

spice# and MNA

- Numerical Inverse Laplace Transform
- MNA WITH STAMPS
- Spice
- MNA Method

MNA Method

Given a linear circuit with n Nodes and m sources, solve the system $A \cdot X = B$

$$A_{n+m \times n+m} = \begin{bmatrix} G & C \\ C & D \end{bmatrix}$$

G have the passive elements: the diagonal elements G(n,n) are the sum of the conductance of each element connected to the node n, and the others G(i,j) are the negative conductance of the element connected to the nodes i,j.

C have the voltage sources, where if a voltage source is connected to nodes m, n, then C(m,n) = ±1, or zero otherwise.

D is the same of B, but with dependent sources.

$$X_{n+m} = \begin{bmatrix} V_{\#N} \\ I_{\#M} \end{bmatrix}$$

X have the node voltages and currents through voltage sources.

$$B_{n+m} = \begin{bmatrix} \Sigma(I_X) \\ V_m \end{bmatrix}$$

For b, b(n) = sum of the currents through the passive elements into the node n, and b(m) = value of the independent voltage sources m.

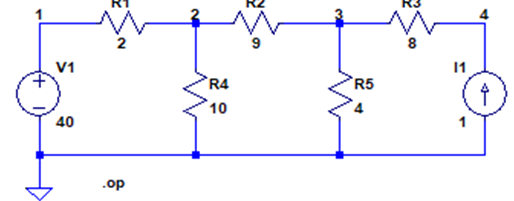
Example MNA Method

Vb	1	0
R1	1	2
R2	2	3
R3	3	4
R4	2	0
R5	3	0
Is	0	4

$\chi_S := 1$

Vb	1	0	40
R1	1	2	2
R2	2	3	9
R3	3	4	8
R4	2	0	10
R5	3	0	4
Is	0	4	1

$\chi := 1$



$$G1 := \frac{1}{R1} \quad G2 := \frac{1}{R2} \quad G3 := \frac{1}{R3} \quad G4 := \frac{1}{R4} \quad G5 := \frac{1}{R5}$$

$$G := \begin{bmatrix} G1 & -G1 & 0 & 0 \\ -G1 & G1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & G2 & -G2 & 0 \\ 0 & -G2 & G2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & G3 & -G3 \\ 0 & 0 & -G3 & G3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & G4 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & G5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad D := [0]$$

$$C = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad V_b \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -Is \\ Vb \end{bmatrix} \quad \text{at node 4} \quad \text{First V source}$$

$$[A \ B \ X \ V \ I] := \text{MNA}(\chi_S) \quad X = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ I_{Vb} \end{bmatrix}$$

$$A = \begin{bmatrix} \frac{1}{R1} & -\frac{1}{R1} & 0 & 0 & 1 \\ -\frac{1}{R1} & \frac{(R2+R4) \cdot R1 + R2 \cdot R4}{R1 \cdot R2 \cdot R4} & -\frac{1}{R2} & 0 & 0 \\ 0 & -\frac{1}{R2} & \frac{(R3+R5) \cdot R2 + R3 \cdot R5}{R2 \cdot R3 \cdot R5} & -\frac{1}{R3} & 0 \\ 0 & 0 & -\frac{1}{R3} & \frac{1}{R3} & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ Is \\ Vb \end{bmatrix}$$

Clear(D, G, G1, G2, G3, G4, G5) = 1

Numerically: $[A \ B \ X \ V \ I] := MNA ("X")$

$$V = \begin{bmatrix} 40 \\ 30 \\ 12 \\ 20 \end{bmatrix} \quad A^{-1} \cdot B = \begin{bmatrix} 40 \\ 30 \\ 12 \\ 20 \\ -5 \end{bmatrix} \quad Spice \left("X", ".OP", \begin{bmatrix} "V(1)" \\ "V(2)" \\ "V(3)" \\ "V(4)" \\ "I(Vb)" \end{bmatrix} \right) = \begin{bmatrix} 0 & 40 \\ 0 & 30 \\ 0 & 12 \\ 0 & 20 \\ 0 & -5 \end{bmatrix}$$

Example spice# MNA Method

spice sharp also uses MNA. From:

https://spicesharp.github.io/SpiceSharp/articles/tutorials/writing_behaviors/modified_nodal_analysis.html

$$\begin{pmatrix} \frac{1}{5\Omega} & -\frac{1}{5\Omega} & 0 & 1 \\ -\frac{1}{5\Omega} & \frac{1}{5\Omega} + \frac{1}{10\Omega} + \frac{1}{7\Omega} & -\frac{1}{7\Omega} & 0 \\ 0 & -\frac{1}{7\Omega} & \frac{1}{7\Omega} & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ i_v \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1.5A \\ 1V \end{pmatrix}$$

```
V1 1 0 1
R1 1 2 5
R2 2 3 7
R3 2 0 10
I1 0 3 1.5
```

