Binary Distillation with the McCabe-Thiele Method

1. Introduction

A Benzene - Water feed at 1 atm is to be separated in a multistage distillation unit to a required purity of both components.

This worksheet will calculate the required number of theoretical stages required. Data describing propane-pentane equilibrium composition is supplied.

2. Equilibrium Data and Required Purity

The following data gives the mole fraction of propane in the liquid and vapour phases in a Benzene - Water mixture at equilibrium at 1 Atm.

$$xPropLiq := \begin{bmatrix} 0 \\ 0.0917 \\ 0.1842 \\ 0.2779 \\ 0.3732 \\ 0.4703 \\ 0.5699 \\ 0.6721 \\ 0.7776 \\ 0.8867 \\ 1 \end{bmatrix} \qquad xPropVap := \begin{bmatrix} 0 \\ 0.1543 \\ 0.2943 \\ 0.4188 \\ 0.5315 \\ 0.6327 \\ 0.7236 \\ 0.8049 \\ 0.8776 \\ 0.9424 \\ 1 \end{bmatrix}$$

Required purity of Benzene in distillate $x_D := 97\%$

Required purity of Benzene in bottoms $x_B := 2\%$

Fraction of Benzene in Feed $x_F := 40\%$

q value $q := 1.5$

Reflux ratio $R := 3.5$

3. The Equilibrium Line

This is a continuous function which describes the equilibrium line

$$g(x) := \text{interp}\left(\text{cspline}\left(xPropLiq, xPropVap\right), xPropLiq, xPropVap, x\right)$$

4. The Top and Bottom Operating Lines and the q-Line

q-line $$qline(x) := x \cdot \frac{q}{q-1} - \frac{x_F}{q-1}$$

Top operating line $$top\_op(x) := x \cdot \frac{R}{1+R} + \frac{x_D}{1+R}$$

Bottom operating line $$bottom\_op(x) := x \cdot \frac{R \cdot (x_F - x_B) + q \cdot (x_D - x_B)}{q \cdot (x_D - x_B) + R \cdot (x_F - x_B) - x_D + x_F} + \frac{x_B \cdot (x_F - x_D)}{q \cdot (x_D - x_B) + R \cdot (x_F - x_B) - x_D + x_F}$$

Hence a function can be defined that described the combined operating line

$$operating\_line(x) := \begin{cases} bottom\_op(x) & \text{if } x < \dfrac{x_D \cdot (q-1) + x_F \cdot (R+1)}{q+R} \\ top\_op(x) & \text{else} \end{cases}$$

5. Required Theoretical Stages

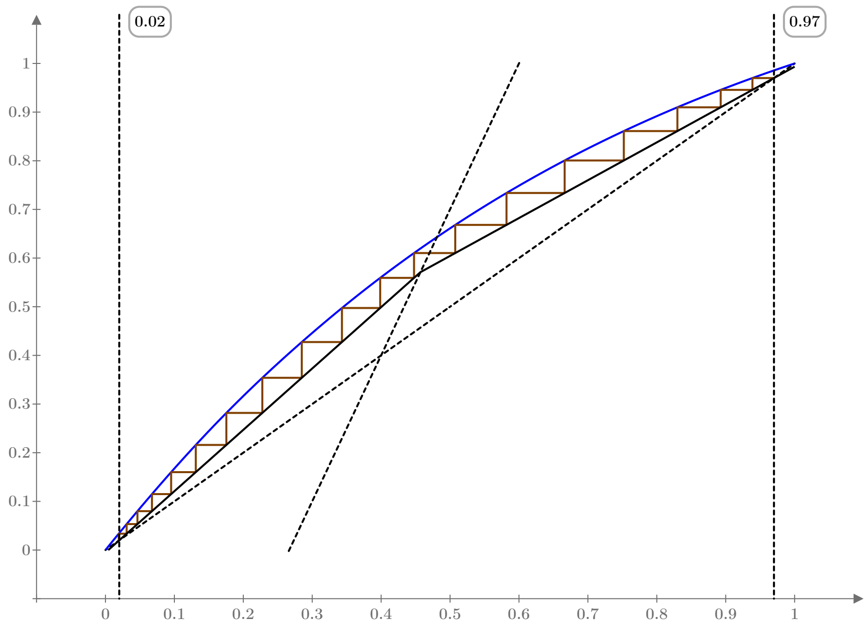
The following function steps off the required number of stages given a function f that describes the operating line and a function g that describes the equilibrium line

$$
MC(f,g) := \left\|
\begin{array}{l}
xguess \leftarrow 0 \\
X_1 \leftarrow x_D \\
Y_1 \leftarrow x_D \\
X_2 \leftarrow \textbf{root}\left(g(xguess) - Y_1, xguess\right) \\
Y_2 \leftarrow Y_1 \\
i \leftarrow 3 \\
\text{while } X_{i-2} > x_B \\
\quad \left\|
\begin{array}{l}
X_{i-1} \leftarrow X_{i-2} \\
Y_{i-1} \leftarrow f\left(X_{i-1}\right) \\
X_i \leftarrow \textbf{root}\left(g(xguess) - Y_{i-1}, xguess\right) \\
Y_i \leftarrow Y_{i-1} \\
i \leftarrow i + 2
\end{array}
\right. \\
X_{i-1} \leftarrow X_{i-2} \\
Y_{i-1} \leftarrow X_{i-2} \\
\text{return augment}(X, Y)
\end{array}
\right.
$$

6. Results

$x := 0, 0.001 .. 1$

Benzene - Water Equilibrium @ 1 atm



The required number of stages is:

$$stages := \text{floor}\left(\frac{\text{rows}\left(MC(operating\_line, g)^{\langle 1 \rangle}\right)}{2}\right) - 1$$

$$stages = 19$$

The efficiency of real stages will be about 0.5-0.7. Hence the actual required number of stages is about $\frac{stages}{0.5} = 38$.

$$MC(operating\_line, g) = \begin{bmatrix} 0.97 & 0.97 \\ 0.97 & 0.97 \\ 0.939 & 0.97 \\ 0.939 & 0.946 \\ 0.893 & 0.946 \\ 0.893 & 0.91 \\ 0.83 & 0.91 \\ 0.83 & 0.861 \\ 0.752 & 0.861 \\ 0.752 & 0.801 \\ 0.666 & 0.801 \\ 0.666 & 0.734 \\ 0.582 & 0.734 \\ 0.582 & 0.668 \\ 0.508 & 0.668 \\ 0.508 & 0.61 \\ 0.448 & 0.61 \\ 0.448 & 0.559 \\ 0.399 & 0.559 \\ 0.399 & 0.497 \\ 0.343 & 0.497 \\ 0.343 & 0.427 \\ 0.285 & 0.427 \\ 0.285 & 0.354 \\ 0.228 & 0.354 \\ 0.228 & 0.282 \\ & \vdots \end{bmatrix}$$