Constructing Mathematics from a Single Function

$\mu(x, y) = e^{x} - \ln(y)$

The exponential minus logarithm function, described by Andrzej Odrzywołek in https://arxiv.org/html/2603.21852v2

Functions

Exponential, Logarithm, identity, oposite, inverse, power ...

$E(x) := \mu(x, 1)$

$L(x) := \mu(1, \mu(\mu(1, x), 1))$

$id(x) := L(E(x)) = \mu(1, \mu(\mu(1, \mu(x, 1)), 1))$

$op(x) := \mu(\mu(1, \mu(1, 0)), \mu(x, 1))$

$inv(x) := \mu(\mu(\mu(1, \mu(1, 0)), x), 1)$

$pow(x, y) := E(P(y, L(x)))$

Product, Sum, cos, sin, acos, asin

$P(x, y) := \mu(\mu(1, \mu(\mu(L(\mu(1, x)), L(\mu(y, 1))), 1)), 1)$

$S(x, y) := L(P(E(x), E(y)))$

$c(x) := P(inv(2), S(E(P(i, x)), E(op(P(i, x)))))$

$s(x) := P(inv(P(2, i)), S(E(P(i, x)), op(E(op(P(i, x))))))$

$ac(x) := P(op(i), L(S(x, P(i, pow(S(1, op(P(x, x))), inv(2))))))$

$as(x) := P(op(i), L(S(P(i, x), pow(S(1, op(P(x, x))), inv(2)))))$

Constants

$0 := \mu(1, \mu(\mu(1, 1), 1))$ $\quad$ $i := op(pow(op(1), inv(2)))$

$2 := S(1, 1)$ $\quad$ $\pi := P(i, L(op(1)))$ $\quad$ $1 := 1$

SMath numerical implementation

$$\mu(x, y) := e^{x} - \begin{cases} -\infty & \text{if } y = 0 \\ \ln(y) & \text{otherwise} \end{cases}$$

This fails for some zero values, but gives good results in almost cases

$0 = 0$ $\quad$ $2 = 2$

$i = 2.2204 \cdot 10^{-16} + i$ $\quad$ $\pi = 3.1416 - 3.1416 \cdot 10^{-15} \cdot i$

usual round-off errors

Basic functions

$[a \ b] := [2 \ 3]$ $\quad$ $c := a + b \cdot i$

$\ln(c) = 1.2825 + 0.9828 \cdot i$ $\quad$ $L(c) = 1.2825 + 0.9828 \cdot i$

$\exp(c) = -7.3151 + 1.0427 \cdot i$ $\quad$ $E(c) = -7.3151 + 1.0427 \cdot i$

$c = 2 + 3 \cdot i$ $\quad$ $id(c) = 2 + 3 \cdot i$

$-c = -2 - 3 \cdot i$ $\quad$ $op(c) = -2 - 3 \cdot i$

$\frac{1}{a} = 0.5$ $\quad$ $inv(a) = 0.5$

$a^{b}=8$ $\qquad$ $pow(a, b)=8$

$a+b=5$ $\qquad$ $S(a, b)=5$

$a \cdot b=6$ $\qquad$ $P(a, b)=6$

Trigonometric functions

$[\alpha \ a] := [30\ \text{deg}\ 0.2]$

$\cos(\alpha)=0.866$ $\qquad$ $c(\alpha)=0.866$

$\sin(\alpha)=0.5$ $\qquad$ $s(\alpha)=0.5-1 \cdot 10^{-15} \cdot i$

$\text{acos}(a)=1.3694$ $\qquad$ $ac(a)=1.3694+1.3694 \cdot 10^{-15} \cdot i$

$\text{asin}(a)=0.2014$ $\qquad$ $as(a)=0.2014-8.0543 \cdot 10^{-16} \cdot i$

$\mu(x, y) := (x \oplus y)$ $\qquad$ showing eml as binary operator

$E(x)=x \oplus 1$

$L(y)=1 \oplus ((1 \oplus y) \oplus 1)$

$id(x)=1 \oplus ((1 \oplus (x \oplus 1)) \oplus 1)$

$op(x)=(1 \oplus (1 \oplus (1 \oplus ((1 \oplus 1) \oplus 1)))) \oplus (x \oplus 1)$

$inv(x)=((1 \oplus (1 \oplus (1 \oplus ((1 \oplus 1) \oplus 1)))) \oplus x) \oplus 1$

$pow(x, y)=((1 \oplus (((1 \oplus ((1 \oplus (1 \oplus y)) \oplus 1)) \oplus (1 \oplus ((1 \oplus ((1 \oplus ((1 \oplus x) \oplus 1)) \oplus 1)) \oplus 1))) \oplus 1)) \oplus 1) \oplus 1$

$P(x, y)=(1 \oplus (((1 \oplus ((1 \oplus (1 \oplus x)) \oplus 1)) \oplus (1 \oplus ((1 \oplus (y \oplus 1)) \oplus 1))) \oplus 1)) \oplus 1$

$S(x, y)=1 \oplus ((1 \oplus ((1 \oplus (((1 \oplus ((1 \oplus (1 \oplus (x \oplus 1))) \oplus 1)) \oplus (1 \oplus ((1 \oplus ((y \oplus 1) \oplus 1)) \oplus 1))) \oplus 1)) \oplus 1)) \oplus 1)$

$0=1 \oplus ((1 \oplus 1) \oplus 1)$

$2=1 \oplus ((1 \oplus ((1 \oplus (((1 \oplus ((1 \oplus (1 \oplus (1 \oplus 1))) \oplus 1)) \oplus (1 \oplus ((1 \oplus ((1 \oplus 1) \oplus 1)) \oplus 1))) \oplus 1)) \oplus 1)) \oplus 1)$

etc ...

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