$\chi := 1$

$$[A \ B \ X \ V \ I] := MNA ("X")$$

$$A = \begin{bmatrix} 0.2 & -0.2 & 0 & 1 \\ -0.2 & 0.44 & -0.14 & 0 \\ 0 & -0.14 & 0.14 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad X = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ I_V1 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1.5 \\ 1 \end{bmatrix}$$

$$V = \begin{bmatrix} 1 \\ 5.67 \\ 16.17 \end{bmatrix} \quad Spice \left("X", ".OP", \begin{bmatrix} "V(1)" \\ "V(2)" \\ "V(3)" \\ "I(V1)" \end{bmatrix} \right) = \begin{bmatrix} 0 & 1 \\ 0 & 5.67 \\ 0 & 16.17 \\ 0 & 0.93 \end{bmatrix}$$

☐ Ohm's Law

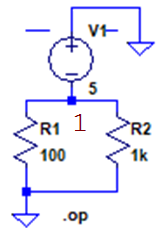
Ohm's Law

```
V1 1 0
R1 1 0
R2 1 0
```

$\chi S := 1$

```
V1 1 0 5
R1 1 0 100
R2 1 0 1k
```

$\chi := 1$



$$[A \ B \ X \ V \ I] := MNA ("X S") \quad V = [V1] \quad I = \left[-\frac{(R1 + R2) \cdot V1}{R1 \cdot R2} \right]$$

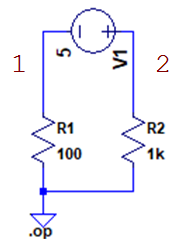
$$[A \ B \ X \ V \ I] := MNA ("X") \quad V = [5] \quad I = [-0.06] \quad Spice ("X", ".OP", "I(V1)") = [0 \ -0.06]$$

```
V1 1 2
R1 1 0
R2 2 0
```

$\chi S := 1$

```
V1 1 2 5
R1 1 0 100
R2 2 0 1k
```

$\chi := 1$

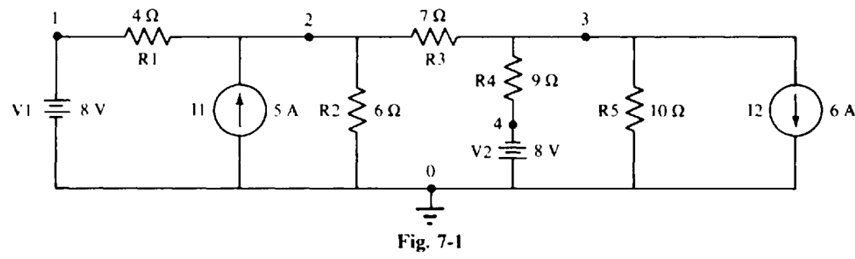


$$[A \ B \ X \ V \ I] := MNA ("X S") \quad V = \begin{bmatrix} \frac{R1 \cdot V1}{R1 + R2} \\ \frac{R2 \cdot V1}{R1 + R2} \\ -\frac{R2 \cdot V1}{R1 + R2} \end{bmatrix} \quad I = \left[-\frac{V1}{R1 + R2} \right]$$

$$[A \ B \ X \ V \ I] := MNA ("X") \quad V = \begin{bmatrix} 0.45 \\ -4.55 \end{bmatrix} \quad Spice \left("X", ".OP", \begin{bmatrix} "V(1)" \\ "V(2)" \\ "I(V1)" \end{bmatrix} \right) = \begin{bmatrix} 0 & 0.45 \\ 0 & -4.55 \\ 0 & 0 \end{bmatrix}$$

