm{augment}(\mathrm{col}(U, 1), 1.0 \cdot \mathrm{col}(U, 4)) \\ \mathrm{augment}(\mathrm{col}(U, 1), 1.0 \cdot \mathrm{col}(U, 5)) \end{array} \right.$

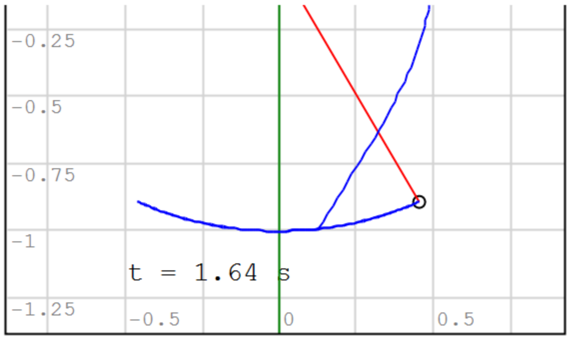


$L_{vf}(D(t, u), U, 3, 4, 2 \cdot NN, \text{"0.5*t"})$

$\left| \begin{array}{l} [\,X1\ X2\ Y2\,] := L_{At}([\,x1\ x2\ y2\,], gc, U) \\ [\,Fn\ Tn\,] := L_{Frames}(U, 100, 2) \end{array} \right.$



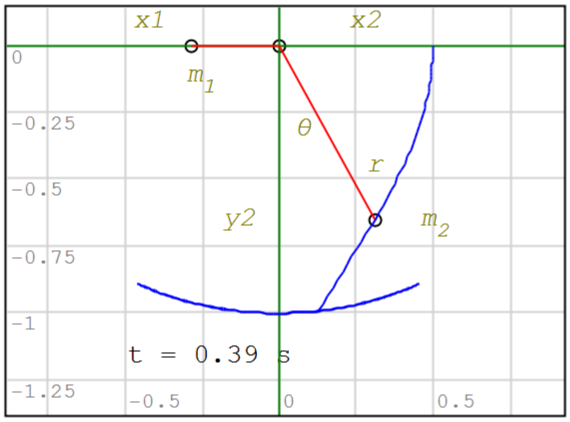
$$A(t) := \begin{cases} \text{eval}(\text{augment}(X2, Y2)) \\ \text{eval}\left(\begin{bmatrix} X1_t & 0 \\ 0 & 0 \\ X2_t & Y2_t \end{bmatrix}\right) \\ \text{eval}([\,0\ 0\ \text{"+"}\ 1000\ \text{"green"}\,]) \\ \text{eval}\left(\text{augment}\left(\begin{bmatrix} X1_t & 0 \\ X2_t & Y2_t \\ 0 & 0 \end{bmatrix}, \text{"o"}\right)\right) \\ \text{eval}([\,-a\ \ -2.2 \cdot a^{Tn}{}_t\,]) \end{cases}$$



A(t)

Generalized coordinates

$$gc = [\,t\ \ r\ \ r'\ \ \theta\ \ \theta'\,]$$



A(40)

Parameters

$$a = 50\ \text{cm}$$

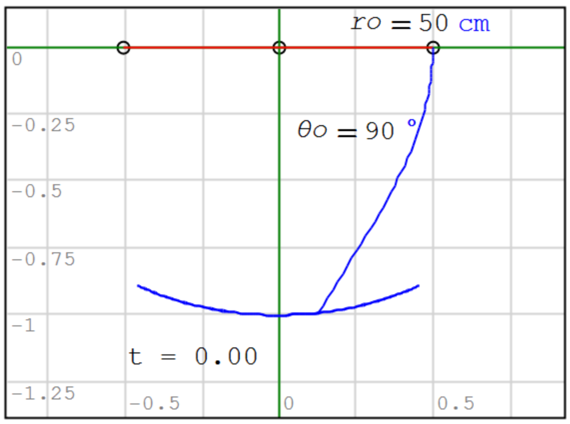
$$m_1 = 50\ \text{g}$$

$$m_2 = 50\ \text{g}$$

Initial conditions $[\,ro\ \ vo\ \ \theta o\ \ \omega o\,] := ic$

$$vo = 1 \cdot 10^{-33}\ \frac{\text{m}}{\text{s}}$$

$$\omega o = 1 \cdot 10^{-33}\ \text{rpm}$$



A(1)

Spherical Pendulum

Spherical Pendulum

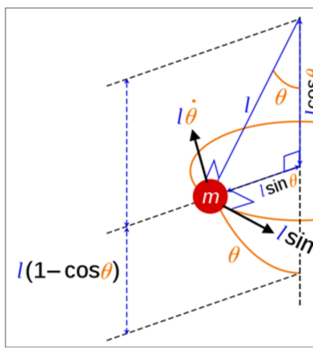
Gen coords, constants & position

$$\text{str2num}\left(\partial Vars\left(\text{"gc"}, [\,\text{"θ"}\ \text{"φ"}\,], [\,\text{"r"}\ \text{"m"}\ \text{"g"}\,]\right)\right) = [\,t\ \ \theta\ \ \theta'\ \ \varphi\ \ \varphi'\,]$$

$$sph(\theta, \varphi) := [\,r \cdot \sin(\theta) \cdot \cos(\varphi)\ \ r \cdot \sin(\theta) \cdot \sin(\varphi)\ \ r \cdot (1 - \cos(\theta))\,]$$

$$[\,x\ \ y\ \ z\,] := sph(\theta, \varphi)$$

$$x' := \frac{d(x)}{\partial t} \qquad y' := \frac{d(y)}{\partial t} \qquad z' := \frac{d(z)}{\partial t}$$



Lagrangian

$$L := \frac{1}{2} \cdot m \cdot \left(x'^2 + y'^2 + z'^2\right) - m \cdot g \cdot z$$

Symplified to

$$L := \frac{1}{2} \cdot m \cdot r^2 \cdot \left(\varphi'^2 \cdot \left(1 - (\cos(\theta))^2\right) + \theta'^2\right) + m \cdot r \cdot g \cdot (-1 + \cos(\theta))$$

Equations of motion

$$\theta''_{eq} := L_{eq}(L, \theta)$$

$$\theta''_{eq} = \frac{\sin(\theta) \cdot \left(-g + \cos(\theta) \cdot \varphi'^2 \cdot r\right)}{r}$$

$\varphi''_{eq} := L_{eq}(L, \varphi)$ $\qquad$ $\varphi''_{eq} = -\dfrac{2 \cdot \varphi' \cdot \sin(\theta) \cdot \cos(\theta) \cdot \theta'}{1 - \cos(\theta)^2}$

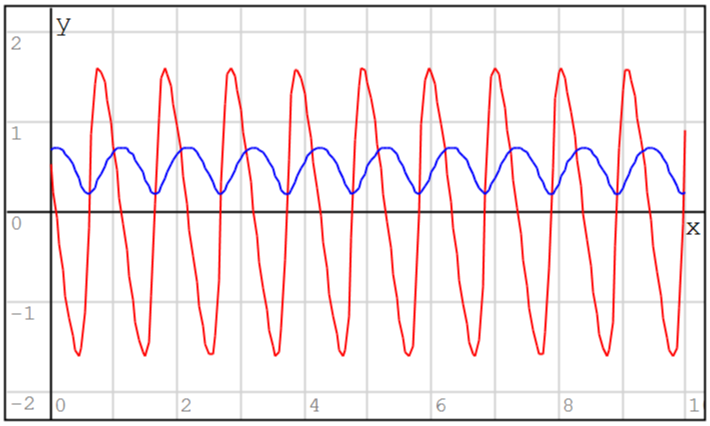
Cyclic coord & conservation $\qquad$ $\dfrac{d}{d\,\varphi} L = 0 \quad \text{->} \quad p_\varphi := \dfrac{d}{d\,\varphi'} L$ $\qquad$ $p_\varphi = m \cdot r^2 \cdot \varphi' \cdot \left(1 - \cos(\theta)^2\right)$

Solutions $\qquad$ $N := 200 \qquad te := 10\ s \qquad [\,r\ m\ g\,] := [\,1\ m\ 1\ kg\ g_e\,]$

