RK23

Runge-Kutta 2nd and 3rd order.

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RK23Adapt(D_v(2), to_v, xo_v, N_v, E_v) :=
:= | [ ut := num2str(UnitsOf(to_v_1 - to_v_2)) ux := num2str(UnitsOf(xo_v)) ]
   | [ m kg s K A cd ] := [ 1 1 1 1 1 1 ]
   | [ nx.. := length(xo_v) c.. := [1..nx..] x.. := xo_v_c.. X.. := x..^T ]
   | [ t.. := eval(min(to_v)) h.. := eval((max(to_v) - t..)/N_v) ]
   | [ n.. := [1..(N_v + 1)] k.. := 1 T.. := [ t.. ] ho.. := h.. ]
   | K..(t.., x.., c.., h..) := | try
   |                            |   eval(h.. · D_v(t.., x..))
   |                            | on error
   |                            |   c..
   | while t.. ≤ to_v_2
   |   | k..1 := K..(t.., x.., x.., h..)
   |   | k..2 := K..(t.. + h.., x.. + k..1, k..1, h..)
   |   | k..3 := K..(t.. + 0.5·h.., x.. + 0.25·k..1 + 0.25·k..2, k..2, h..)
   |   | d.. := normi((k..1 - 2·k..3 + k..2)/3)
   |   | q.. := E_v · max([ normi(x..) ho.. ])
   |   | if d.. ≤ q..
   |   |   | [ t.. := t.. + h..  T..  k.. := k.. + 1 := t.. ]
   |   |   | [ x.. := eval(x.. + (k..1 + 4·k..3 + k..2)/6) X..  k..  c.. := x..  c.. ]
   |   | else
   |   |   ""
   |   | if d.. ≠ 0
   |   |   h.. := eval(min([[ ho..  0.9·h.. · ∛(q../d..) ]]))
   |   | else
   |   |   ""
   | [ t..  n.. := T..1 + (n.. - 1)·ho..  x.. := 0 ]
   | [ Clear(m, kg, s, K, A, cd) ux := str2num(ux) X2.. := 0 ]
   | x..  n..  c.. := cinterp(T.., col(X.., c..), t..  n..)·ux  c..
   | t.. := t.. · str2num(ut)
   | eval(augment(t.., x.., X2..  n.. := D_v(t..  n.., x..  n..  c..)  nx..))
```

Runge-Kutta 2nd and 3rd order.

```
RK23Fix(D_v(2), to_v, xo_v, N_v) :=
:= | [ ut := num2str(UnitsOf(to_v_1 - to_v_2)) ux := num2str(UnitsOf(xo_v)) ]
   | [ m kg s K A cd ] := [ 1 1 1 1 1 1 ]
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```
[m kg s K A cd]:=[1 1 1 1 1 1]
[nx.. := length(xo◡) c.. := [1..nx..] x.. := xo◡ c.. X.. := x..^T]
[t.. := eval(min(to◡)) h.. := eval((max(to◡) − t..)/N◡)]
[n.. := [1..(N◡ + 1)] T.. n.. := t.. + (n.. − 1)·h..]
K..(t.., x.., c..) := |try
                      |  eval(h.. · D◡(t.., x..))
                      |on error
                      |  c..
for k.. ∈ [2..(N◡ + 1)]
  |k..1 := K..(T.. k.., x.., x..)
  |k..2 := K..(T.. k.. + h.., x.. + k..1, k..1)
  |k..3 := K..(T.. k.. + 0.5·h.., x.. + 0.25·k..1 + 0.25·k..2, k..2)
  |[x.. := eval(x.. + (k..1 + 4·k..3 + k..2)/6) X.. k.. c.. := x.. c..]
[Clear(m, kg, s, K, A, cd) ux := str2num(ux) X2.. := 0]
[T.. := T.. · str2num(ut) X.. n.. c.. := X.. n.. c.. · ux c..]
eval(augment(T.., X.., X2.. n.. := D◡(T.. n.., X.. n.. c..) nx..))