Resistive networks

Resistive networks



V1	1	0	8
R1	1	2	4
I1	0	2	5
R2	2	0	6
R3	2	3	7
R4	3	4	9
V2	0	4	8
R5	3	0	10
I2	3	0	6

Fig. 7-1

$\chi := 1$

$[A \ B \ X \ V \ I] := MNA(\chi)$

$$V = \begin{bmatrix} 8 \\ 8.41 \\ -16.07 \\ -8 \end{bmatrix}$$

$$I = \begin{bmatrix} 0.1 \\ 0.9 \end{bmatrix}$$

Spice $\chi, ".OP",$

"V(1)"	0	8
"V(2)"	0	8.41
"V(3)"	0	-16.07
"V(4)"	0	-8
"I(V1)"	0	0.1
"I(V2)"	0	0.9

RL Circuit

RL Circuit

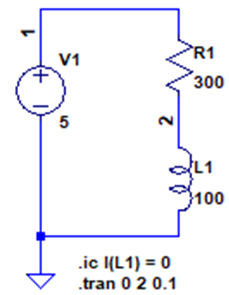
Symbolic and Numerical analysis

V1	1	0
R1	1	2
L1	2	0

$\chi s := 1$

V1	1	0	5
R1	1	2	300
L1	2	0	100

$\chi := 1$



$[A \ B \ X \ V \ I] := MNA(\chi s)$

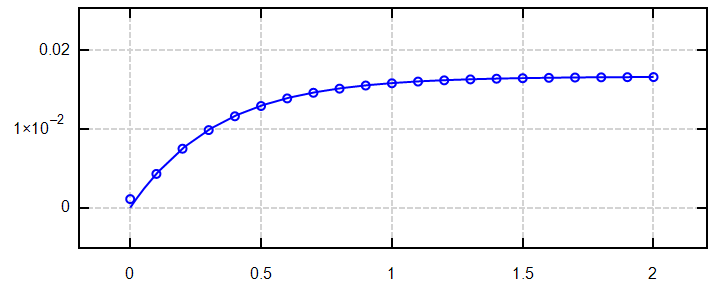
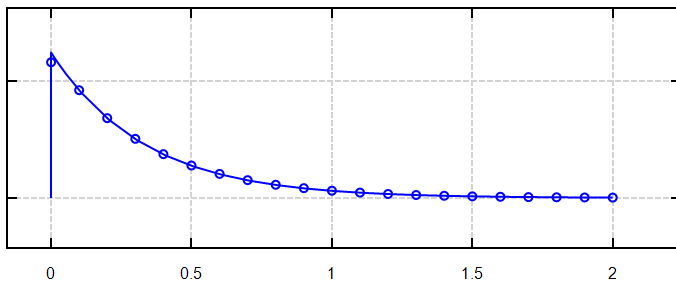
$$I = \left[-\frac{V1}{R1 + s \cdot L1} \right]$$

$$V = \left[\frac{V1 \cdot s \cdot L1}{R1 + s \cdot L1} \right]$$

$[A \ B \ X \ V \ I] := MNA(\chi)$

$O := \left[\begin{array}{l} ".IC \ V(2) = 0" \\ ".TRAN \ 0 \ 2 \ 0.1" \end{array} \right]$

"IC I(L1) = 0" ?



$$\left\{ \begin{array}{l} \text{Spice}(\chi, O, "V(2)") \\ \mathcal{L}\left[\frac{V_2}{s}, [0, 0.1..2]\right] \end{array} \right.$$

$$\left\{ \begin{array}{l} \text{Spice}(\chi, O, "I(L1)") \\ \mathcal{L}\left[-\frac{I_1}{s}, [0, 0.1..2]\right] \end{array} \right.$$

RC Circuit

RC Circuit

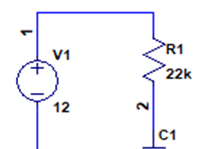
Symbolic and Numerical analysis

V1	1	0
R1	1	2
C1	2	0

$\chi s := 1$

V1	1	0	12
R1	1	2	22k
C1	2	0	0.9m

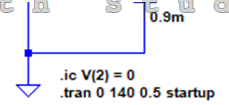
$\chi := 1$



X 0.9m is wrong, it correct expression is 900u or 900μ

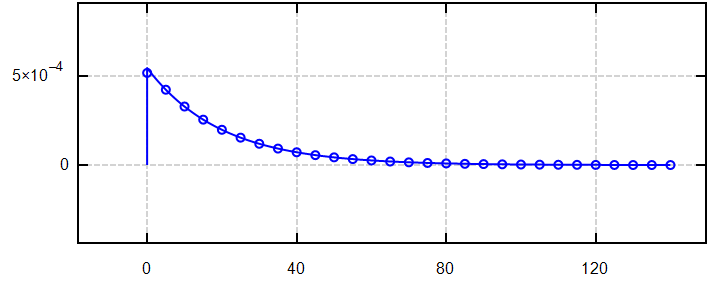
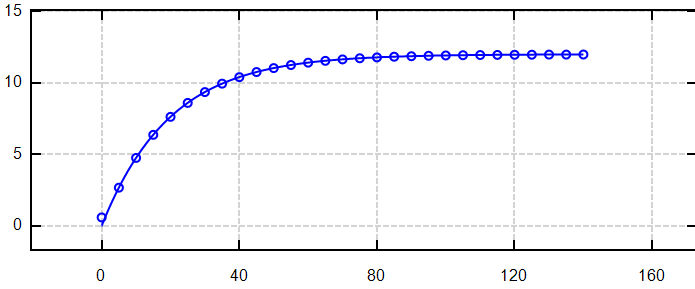
```
[ A B X V I ] := MNA ( "X S" )
```

$$I = \left[-\frac{s \cdot C1 \cdot V1}{1 + s \cdot C1 \cdot R1} \right] \quad V = \left[\frac{V1}{1 + s \cdot C1 \cdot R1} \right]$$



```
[ A B X V I ] := MNA ( "X" )
```

```
O := [ ".IC V(2) = 0"
      ".TRAN 0 140 0.5" ]
```



```
{ Spice ( "X", O, "V(2)" )
  { gL ( V2 / s, [ 0, 5..140 ] )
```