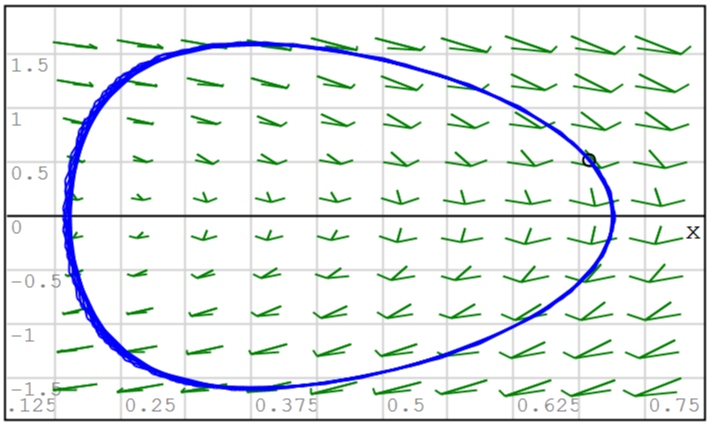
$ic := [\,40\ °\ 5\ rpm\ 30\ °\ (-10)\ rpm\,] \qquad [\,\theta o\ \theta o'\ \varphi o\ \varphi o'\,] := ic$

$D(t, u) := \left|\begin{array}{l} [\,\theta\ \theta'\ \varphi\ \varphi'\,] := [\,u_1\ u_2\ u_3\ u_4\,] \\ \text{stack}\left(\theta', \theta''_{eq}, \varphi', \varphi''_{eq}\right) \end{array}\right.$

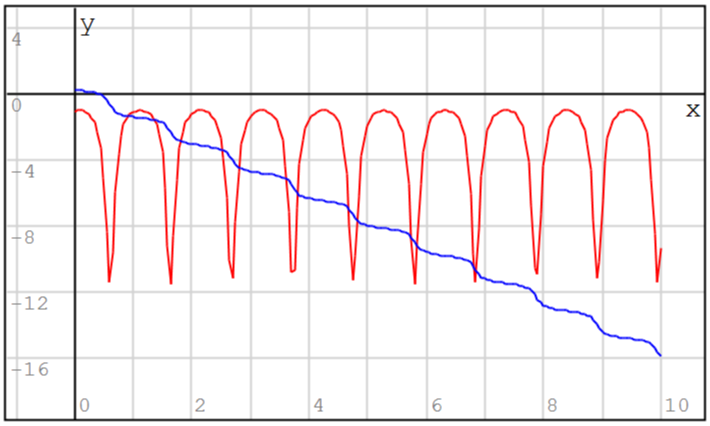
$U := L_{ode}\left(D(t, u), [\,0\ te\,], ic, N, \text{"Rkadapt"}\right)$



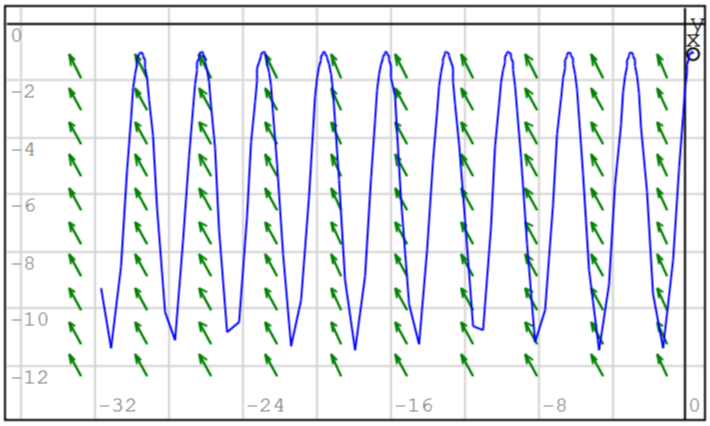
$\left\{\begin{array}{l} \text{augment}(\text{col}(U, 1), 1.0 \cdot \text{col}(U, 2)) \\ \text{augment}(\text{col}(U, 1), 1.0 \cdot \text{col}(U, 3)) \end{array}\right.$



$L_{vf}(D(t, u), U, 1, 2, NN, \text{"0.5*t"})$



$\left\{\begin{array}{l} \text{augment}(\text{col}(U, 1), 0.5 \cdot \text{col}(U, 4)) \\ \text{augment}(\text{col}(U, 1), 1.0 \cdot \text{col}(U, 5)) \end{array}\right.$



$L_{vf}(D(t, u), U, 3, 4, NN, \text{"1"})$

https://handwiki.org/wiki/Physics:Spherical_pendulum

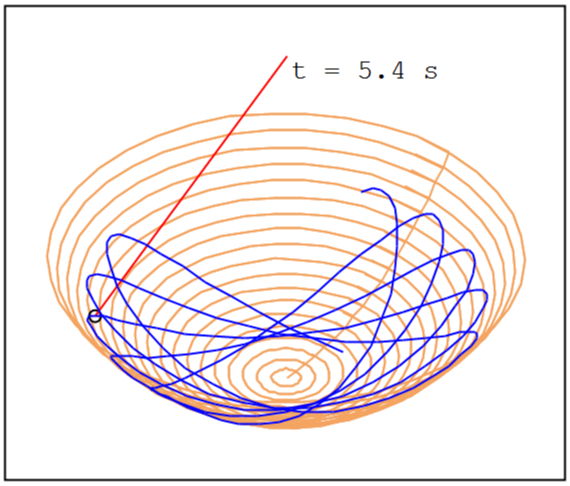
$Y_2 := pView2(-37.5\ °, 30\ °)$

$G := pGrid(sph, pR(0, 0.3 \cdot \pi, 20), pR(0, 2.1 \cdot \pi, 40))$

$[\,X\ Y\ Z\ P\varphi\,] := L_{At}\left([\,x\ y\ z\ P_\varphi\,], gc, U\right)$

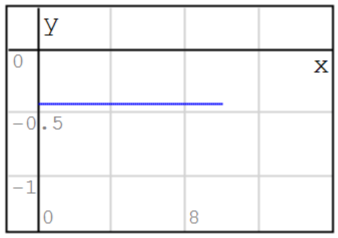
$[\,Fn\ Tn\,] := L_{Frames}(U, 100, 1)$

$A := \left\{\begin{array}{l} \text{eval}\left(\text{augment}(X, Y, Z) \cdot Y_2\right) \\ \text{eval}\left(\begin{bmatrix} 0 & 0 & r \\ X_t & Y_t & Z_t \end{bmatrix} \cdot Y_2\right) \\ \text{eval}\left(\text{augment}\left([\,X_t\ Y_t\ Z_t\,] \cdot Y_2, \text{"o"}\right)\right) \\ \text{eval}\left(\text{augment}\left([\,0\ 0\ r\,] \cdot Y_2, Tn_t\right)\right) \\ \text{eval}\left(pMesh(G, [\,0\ 0\,], [\,1\ 1\,]) \cdot Y_2\right) \end{array}\right.$



$[\,\Phi\ \ P\varphi\,] := L_{At}\left([\,\varphi\ \ P_\varphi\,],\ gc,\ U\right)$

A



augment(col(U, 1), Pφ)

Elastic Pendulum

Elastic Pendulum

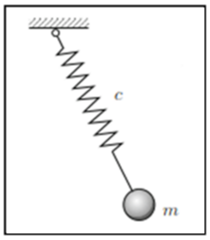
Gen coords, constants & position

$\text{str2num}\left(\partial Vars\left(\text{"gc"}, [\text{"x" "y"}], [\text{"lo" "m" "κ" "g"}]\right)\right) = [\,t\ \ x\ \ x'\ \ y\ \ y'\,]$

x,y

Lagrangian

$L := \frac{1}{2}\cdot m\cdot\left(x'^2 + y'^2\right) - \left(m\cdot g\cdot y + \frac{1}{2}\cdot\kappa\cdot\left(\sqrt{x^2+y^2} - lo\right)^2\right)$