```

RK23

$0 := 10^{-15}$ For avoid loosing units with zero appVersion(4) = "0.99.7921.69"

$x_1 = x(t)$ $x(t) := A \cdot t \cdot \sin(\omega \cdot t + \varphi)$

$x_2 = x'(t)$ $x'(t) := \frac{d}{dt} x(t) = A \cdot (t \cdot \omega \cdot \cos(\varphi + \omega \cdot t) + \sin(\varphi + \omega \cdot t))$

$D(x_2) = x''(t)$ $x''(t) := \frac{d^2}{dt^2} x(t) = \omega \cdot (-t \cdot \omega \cdot \sin(\varphi + \omega \cdot t) + 2 \cdot \cos(\varphi + \omega \cdot t)) \cdot A$

$A := 200 \frac{cm}{min}$ $\omega := 2\ Hz$ $\varphi := 0.5$

$t_o := 0\ s$ $t_e := \frac{2 \cdot \pi}{\omega}$

$x_o := x(t_o) = 1.5981 \cdot 10^{-17}\ m$ $x'_o := x'(t_o) = 0.016 \frac{m}{s}$ $x''_o := x''(t_o) = 0.117 \frac{m}{s^2}$

$D(t, x) := \begin{bmatrix} x'(t) \\ x''(t) \end{bmatrix}$ $D(t_e, [x(t_e)\ x'(t_e)]) = \begin{bmatrix} 0.1998 \frac{m}{s} \\ -0.0838 \frac{m}{s^2} \end{bmatrix}$

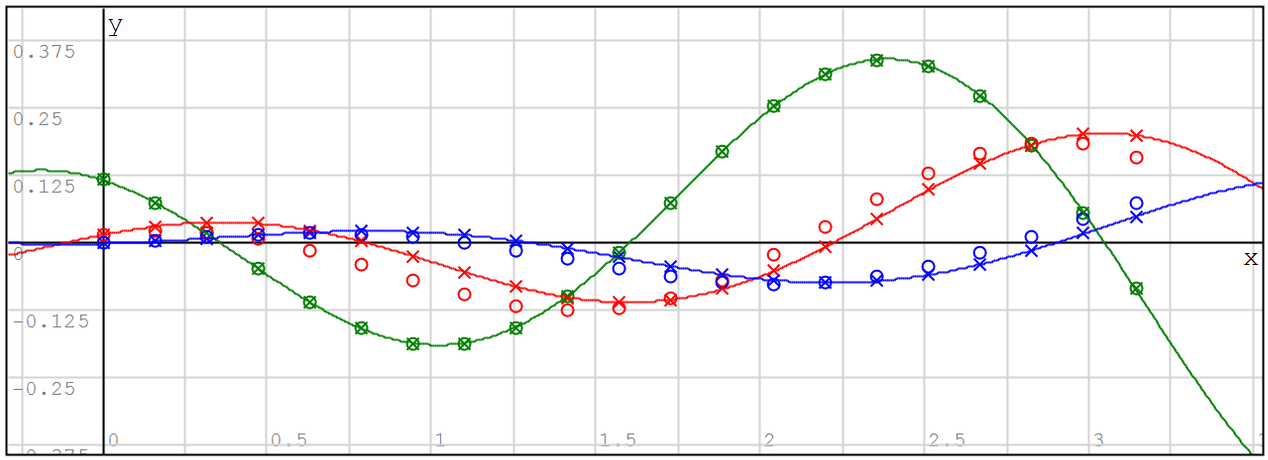
$[N := 20 \quad \varepsilon := 0.00001]$

$RK_F := RK23Fix\left(D(t, x), \left[t_o \; t_e\right], \left[x_o \; x'_o\right], N\right)$

$RK_A := RK23Adapt\left(D(t, x), \left[t_o \; t_e\right], \left[x_o \; x'_o\right], N, \varepsilon\right)$

This have an "extra" column with the acc

$$RK_F = \begin{bmatrix} 1\cdot10^{-15}\,s & 1.6\cdot10^{-17}\,m & 0.02\,\frac{m}{s} & 0.12\,\frac{m}{s^2} \\ 0.16\,s & 0.01\,m & 0.02\,\frac{m}{s} & 0.08\,\frac{m}{s^2} \\ 0.31\,s & 0.01\,m & 0.02\,\frac{m}{s} & 0.02\,\frac{m}{s^2} \\ \vdots & & & \end{bmatrix} \qquad RK_A = \begin{bmatrix} 1\cdot10^{-15}\,s & 1.6\cdot10^{-17}\,m & 0.02\,\frac{m}{s} & 0.12\,\frac{m}{s^2} \\ 0.16\,s & 0\,m & 0.03\,\frac{m}{s} & 0.08\,\frac{m}{s^2} \\ 0.31\,s & 0.01\,m & 0.04\,\frac{m}{s} & 0.02\,\frac{m}{s^2} \\ \vdots & & & \end{bmatrix}$$



$x\,(x\;s)$
$x'\,(x\;s)$
$x''\,(x\;s)$
$\text{augment}\left(\text{col}\left(RK_F, 1\right), \text{col}\left(RK_F, 2\right), \text{"o"}, 10, \text{"blue"}\right)$
$\text{augment}\left(\text{col}\left(RK_F, 1\right), \text{col}\left(RK_F, 3\right), \text{"o"}, 10, \text{"red"}\right)$