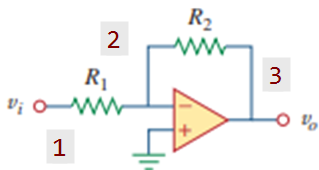
```
{ Spice ( "X", O, "I(C1)" )
  { gL ( -I1 / s, [ 0, 5..140 ] )
```

□ opamps

Summary of basic op amp circuits

Shorthand:

$$V(n) := \left| MNA("X") \right|_{4n}$$

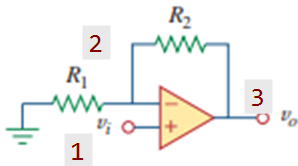


Inverter amplifier

```
V.i 1 0
R.1 1 2
R.2 2 3
O 0 3 2
```

$$V_o := V(3) = -\frac{V_i \cdot R_2}{R_1}$$

$\chi := 1$

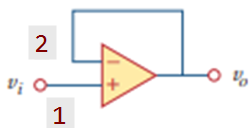


Non-inverter amplifier

```
V.i 1 0
R.1 0 2
R.2 2 3
O 1 3 2
```

$$V_o := V(3) = \frac{(R_1 + R_2) \cdot V_i}{R_1}$$

$\chi := 1$

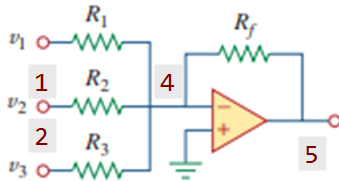


Voltage Follower

```
V 1 0 V.i
O 1 2 2
```

$$V_o := V(2) = V_i$$

$\chi := 1$



Summer

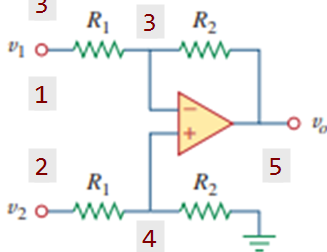
```
V 1 0 V.1
V 2 0 V.2
V 3 0 V.3
R 1 4 R.1
R 2 4 R.2
R 3 4 R.3
R 4 5 R.f
O 0 5 4
```

$$V_o := V(5) = -\frac{R_f \cdot ((V_1 \cdot R_2 + V_2 \cdot R_1) \cdot R_3 + V_3 \cdot R_2 \cdot R_1)}{R_3 \cdot R_2 \cdot R_1}$$

or

$$V_o = -\left(\frac{R_f}{V_1} + \frac{R_f}{V_2} + \frac{R_f}{V_3} \right)$$

$\chi := 1$



Difference Amplifier

```
V 1 0 V.1
V 2 0 V.2
R 1 3 R.1
R 2 4 R.2
R 3 4 R.3
R 4 5 R.f
O 0 5 4
```

$$V_o := V(5) = -\frac{R_f \cdot (V_1 \cdot R_2 + (R_1 + R_3) \cdot V_2)}{(R_1 + R_3) \cdot R_2}$$

or

$$V_o = \frac{R_2}{R_1} \cdot (V_2 - V_1)$$

$\chi := 1$

Opamp for spice#

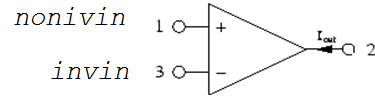
$$E(x) := 10^x$$

```
.subckt opamp 1 2 3
G1 0 2 1 3 100k
R3 2 0 1
C3 2 0 0.001591549430919
.ends opamp
```

$$_Aol := 100 \cdot E(3) \quad _GBW := 10 \cdot E(6)$$

$$\frac{_Aol}{2 \cdot \pi \cdot _GBW} = 0.001591549430919$$

```
opamp := description(opamp)
```



```
opamp := 1
```

Example with spice#: Inverter amplifier

```
V.i 1 0 5
R.1 1 2 10
R.2 2 3 20
X1 0 3 2 opamp
```

$$V_3 = -\frac{V_i \cdot R_2}{R_1}$$

$$\text{Spice} \left("X", \left[".OP" \right], \left[\begin{matrix} "V(1)" \\ "V(2)" \\ "V(3)" \end{matrix} \right] \right) = \begin{bmatrix} 0 & 5 \\ 0 & 0 \\ 0 & -10 \end{bmatrix}$$