Equations of motion

$x''_{eq} := L_{eq}(L, x)$ $\quad x''_{eq} = -\frac{x\cdot\left(-lo + \sqrt{x^2+y^2}\right)\cdot\kappa}{m\cdot\sqrt{x^2+y^2}}$

$y''_{eq} := L_{eq}(L, y)$ $\quad y''_{eq} = -\frac{m\cdot\sqrt{x^2+y^2}\cdot g + y\cdot\left(-lo+\sqrt{x^2+y^2}\right)\cdot\kappa}{m\cdot\sqrt{x^2+y^2}}$

Solutions

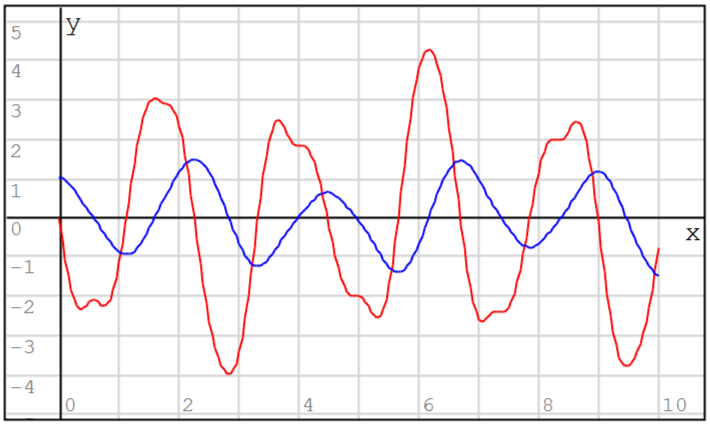
$N := 200 \qquad te := 10\ \text{s}$

$[\,lo\ \ m\ \ \kappa\ \ g\,] := \left[\,1\ \text{m}\ \ 1\ \text{kg}\ \ 20\ \frac{\text{N}}{\text{m}}\ \ g_e\,\right]$

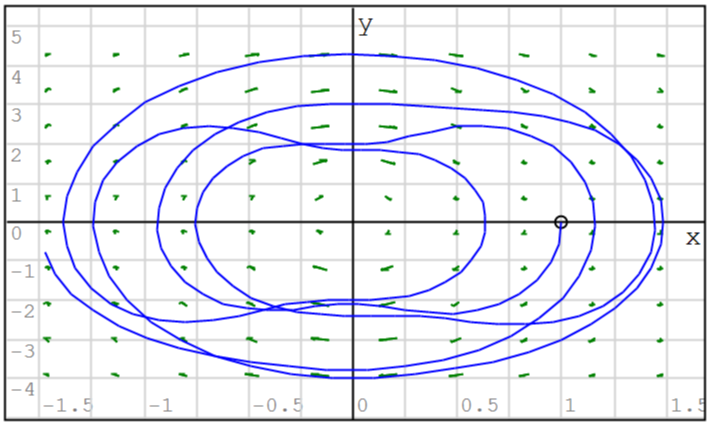
$ic := \left[\,1\ \text{m}\ \ 0\ \frac{\text{m}}{\text{s}}\ \ -2\ \text{m}\ \ 2\ \frac{\text{m}}{\text{s}}\,\right] \qquad [\,xo\ \ vxo\ \ yo\ \ vyo\,]$

$D(t, u) := \begin{array}{|l} [\,x\ \ x'\ \ y\ \ y'\,] := [\,u_1\ \ u_2\ \ u_3\ \ u_4\,] \\ \text{stack}\left(x', x''_{eq}, y', y''_{eq}\right) \end{array}$

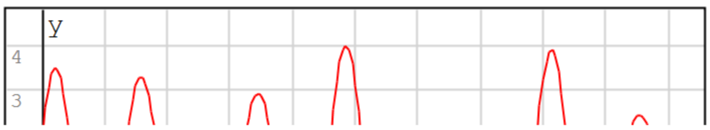
$U := L_{ode}\left(D(t, u), [\,0\ \ te\,], ic, N, \text{"Rkadapt"}\right)$

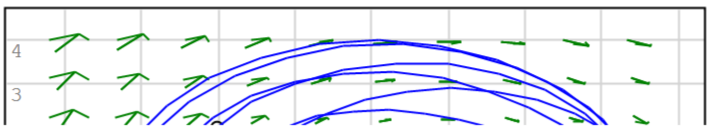


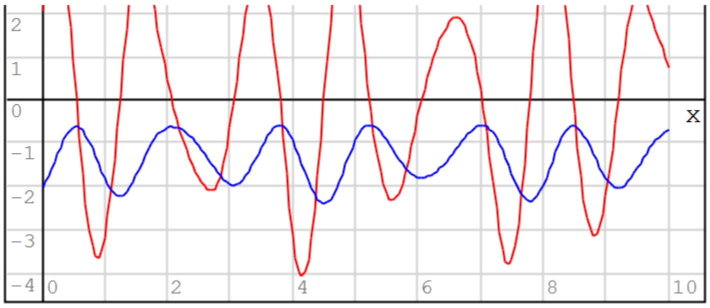
augment(col(U, 1), 1.0·col(U, 2))
augment(col(U, 1), 1.0·col(U, 3))



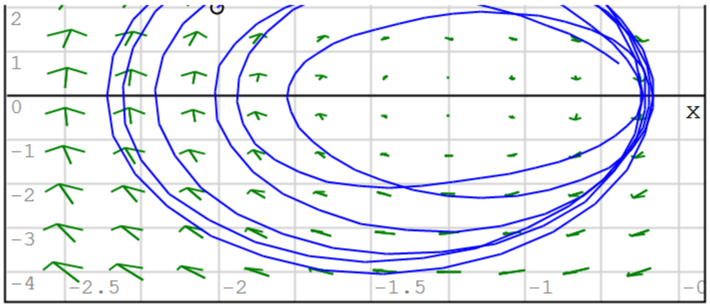
$L_{vf}(D(t, u), U, 1, 2, NN, \text{"0.1"})$







$$\begin{cases} \text{augment}(\text{col}(U,1), 1.0\cdot\text{col}(U,4)) \\ \text{augment}(\text{col}(U,1), 1.0\cdot\text{col}(U,5)) \end{cases}$$

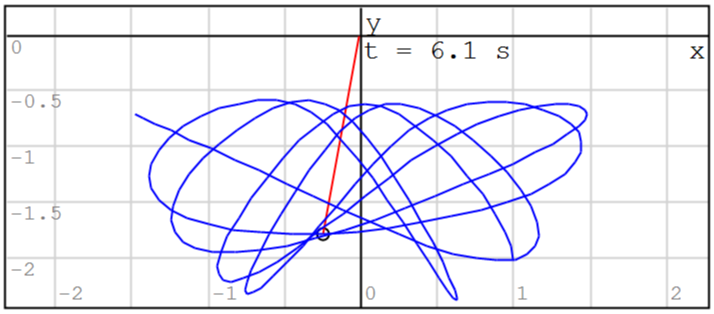


$L_{vf}(D(t,u), U, 3, 4, NN, \text{"0.5*t"})$

$$[\,T\ X\ X'\ Y\ Y'\,] := Cols(U)$$

$$[\,Fn\ Tn\,] := L_{Frames}(U, 100, 1)$$

$$A := \begin{cases} \text{augment}(X, Y) \\ \begin{bmatrix} 0 & 0 \\ X_t & Y_t \end{bmatrix} \\ \text{augment}([\,X_t\ Y_t\,], \text{"o"}) \\ \text{augment}(0, 0, Tn_t) \end{cases}$$



A

⊟— Double Pendulum

Double Pendulum

Gen coords, constants & position

$$\text{str2num}(\partial Vars(\text{"gc"}, [\text{"α"}\ \text{"β"}], [\text{"m.1"}\ \text{"m.2"}\ \text{"g"}\ \text{"a"}\ \text{"b"}])) = [\,t\ \alpha\ \alpha'\ \beta\ \beta'\,]$$

$$x1 := a\cdot\sin(\alpha) \qquad y1 := -a\cdot\cos(\alpha)$$

$$x2 := x1 + b\cdot\sin(\beta) \qquad y2 := y1 - b\cdot\cos(\beta)$$

$$x1' := \frac{d(x1)}{\partial t} \qquad y1' := \frac{d(y1)}{\partial t} \qquad x2' := \frac{d(x2)}{\partial t} \qquad y2' := \frac{d(y2)}{\partial t}$$

