

⊕ isol

Coupled ODE

$$D_1 := (x + \mu)^2 + y^2$$

$$D_2 := (x - \mu p)^2 + y^2$$

$$x'''_{eq} := x + 2 \cdot y' - \frac{\mu p}{D_1 \cdot \sqrt{D_1}} \cdot (x + \mu) - \frac{\mu}{D_2 \cdot \sqrt{D_2}} \cdot (x - \mu p)$$

$$y'''_{eq} := y \cdot \left(1 - \frac{\mu p}{D_1 \cdot \sqrt{D_1}} - \frac{\mu}{D_2 \cdot \sqrt{D_2}} \right) - 2 \cdot x'$$

Push y''
into x'

$$x'''_{eq} := isol \left(x''' = x'''_{eq} \middle| y''' = y'''_{eq}, x''' \right)$$

Now $x'''.eq$ is free of y'' .
We use that in D

ODE System

$$D(t, u) := \begin{cases} [x \ x' \ y \ y'] := [u_1 \ u_2 \ u_3 \ u_4] \\ x''' := x'''_{eq} \\ [x' \ x''' \ y' \ y'''_{eq}]^T \end{cases}$$

Numerical
example

$$ic := stack(0.994, 0, 0, -2.001585106)$$

$$steps := 1000$$

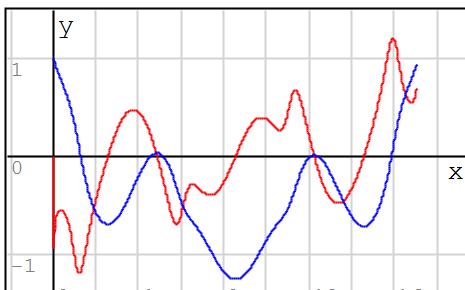
$$\mu := 0.012277471$$

$$\mu p := 1 - \mu$$

$$t_{max} := 17$$

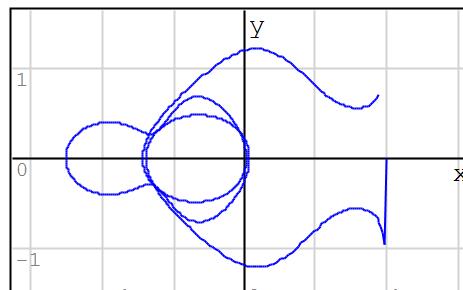
$$RK := \text{Rkadapt}(ic, 0, t_{max}, steps, D)$$

x(t) & y(t)



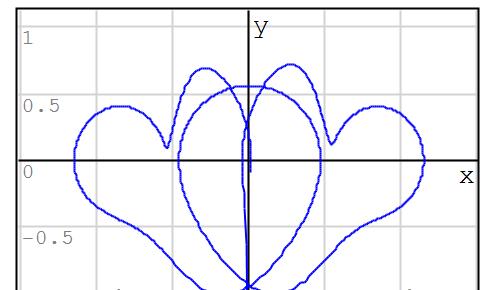
$$\begin{cases} \text{augment}(\text{col}(RK, 1), \text{col}(RK, 2)) \\ \text{augment}(\text{col}(RK, 1), \text{col}(RK, 3)) \end{cases}$$

x''' vs x'



$$\text{augment}(\text{col}(RK, 2), \text{col}(RK, 3))$$

y''' vs y'



$$\text{augment}(\text{col}(RK, 4), \text{col}(RK, 5))$$

Checking that $x'''.eq$
is free of y''

$$\text{Clear}(\mu, \mu p) = 1$$

$$\frac{d}{d y'''} x'''_{eq} = 0$$

$$D := D \left(t, \begin{bmatrix} x \\ x' \\ y \\ y' \end{bmatrix} \right)$$

$$D = \begin{bmatrix} x' \\ -\frac{-2\sqrt{(x - \mu p)^2 + y^2}^3 \cdot \left(2\sqrt{(x + \mu)^2 + y^2}^3 \cdot (x + 2 \cdot y') - \mu p \cdot (x + \mu) \right) + \mu \cdot (x - \mu p) \cdot 2\sqrt{(x + \mu)^2 + y^2}^3}{\sqrt{(x - \mu p)^2 + y^2}^3 \cdot \sqrt{(x + \mu)^2 + y^2}^3} \\ y \cdot \left(\left(-\mu p + 2\sqrt{(x + \mu)^2 + y^2}^3 \right) \cdot 2\sqrt{(x - \mu p)^2 + y^2}^3 - \mu \cdot 2\sqrt{(x + \mu)^2 + y^2}^3 \right) - 2 \cdot x' \cdot 2\sqrt{(x - \mu p)^2 + y^2}^3 \cdot 2\sqrt{(x + \mu)^2 + y^2}^3 \end{bmatrix}$$

Alvaro