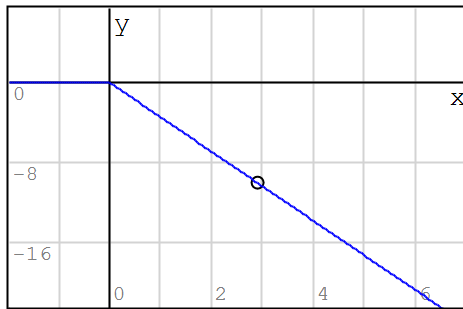
Sweep analysis

```
V 1 0 5V
V 2 0 3V
V 3 0 4V
R 1 4 2Ω
R 2 4 8Ω
R 3 4 7Ω
R 4 5 R.f
O 0 5 4
```

```
χ := 1
```

$$V_o(R_f) := V(5) \quad V_o := -10 \text{ V}$$

$$R_f := \text{roots} \left(V_o(R_f \Omega) = V_o, R_f \right) \Omega = 2.9 \Omega$$



Sanity check

$$V := \begin{bmatrix} 5 \\ 3 \\ 4 \end{bmatrix} \text{ V} \quad R := \begin{bmatrix} 2 \\ 8 \\ 7 \end{bmatrix} \Omega$$

$$R_f := -\frac{V_o}{\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}} = 2.9 \Omega$$

```
{ V_o(x Ω) · (x > 0)
  augment(R_f, V_o, "o")
```

TODO: Sweep analysis with spice#

opamp: Capacitance multiplier

Capacitance multiplier

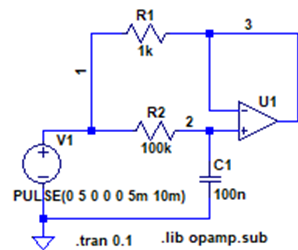
```
Vb 1 0
R1 1 3 1k
R2 1 2 100k
C1 2 0 100n
O1 2 3 3
```

$$[A \ B \ X \ V \ I] := MNA("X")$$

```
χ := 1
```

Laplace transform of a pulse:

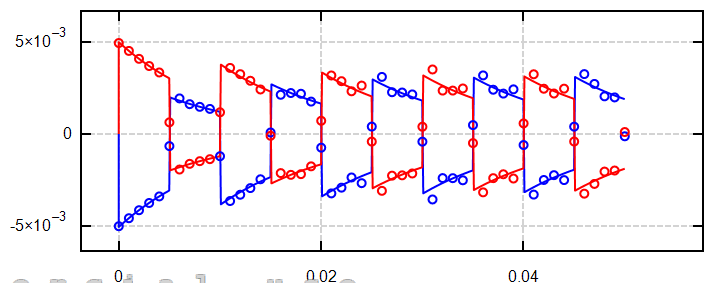
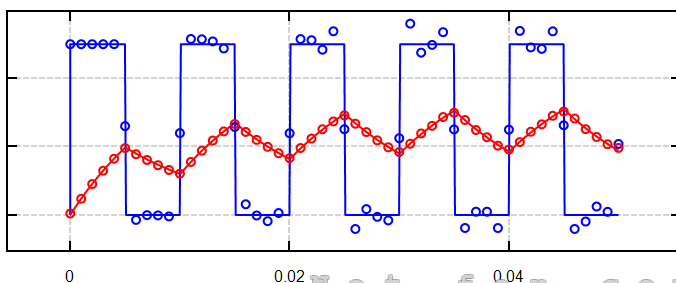
$$T_o := 5 \cdot 10^{-3} \quad V_b := \frac{5}{s \cdot (1 + e^{-T_o \cdot s})}$$



```
Vb 1 0 PULSE(0 5 0 0 0 5m 10m)
R1 1 3 1k
R2 1 2 100k
C1 2 0 100n
X1 2 3 3 opamp
```

$$O := \left[\begin{matrix} \text{opamp} \\ ".TRAN 0 0.05 0.1m" \end{matrix} \right] \quad T := [0, 0.001 \dots 0.05] \quad \text{rows}(T) = 51$$

```
χ := 1
```



```

{
  Spice("X", 0, "V(1)")
  Spice("X", 0, "V(2)")
   $\mathcal{L}\{V_1, T\}$ 
   $\mathcal{L}\{V_2, T\}$ 
}
{
  Spice("X", 0, "I(Vb)")
  Spice("X", 0, "I(R1)")
   $\mathcal{L}\{I_1, T\}$ 
   $\mathcal{L}\{I_2, T\}$ 
}