Lagrangian

$$L := \frac{1}{2}\cdot m_1\cdot\left(x1'^2 + y1'^2\right) + \frac{1}{2}\cdot m_2\cdot\left(x2'^2 + y2'^2\right) - m_1\cdot g\cdot y1 - m_2\cdot g\cdot y2$$

Equations of motion

$$\alpha''_{eq} := L_{eq}(L, \alpha) \qquad \beta''_{eq} := L_{eq}(L, \beta)$$

$$L = \frac{m_1\cdot a\cdot\left(a\cdot\alpha'^2\cdot 1 + 2\cdot g\cdot\cos(\alpha)\right) + m_2\cdot\left(\left(a\cdot\cos(\alpha)\cdot\alpha' + b\cdot\cos(\beta)\cdot\beta'\right)^2 + \left(a\cdot\sin(\alpha)\cdot\alpha' + b\cdot\sin(\beta)\cdot\beta'\right)^2\right) + 2\cdot m_2\cdot g\cdot(a \ldots}{2}$$

$$\alpha''_{eq} = -\frac{m_2\cdot\left(b\cdot\left(\beta'^2\cdot(-\sin(\beta)\cdot\cos(\alpha) + \cos(\beta)\cdot\sin(\alpha)) + (\cos(\beta)\cdot\cos(\alpha) + \sin(\beta)\cdot\sin(\alpha))\cdot\beta''\right) + g\cdot\sin(\alpha)\right) + m_1\cdot g \ldots}{a\cdot(m_1 + m_2)}$$

$$\beta''_{eq} = -\frac{a\cdot\left(\alpha'^2\cdot(-\sin(\alpha)\cdot\cos(\beta) + \cos(\alpha)\cdot\sin(\beta)) + (\cos(\alpha)\cdot\cos(\beta) + \sin(\alpha)\cdot\sin(\beta))\cdot\alpha''\right) + g\cdot\sin(\beta)}{b}$$

Push β'' into α''

$$\alpha''_{eq} := isol\left(\alpha'' = \alpha''_{eq}\Big|_{\beta'' = \beta''_{eq}}, \alpha''\right)$$

Now α''.eq is free of β''. We use that in D

$$\alpha''_{eq} = -\frac{m_2\cdot\left(\beta'^2\cdot(-\sin(\beta)\cdot\cos(\alpha) + \cos(\beta)\cdot\sin(\alpha))\cdot b - (\cos(\beta)\cdot\cos(\alpha) + \sin(\beta)\cdot\sin(\alpha))\cdot(\ldots\right.}{a\cdot\left(m_1 + m_2\cdot\left(1 - (\cos(\alpha)\cdot\cos \ldots\right.\right.}$$

Solutions

$N := 200$ $te := 10\ s$

$[\,m_1\ m_2\ a\ b\ g\,] := [\,800\ g\ 1600\ g\ 120\ cm\ 250\ cm\ g_e\,]$

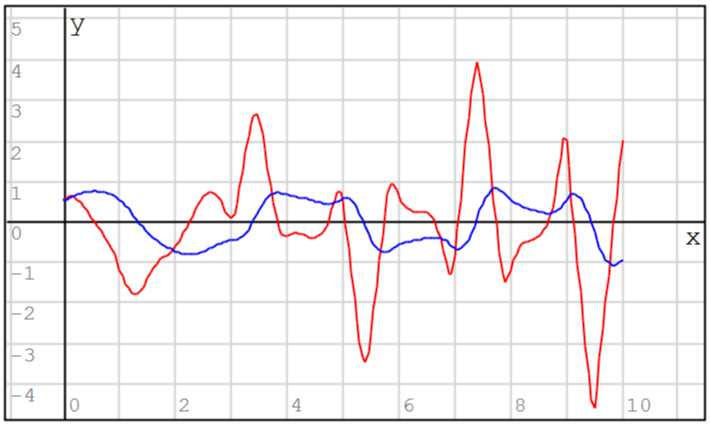
$ic := [\,30\ °\ 5\ rpm\ 60\ °\ 10\ rpm\,]$ $[\,\alpha o\ \alpha o'\ \beta o\ \beta o'\,]$

$D(t, u) :=$ $[\,\alpha\ \alpha'\ \beta\ \beta'\,] := [\,u_1\ u_2\ u_3\ u_4\,]$
$\alpha'' := \alpha''_{eq}$
$\text{eval}\left(\text{stack}\left(\alpha', \alpha'', \beta', \beta''_{eq}\right)\right)$

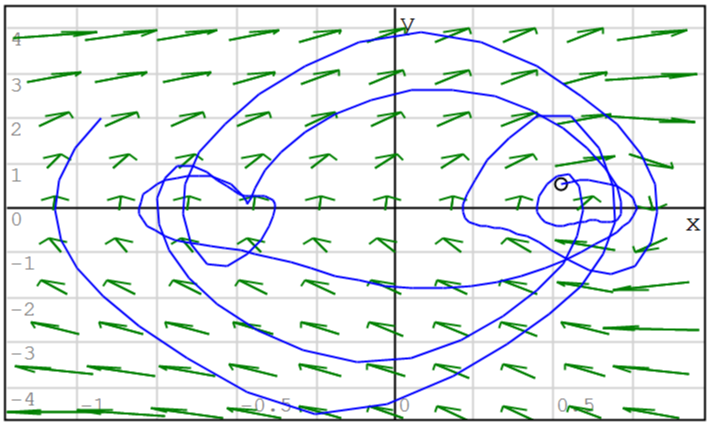
$u =$

$U := L_{ode}\left(D(t, u), [\,0\ te\,], ic, N, \text{"Rkadapt"}\right)$

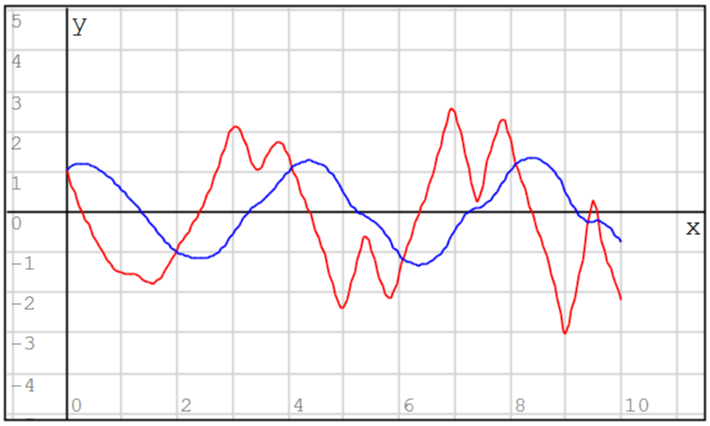
$L_{vf}\left(D(t, u), U, 3, 4, NN\right)$



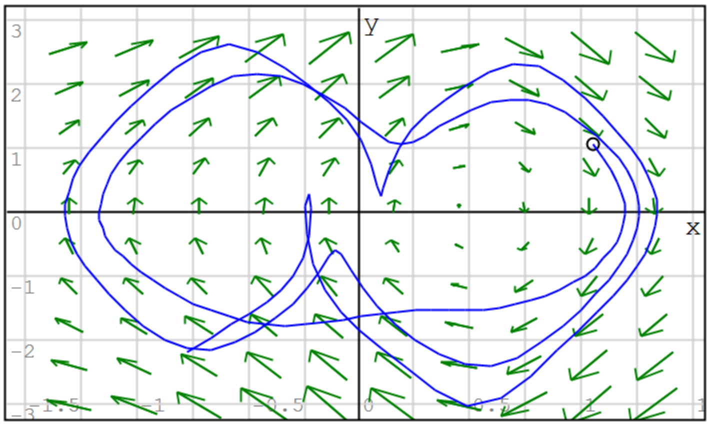
augment (col (U , 1) , 1.0·col (U , 2))
augment (col (U , 1) , 1.0·col (U , 3))