$\text{augment}\left(\text{col}\left(RK_F, 1\right), \text{col}\left(RK_F, 4\right), \text{"o"}, 10, \text{"green"}\right)$
$\text{augment}\left(\text{col}\left(RK_A, 1\right), \text{col}\left(RK_A, 2\right), \text{"x"}, 10, \text{"blue"}\right)$
$\text{augment}\left(\text{col}\left(RK_A, 1\right), \text{col}\left(RK_A, 3\right), \text{"x"}, 10, \text{"red"}\right)$
$\text{augment}\left(\text{col}\left(RK_A, 1\right), \text{col}\left(RK_A, 4\right), \text{"x"}, 10, \text{"green"}\right)$

Double pendulum

$\text{Clear}\left(l_1, l_2, m_1, m_2, g\right) = 1$

$$D(t, y) := \begin{bmatrix} a := \left(m_1 + m_2\right)\cdot l_1 \\ b := m_2 \cdot l_2 \cdot \cos\left(y_1 - y_3\right) \\ c := m_2 \cdot l_1 \cdot \cos\left(y_1 - y_3\right) \\ d := m_2 \cdot l_2 \\ j := -m_2 \cdot l_2 \cdot y_4 \cdot y_4 \cdot \sin\left(y_1 - y_3\right) - g\cdot\left(m_1 + m_2\right)\cdot\sin\left(y_1\right) \\ k := m_2 \cdot l_1 \cdot y_2 \cdot y_2 \cdot \sin\left(y_1 - y_3\right) - m_2 \cdot g \cdot \sin\left(y_3\right) \end{bmatrix}$$

$$\left|\text{stack}\left(y_2, \frac{(j\cdot d - b\cdot k)}{(a\cdot d - c\cdot b)}, y_4, \frac{(a\cdot k - c\cdot j)}{(a\cdot d - c\cdot b)}\right)\right.$$

Double Pendulum without units

$l_1 := 1 \quad l_2 := 2 \quad m_1 := 1 \quad m_2 := 1 \quad g := 9.8$

$Yo := \text{stack}(1.6,\ 0,\ 2.2,\ 0)$

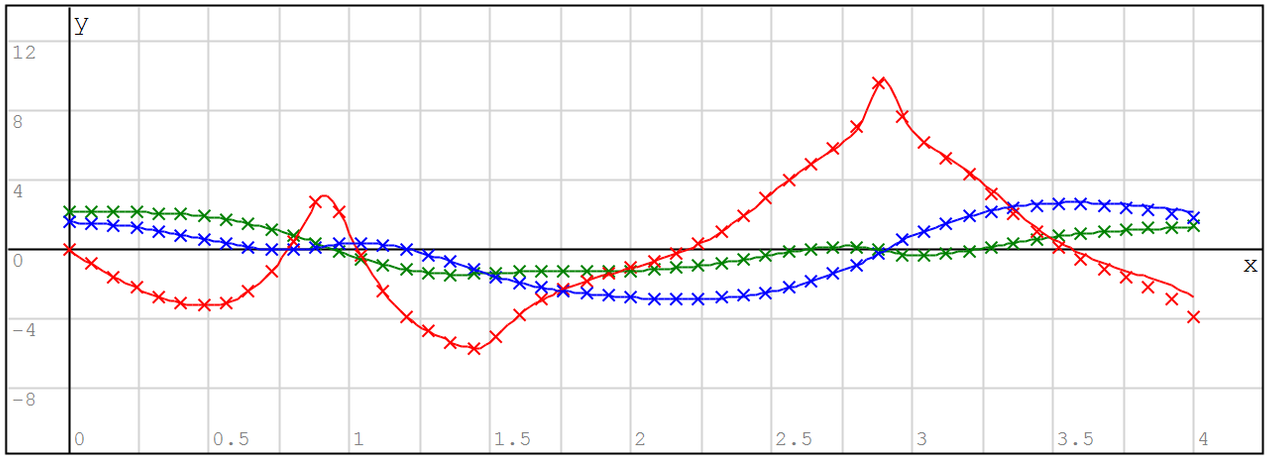
$RK := \text{Rkadapt}(Yo,\ 0,\ 4,\ 200,\ D)$

Double Pendulum wit units

$l_1 := 1\ \text{m} \quad l_2 := 2\ \text{m} \quad m_1 := 1\ \text{kg} \quad m_2 := 1\ \text{kg} \quad g := g_e$

$Yo := \text{stack}\left(1.6,\ 0\ \frac{\text{rad}}{\text{s}},\ 2.2,\ 0\ \frac{\text{rad}}{\text{s}}\right)$

$RK_A := RK23Adapt(D(t, x), [0\ 4]\ \text{s}, Yo, 50, 0.001)$



augment(col(RK, 1), col(RK, 2))
augment(col(RK, 1), col(RK, 3))
augment(col(RK, 1), col(RK, 4))
augment(col(RK_A, 1), col(RK_A, 2), "x", 10, "blue")
augment(col(RK_A, 1), col(RK_A, 3), "x", 10, "red")
augment(col(RK_A, 1), col(RK_A, 4), "x", 10, "green")

Alvaro