```

Clear(Vb)=1

opamp: DC OP Point

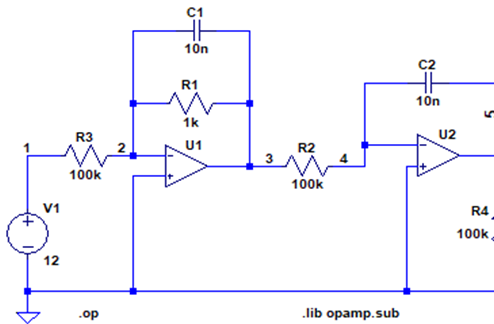
DC OP Point

Vb	1	0
R1	2	3
R2	3	4
R3	1	2
R4	5	0
C1	2	3
C2	4	5
O1	0	2 3
O2	0	4 5

Vb	1	0	12
R1	2	3	1k
R2	3	4	100k
R3	1	2	100k
R4	5	0	100k
C1	2	3	10n
C2	4	5	10n
X1	0	2 3	opamp
X2	0	4 5	opamp

$\chi s := 1$

$\chi := 1$



$$[R1 \ R2 \ R3 \ R4] := \begin{bmatrix} \frac{1}{G1} & \frac{1}{G2} & \frac{1}{G3} & \frac{1}{G4} \end{bmatrix}$$

$$[A \ B \ X] := MNA("Xs", "ABX", \Sigma)$$

$$A \cdot X = B$$

$$A = \begin{bmatrix} G3 & -G3 & 0 & 0 & 0 & 1 & 0 & 0 \\ -G3 & G3 + G1 + s \cdot C1 & -(G1 + s \cdot C1) & 0 & 0 & 0 & 1 & 0 \\ 0 & -(G1 + s \cdot C1) & G2 + G1 + s \cdot C1 & -G2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -G2 & G2 + s \cdot C2 & -s \cdot C2 & 0 & 0 & 1 \\ 0 & 0 & 0 & -s \cdot C2 & G4 + s \cdot C2 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ I_{Vb} \\ I_{O1} \\ I_{O2} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ Vb \\ 0 \\ 0 \end{bmatrix}$$

DC Operating point (C as open circuits):



From Itspice:

$$Spice \left("X", \begin{bmatrix} ".OP" \\ opamp \end{bmatrix}, \begin{bmatrix} "V(1)" \\ "V(2)" \\ "V(3)" \\ "I(R4)" \\ "I(Vb)" \end{bmatrix} \right) = \begin{bmatrix} 0 & 12 \\ 0 & 1.21 \cdot 10^{-9} \\ 0 & 1.2 \cdot 10^{-9} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

--- Operating Point ---

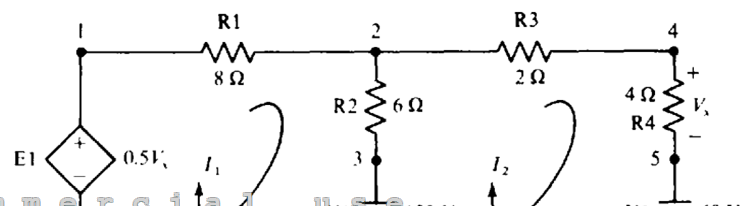
V(1):	12	voltage
V(3):	-0.119999	voltage
V(2):	1.20119e-006	voltage
V(n001):	11999.8	voltage
V(4):	-0.119999	voltage
I(C2):	1.19999e-016	device_current
I(C1):	-1.2e-021	device_current
I(R4):	-0.119998	device_current
I(R3):	-0.00012	device_current
I(R2):	1.19999e-016	device_current
I(R1):	-0.00012	device_current
I(V1):	-0.00012	device_current
Ix(u1:3):	0.00012	subckt_current
Ix(u2:3):	-0.119998	subckt_current

Clear(s, Vb, R1, R2, R3, R4, C1, C2)=1

E Source

Example

E1	1	0	4	5	0.5
R1	1	2	8		
R2	2	3	6		
V1	3	0	120		
R3	2	4	2		
R4	4	5	4		
V2	5	0	60		



```

χ := 1
O := ".DC V1 120 120 0.1"
O := ".op"
[A B X V I] := MNA("χ")
    
```

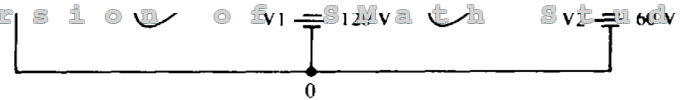


Fig. 7-4

⊡ Different node convention?

$$X = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ I_{E1} \\ I_{V1} \\ I_{V2} \end{bmatrix}$$

$$V = \begin{bmatrix} 38.18 \\ 38.18 \\ 120 \\ 10.91 \\ 60 \end{bmatrix}$$

$$I = \begin{bmatrix} 25.91 \\ -13.64 \\ -12.27 \end{bmatrix}$$

$$\text{Spice}(\chi, O, "V(1)") = [0 \ 2]$$

$$\text{Spice}(\chi, O, "V(2)") = [0 \ 66]$$

$$\text{Spice}(\chi, O, "V(4)") = [0 \ 64]$$

$$\text{Spice}(\chi, O, "I(R1)") = [0 \ -8]$$

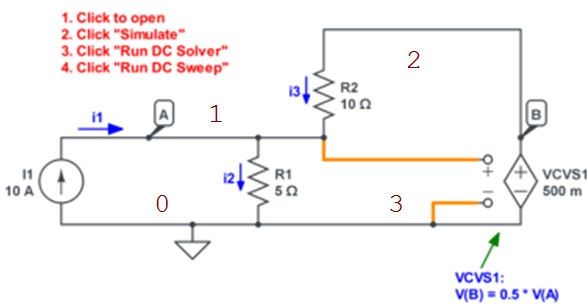
$$\text{Spice}(\chi, O, "I(R4)") = [0 \ 1]$$