$L_{vf}\left(D(t, u), U, 1, 2, NN, \text{"0.3"}\right)$

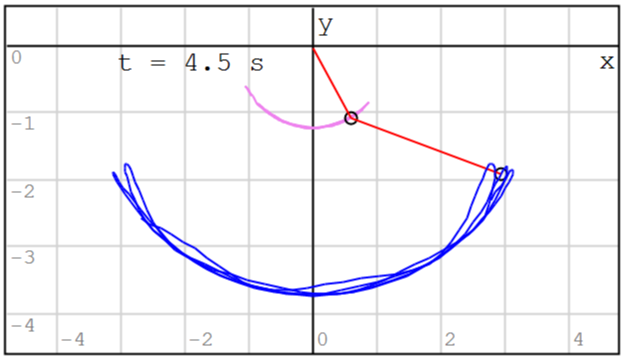


augment (col (U , 1) , 1.0·col (U , 4))
augment (col (U , 1) , 1.0·col (U , 5))

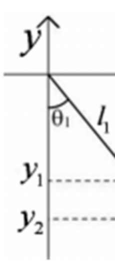


$L_{vf}\left(D(t, u), U, 3, 4, NN, \text{"t"}\right)$

$[\,X1\ X2\ Y1\ Y2\,] := L_{At}\left([\,x1\ x2\ y1\ y2\,], gc, U\right)$
$[\,Fn\ Tn\,] := L_{Frames}(U, 100, 1)$
$M := \begin{bmatrix} X1_t & Y1_t \\ X2_t & Y2_t \end{bmatrix}$
$A :=$ eval (augment (X2 , Y2))
eval (stack ([0 0], M))
eval (augment (M , "o"))
eval (augment (X1 , Y1))
eval (augment (min (X2) , 0 , Tn_t))



A



⊟— Particle in a Cone

Particle in a Cone

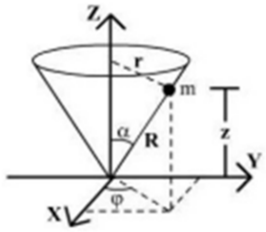
Gen coords, constants & position

$$\text{str2num}\left(\partial Vars\left(\text{"gc"}, [\text{"r" "φ"}], [\text{"α" "m" "g"}]\right)\right) = [\,t\;\; r\;\; r'\;\; \varphi\;\; \varphi'\,]$$

$$cone(r, \varphi) := [\,r\cdot\cos(\varphi)\;\; r\cdot\sin(\varphi)\;\; r\cdot\cot(\alpha)\,]$$

$$[\,x\;\; y\;\; z\,] := cone(r, \varphi)$$

$$x' := \frac{d(x)}{\partial t} \qquad y' := \frac{d(y)}{\partial t} \qquad z' := \frac{d(z)}{\partial t}$$



Lagrangian

$$L := \frac{1}{2}\cdot m\cdot\left(x'^2 + y'^2 + z'^2\right) - m\cdot g\cdot z$$

Equations of motion

$$r''_{eq} := L_{eq}(L, r) \qquad \varphi''_{eq} := L_{eq}(L, \varphi)$$

$$L = \frac{m\cdot\left(\left(\cos(\varphi)\cdot r' - r\cdot\sin(\varphi)\cdot\varphi'\right)^2 + \left(\sin(\varphi)\cdot r' + r\cdot\cos(\varphi)\cdot\varphi'\right)^2 + \cot(\alpha)^2\cdot r'^2 - 2\cdot g\cdot r\cdot\cot(\alpha)\right)}{2}$$

$$r''_{eq} = -\frac{-\varphi'\cdot\left(-\sin(\varphi)\cdot\left(\cos(\varphi)\cdot r' - r\cdot\sin(\varphi)\cdot\varphi'\right) + \cos(\varphi)\cdot\left(\sin(\varphi)\cdot r' + r\cdot\cos(\varphi)\cdot\varphi'\right)\right) + g\cdot\cot(\alpha)}{\cos(\varphi)^2 + \sin(\varphi)^2 + \cot(\alpha)^2}$$

$$\varphi''_{eq} = -\frac{\left(-\sin(\varphi)\cdot\left(\cos(\varphi)\cdot r' - r\cdot\sin(\varphi)\cdot\varphi'\right) + \cos(\varphi)\cdot\left(\sin(\varphi)\cdot r' + r\cdot\cos(\varphi)\cdot\varphi'\right) + r\cdot\varphi'\right)\cdot r'}{r^2}$$

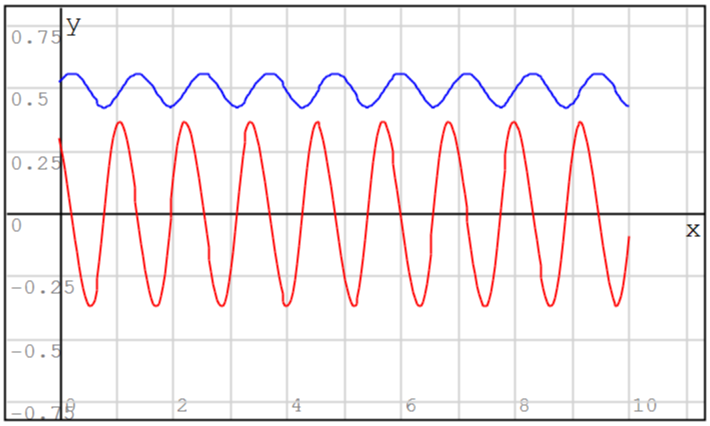
Solutions

$$N := 300 \qquad te := 10\ \text{s} \qquad [\,\alpha\;\; m\;\; g\,] := [\,40\,°\;\; 1200\ \text{g}\;\; g_e\,]$$

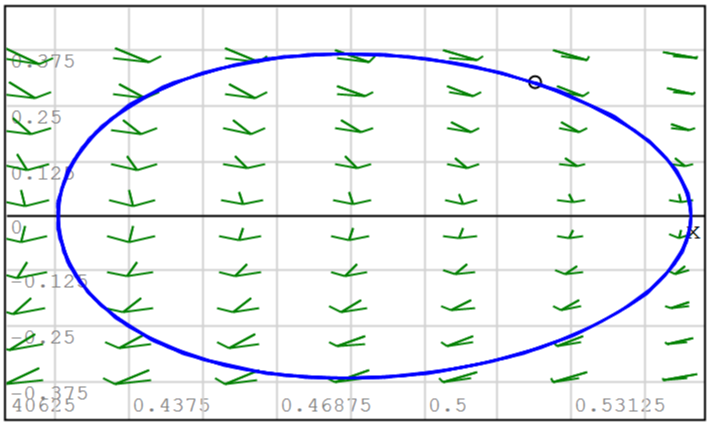
$$5\ \text{rpm} = 0.5236\ \text{Hz} \qquad ic := \left[\,30\,°\;\; 0.3\ \frac{\text{m}}{\text{s}}\;\; 60\,°\;\; 40\ \text{rpm}\,\right] \qquad [\,ro\;\; vo\;\; \varphi o\;\; \omega o\,] := ic$$

$$D(t, u) := \left|\begin{array}{l} [\,r\;\; r'\;\; \varphi\;\; \varphi'\,] := [\,u_1\;\; u_2\;\; u_3\;\; u_4\,] \\ \text{stack}\left(r', r''_{eq}, \varphi', \varphi''_{eq}\right)\end{array}\right.$$

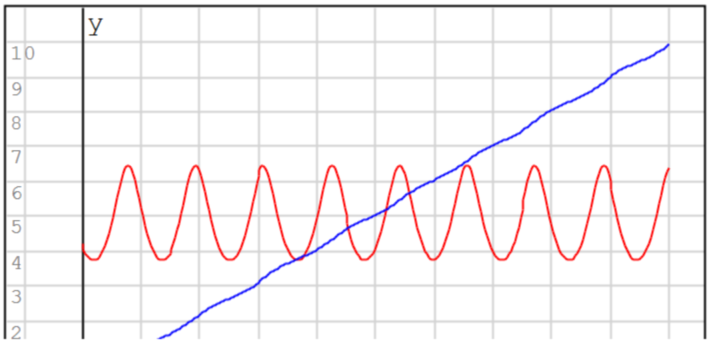
$$U := L_{ode}\left(D(t, u), [\,0\;\; te\,], ic, N, \text{"Rkadapt"}\right)$$