SMATH determinant can't handle units:

```
MInverse(A##) := maple(inverse(A##))
```

⊡ Controlled Sources with Feedback

Controlled Sources with Feedback



1. Click to open
2. Click "Simulate"
3. Click "Run DC Solver"
4. Click "Run DC Sweep"

```

R 0 1 5Ω
R 1 2 10Ω
I 0 1 10A
E 1 2 0 0 0.5
    
```

```

R1 0 1 5
R2 1 2 10
I1 0 1 10
E2 2 1 0 0 0.5
    
```

```
χ := 1
```

```
χ' := 1
```

```
[A B X V I] := MNA("χ")
```

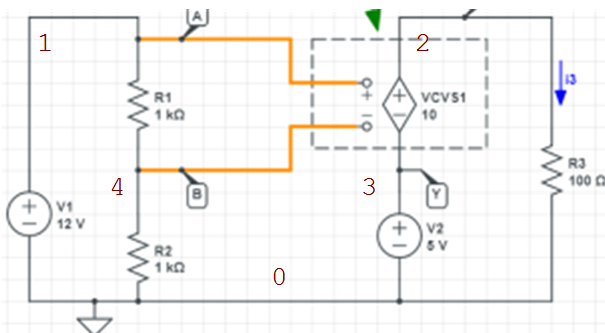
$$V = \begin{bmatrix} 40 \\ 20 \end{bmatrix} \text{ V}$$

$$I = [-2] \text{ A}$$

$$\text{Spice}(\chi', ".OP", \begin{bmatrix} "V(1)" \\ "V(2)" \\ "I(R1)" \end{bmatrix}) = \begin{bmatrix} 0 & 50 \\ 0 & 50 \\ 0 & -10 \end{bmatrix}$$

⊡ E: VCVS

Voltage Controlled Voltage Source (VCVS)



```

V1 1 0 12V
V2 3 0 5V
R 1 4 1kΩ
R 0 4 1kΩ
R 0 2 100Ω
E 1 2 3 4 10
    
```

```

V1 1 0 12
V2 3 0 5
R1 1 4 1k
R2 0 4 1k
R3 0 2 100
E1 1 2 3 4 10
    
```

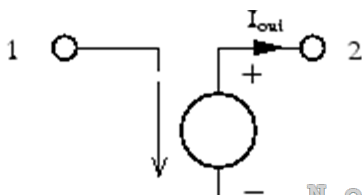
```
χ := 1
```

```
χ' := 1
```

```
[A B X V I] := MNA("χ")
```

$$V = \begin{bmatrix} 12 \\ 65 \\ 5 \\ 6 \end{bmatrix} \text{ V}$$

$$\text{Spice}(\chi', ".OP", \begin{bmatrix} "V(1)" \\ "V(2)" \\ "V(3)" \\ "V(4)" \\ "I(R1)" \end{bmatrix}) = \begin{bmatrix} 0 & 12 \\ 0 & 22 \\ 0 & 5 \\ 0 & 6 \\ 0 & 0.01 \end{bmatrix}$$



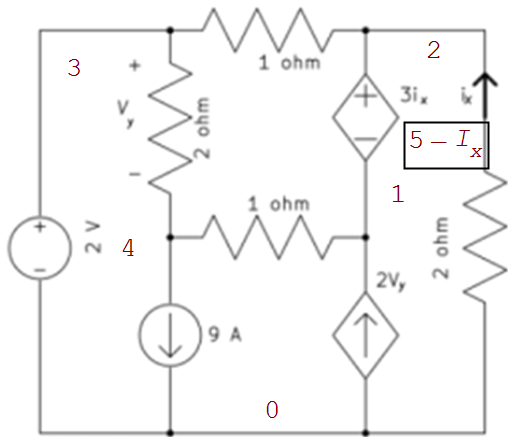
$$X_{[5..7]} = \begin{bmatrix} I_{V1} \\ I_{V2} \\ I_E \end{bmatrix} \quad |A| = 0 \text{ S}$$

$$I = \begin{bmatrix} -6 \\ -650 \\ 650 \end{bmatrix} \text{ mA}$$

$$\begin{bmatrix} I(R2) \\ I(R3) \end{bmatrix} \begin{bmatrix} 0 & -0.01 \\ 0 & -0.22 \end{bmatrix}$$