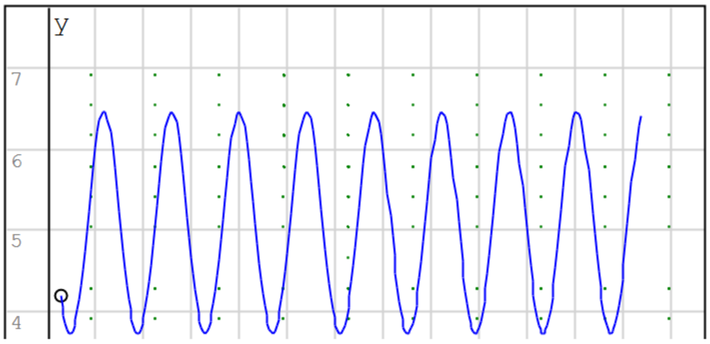


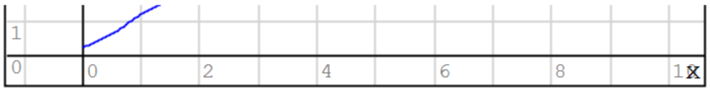
augment (col (U, 1), 1.0·col (U, 2))
augment (col (U, 1), 1.0·col (U, 3))



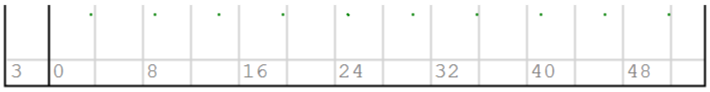
L_{vf} (D (t, u), U, 1, 2, NN, "0.5*t")







$\begin{cases} \text{augment}(\text{col}(U,1),\ 0.2\cdot\text{col}(U,4)) \\ \text{augment}(\text{col}(U,1),\ 1.0\cdot\text{col}(U,5)) \end{cases}$



$L_{vf}(D(t,u),\ U,\ 3,\ 4,\ NN,\ \text{"0.1*t"})$

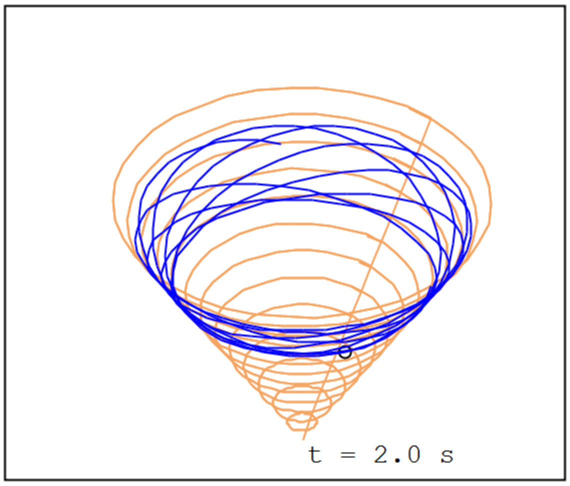
$Y_2 := pView2(-37.5°,\ 30°)$

$G := pGrid(cone,\ pR(0,\ 1.2\cdot\max(\text{col}(U,2)),\ 14),\ pR(0,\ 2.1\cdot\pi,\ 40))$

$[X\ Y\ Z] := L_{At}([x\ y\ z],\ gc,\ U)$

$[Fn\ Tn] := L_{Frames}(U,\ 100,\ 1)$

$A := \begin{cases} \text{eval}(\text{augment}(X,Y,Z)\cdot Y_2) \\ \text{eval}(\text{augment}([X_t\ Y_t\ Z_t]\cdot Y_2,\ \text{"o"})) \\ \text{eval}(\text{augment}([0\ 0\ 0]\cdot Y_2,\ Tn_t)) \\ \text{""} \\ \text{eval}(pMesh(G,\ [0\ 0],\ [1\ 1])\cdot Y_2) \end{cases}$



A

Simple pendulum

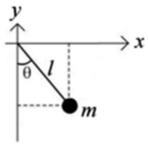
Simple pendulum

Gen coords, constants & position

$\text{str2num}(\partial Vars(\text{"gc"},\ \text{"θ"},\ [\text{"a"}\ \text{"m"}\ \text{"g"}\ \text{"θo"}])) = [t\ \theta\ \theta']$

$x := a\cdot\sin(\theta) \qquad y := -a\cdot\cos(\theta)$

$x' := \frac{d(x)}{\partial t} \qquad y' := \frac{d(y)}{\partial t}$



Lagrangian

$L := \frac{1}{2}\cdot m\cdot\left(x'^2 + y'^2\right) - m\cdot g\cdot y \qquad L = \frac{m\cdot a\cdot\left(a\cdot\theta'^2\cdot 1 + 2\cdot g\cdot\cos(\theta)\right)}{2}$

Equations of motion

$\theta''_{eq} := L_{eq}(L,\ \theta) = -\frac{g\cdot\sin(\theta)}{a} \qquad \theta''_{eq} = -\frac{g\cdot\sin(\theta)}{a}$

Steps

$N := 200 \qquad te := 10\ s$

Solutions

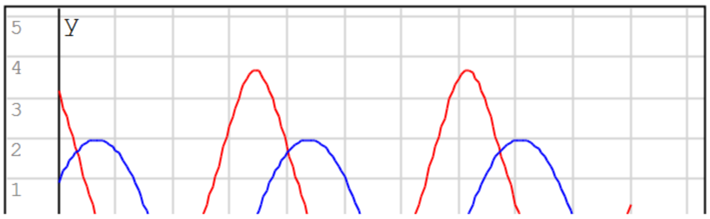
$[a\ m\ g\ \theta o\ \omega o] := [2\ 1\ 10\ 45°\ 3]$

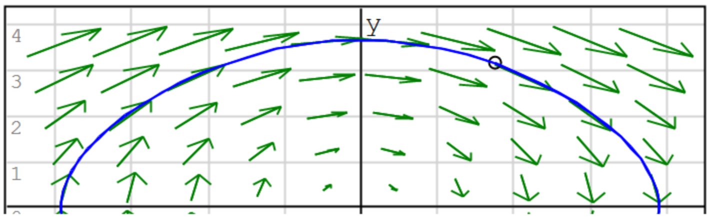
$[a\ m\ g] := [200\ cm\ 1200\ g\ g_e]$

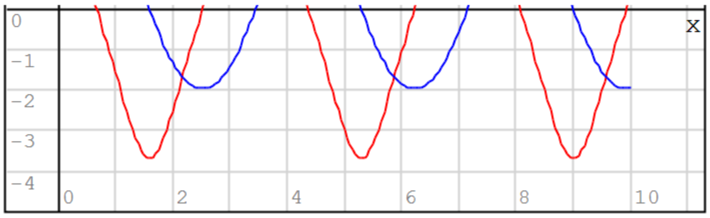
$ic := [50°\ 30\ rpm] \qquad [\theta o\ \omega o] := ic$

$D(t,u) := \begin{cases} [\theta\ \theta'] := [u_1\ u_2] \\ \text{stack}(\theta',\ \theta''_{eq}) \end{cases}$

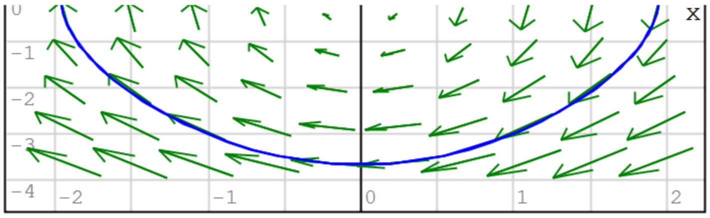
$U := L_{ode}(D(t,u),\ [0\ te],\ ic,\ N,\ \text{"Rkadapt"})$