G: CCCS

Current Controlled Current Source (CCCS)



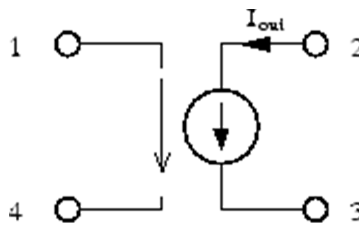
I	4	0	9A
V	3	0	2V
R	1	4	1Ω
R	3	4	2Ω
R	2	3	1Ω
R	0	5	2Ω
Hx	5	2	1 2 3Ω
Gy	3	0	1 4 2S

$$[A \ B \ X \ V \ I] := MNA("X")$$

$$V = \begin{bmatrix} 5 \\ 2 \\ 2 \\ -2 \\ 2 \end{bmatrix} \text{ V}$$

$$I = \begin{bmatrix} -2 \\ -1 \\ 1 \\ 8 \end{bmatrix} \text{ A}$$

$$X := 1$$

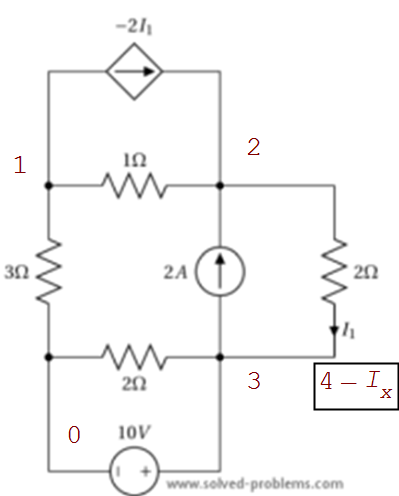


$$X_{[6..9]} = \begin{bmatrix} I_V \\ I_{in_Hx} \\ I_{out_Hx} \\ I_{Gy} \end{bmatrix}$$

$$|A| = 4.25 \text{ S}$$

F: CCCS

Current Controlled Current Source (CCCS)



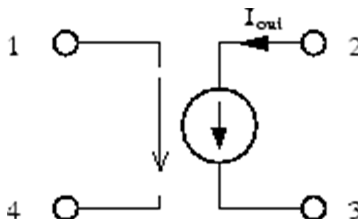
I	3	2	2A
V	3	0	10V
R	0	3	2Ω
R	0	1	3Ω
R	2	4	2Ω
R	1	2	1Ω
Fx	4	1	2 3 -2

$$[A \ B \ X \ V \ I] := MNA("X")$$

$$V = \begin{bmatrix} 6.75 \\ 9.5 \\ 10 \\ 10 \end{bmatrix} \text{ V}$$

$$I = \begin{bmatrix} -7.25 \\ 0.5 \end{bmatrix} \text{ A}$$

$$X := 1$$



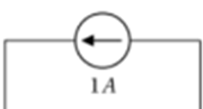
$$X = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ I_V \\ I_{Fx} \end{bmatrix}$$

LOS VOLTAJES ESTÁN BIEN, CHEKAR LAS CORRIENTES.

$$\frac{|A|}{2} = -0.67 \text{ S}$$

H: CCVS

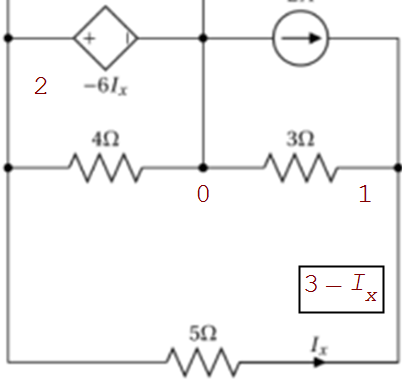
Current Controlled Voltage Source (CCVS)



I	0	2	1A
I	0	1	2A
R	0	2	4Ω
R	0	1	3Ω

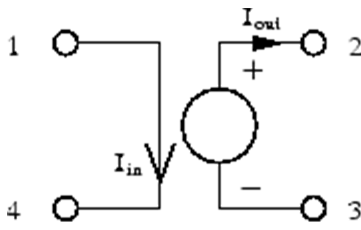
$$[A \ B \ X \ V \ I] := MNA("X")$$

$$[4.71] \quad [-0.43]$$



$$\begin{matrix} R & 2 & 3 & 5\Omega \\ H & 3 & 2 & 0 & 1 & -6\Omega \end{matrix} \quad V = \begin{bmatrix} 2.57 \\ -0.79 \\ 4.71 \end{bmatrix} V$$

$$X := 1$$



$$X = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ Iin_H \\ Iout_H \end{bmatrix}$$

Alvaro

appVersion(4) = "1.2.9018.0"