$\left\{\begin{array}{l}\text{augment}(\text{col}(U,1), 1.0\cdot\text{col}(U,2))\\ \text{augment}(\text{col}(U,1), 1.0\cdot\text{col}(U,3))\end{array}\right.$

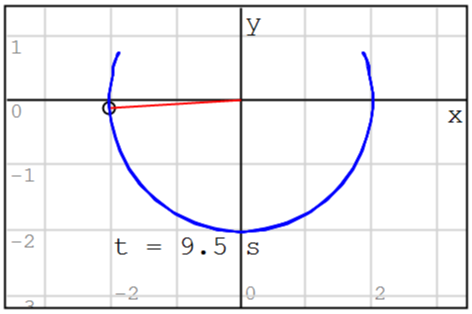


$L_{vf}(D(t,u), U, 1, 2, NN, \text{"t"})$

$[X\ Y] := L_{At}([x\ y], gc, U)$

$[Fn\ Tn] := L_{Frames}(U, 100, 1)$

$A := \left\{\begin{array}{l}\text{eval}(\text{augment}(X,Y))\\ \begin{bmatrix}0 & 0\\ X_t & Y_t\end{bmatrix}\\ \text{augment}(X_t, Y_t, \text{"o"})\\ \text{augment}(\min(X), \min(Y), Tn_t)\end{array}\right.$



A

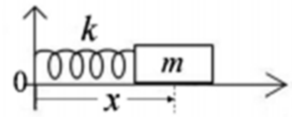
⊟— Harmonic Oscillator

Harmonic Oscillator

Gen coords, constants & position

$\text{str2num}(\partial Vars(\text{"gc"}, \text{"x"}, [\text{"κ"}\ \text{"m"}])) = [t\ x\ x']$

x



Lagrangian

$L := \frac{1}{2}\cdot m\cdot x'^2 - \frac{1}{2}\cdot \kappa\cdot x^2$ $\qquad L = \frac{m\cdot x'^2 - \kappa\cdot x^2}{2}$

Equations of motion

$x''_{eq} := L_{eq}(L, x)$ $\qquad x''_{eq} = -\frac{x\cdot\kappa}{m}$

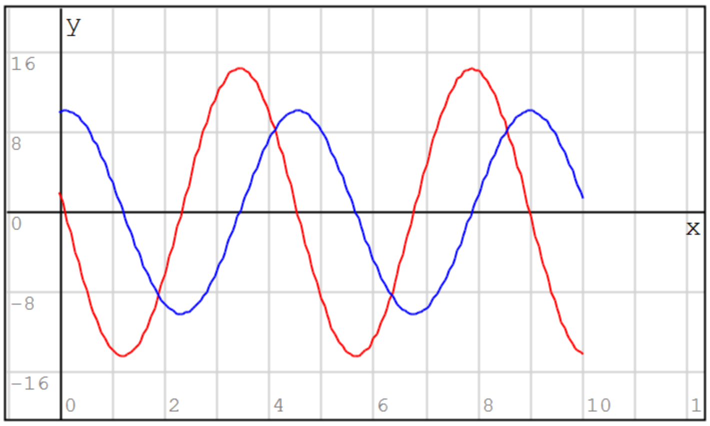
Solutions

$N := 200 \qquad te := 10\ \text{s}$

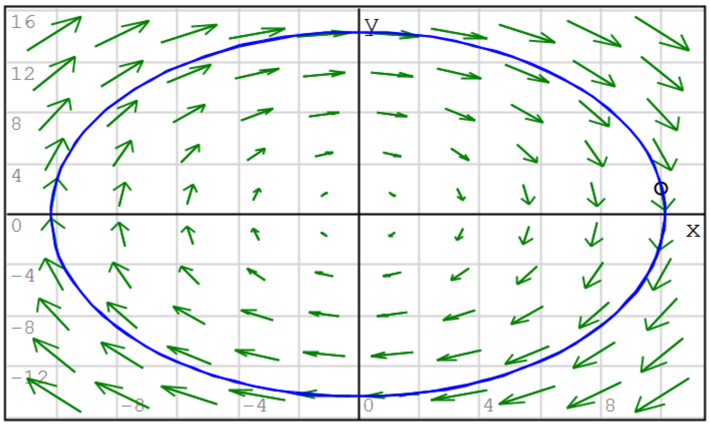
$[\kappa\ m] := \left[2\ \frac{\text{N}}{\text{m}}\ \ 1\ \text{kg}\right] \qquad ic := \left[10\ \text{m}\ \ 2\ \frac{\text{m}}{\text{s}}\right] \qquad [xo\ vo]$

$D(t,u) := \left|\begin{array}{l}[x\ x'] := [u_1\ u_2]\\ \text{stack}(x', x''_{eq})\end{array}\right.$ $\qquad D(te, ic) = \begin{bmatrix}2\ \frac{\text{m}}{\text{s}}\\ -20\ \frac{\text{m}}{\text{s}^2}\end{bmatrix}$

$U := L_{ode}(D(t,u), [0\ te], ic, N, \text{"Rkadapt"})$



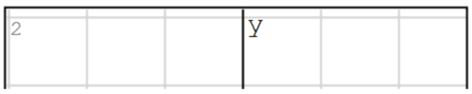
$\left\{\begin{array}{l}\text{augment}(\text{col}(U,1), 1.0\cdot\text{col}(U,2))\\ \text{augment}(\text{col}(U,1), 1.0\cdot\text{col}(U,3))\end{array}\right.$



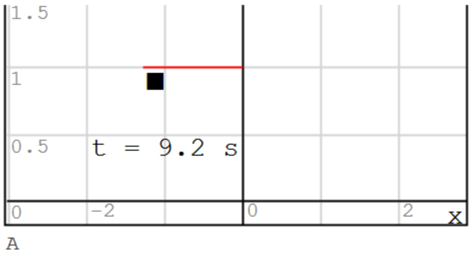
$L_{vf}(D(t,u), U, 1, 2, NN, \text{"t"})$

$[Fn\ Tn] := L_{Frames}(U, 100, 1)$

""



$$A := \begin{cases} \begin{bmatrix} 0 & 1 \\ X_t & 1 \end{bmatrix} \\ \text{augment}\left(X_t, 1, \text{"■"}\right) \\ \text{augment}\left(\min(X), 0.5, Tn_t\right) \end{cases}$$



⊟— Pendulum with Horizontal support —

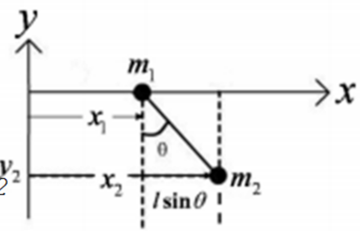
Pendulum with Horizontal support

Gen coords, constants & position

$$\text{str2num}\left(\partial Vars\left(\text{"gc"}, [\text{"x"}\ \text{"θ"}], [\text{"a"}\ \text{"m.1"}\ \text{"m.2"}\ \text{"g"}]\right)\right) = [t\ x\ x'\ \theta\ \theta']$$

$$x2 := x + a \cdot \sin(\theta) \qquad y2 := -a \cdot \cos(\theta)$$

$$x2' := \frac{d(x2)}{\partial t} \qquad y2' := \frac{d(y2)}{\partial t}$$



Lagrangian

$$L := \frac{1}{2} \cdot m_1 \cdot x'^2 + \frac{1}{2} \cdot m_2 \cdot \left(x2'^2 + y2'^2\right) - m_2 \cdot g \cdot y2$$

$$L = \frac{m_1 \cdot x'^2 + m_2 \cdot \left(\left(x' + a \cdot \cos(\theta) \cdot \theta'\right)^2 + a^2 \cdot \sin(\theta)^2 \cdot \theta'^2\right) + 2 \cdot m_2 \cdot g \cdot a \cdot \cos(\theta)}{2}$$

Equations of motion

$$x''_{eq} := L_{eq}(L, x) \qquad x''_{eq} = -\frac{a \cdot m_2 \cdot \left(-\sin(\theta) \cdot \theta'^2 + \cos(\theta) \cdot \theta''\right)}{m_1 + m_2}$$

$$\theta''_{eq} := L_{eq}(L, \theta) \qquad \theta''_{eq} = -\frac{\cos(\theta) \cdot x'' + \sin(\theta) \cdot g}{a}$$

Cyclic coord & conservation

$$\boxed{\frac{d}{d\ x} L = 0} \quad \text{thus} \quad p_x := \frac{d}{d\ x'} L \qquad p_x = x' \cdot m_1 + \left(x' + a \cdot \cos(\theta) \cdot \theta'\right) \cdot m_2$$

Push x'' into θ''

$$\theta''_{eq} := isol\left(\theta'' = \theta''_{eq} \Big|_{x'' = x''_{eq}}, \theta''\right)$$

$$\theta''_{eq} = -\frac{\sin(\theta) \cdot \left(\cos(\theta) \cdot a \cdot m_2 \cdot \theta'^2 + g \cdot \left(m_1 + m_2\right)\right)}{a \cdot \left(m_1 + m_2 \cdot \left(1 - \cos(\theta)^2\right)\right)}$$

Solutions

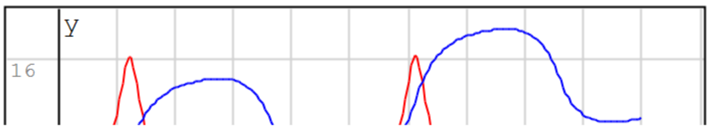
$$N := 200 \qquad te := 10\ \text{s}$$

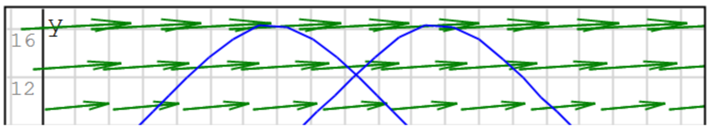
$$[a\ m_1\ m_2\ g] := [8\ \text{m}\ 1\ \text{kg}\ 3\ \text{kg}\ g_e]$$

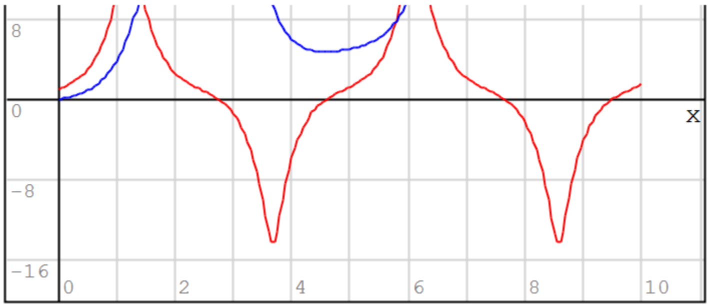
$$ic := \left[0\ \text{m}\ 1\ \frac{\text{m}}{\text{s}}\ 70\ °\ 0\ \text{rpm}\right] \qquad [xo\ vo\ \theta o\ \omega o]$$

$$D(t, u) := \begin{vmatrix} [x\ x'\ \theta\ \theta'] := [u_1\ u_2\ u_3\ u_4] \\ \theta'' := \theta''_{eq} \\ \text{stack}\left(x', x''_{eq}, \theta', \theta''_{eq}\right) \end{vmatrix}$$

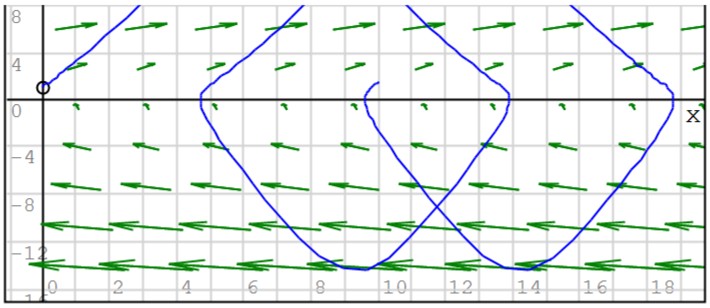
$$U := L_{ode}\left(D(t, u), [0\ te], ic, N, \text{"Rkadapt"}\right)$$



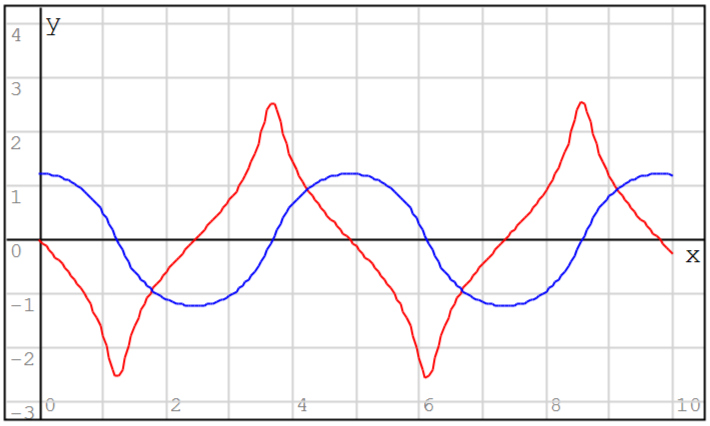




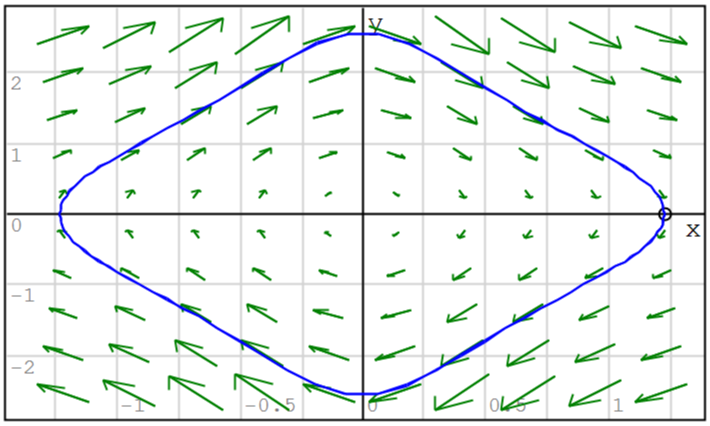
augment (col (U , 1), 1.0 · col (U , 2))
augment (col (U , 1), 1.0 · col (U , 3))



L_{vf} (D (t , u), U , 1, 2, NN , "t")



augment (col (U , 1), 1.0 · col (U , 4))
augment (col (U , 1), 1.0 · col (U , 5))



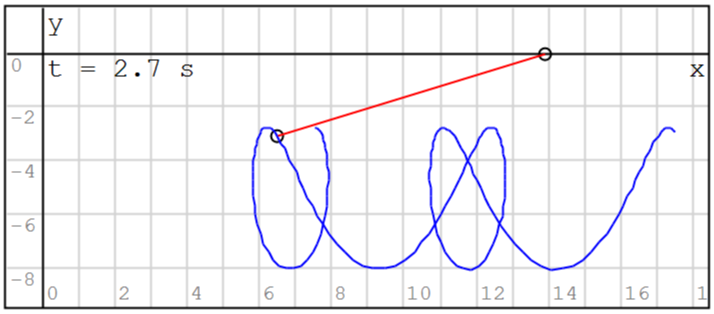
L_{vf} (D (t , u), U , 3, 4, NN , "t")

$[\ X\ \ X2\ \ Y2\ \ Px\ ] := L_{At}\left(\left[\ x\ \ x2\ \ y2\ \ P_x\ \right], gc, U\right)$

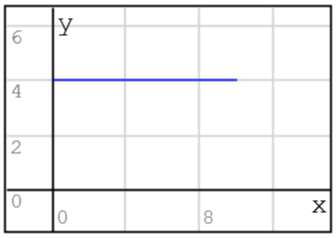
$[\ Fn\ \ Tn\ ] := L_{Frames}(U, 100, 1)$

$M := \begin{bmatrix} X2_t & Y2_t \\ X_t & 0 \end{bmatrix}$

$A := \begin{cases} \text{eval}(\text{augment}(X2, Y2)) \\ M \\ \text{augment}(M, \text{"o"}) \\ \text{augment}(0, 0, Tn_t) \end{cases}$



A



augment (col (U , 1), Px)

Alvaro