Alglib solver.An attempt to automate the finding of roots

AlgLib Solver for a system of nonlinear equations(Uno)

```
result := | for k ∈ [1..rows(f(x))]
          |   u_k := x_k
          | Jac(x) := Jacobian(f(u), u)
          | StepMax := 0
          | Eps := 10^-14
          | u := al_nleqsolve(X0, StepMax, Eps, f(x), Jac(x))
```

Approximation by random numbers

```
X0 := | u_1 := 2
      | for i ∈ [2..rows(f(x))]
      |   u_i := u_(i-1) + (2·Random(1) - 1)_1
      | u
```

1.System of 3 equations

$$f(x) := \begin{bmatrix} 2\cdot x_1 + x_2 + 2\cdot (x_3)^2 - 5 \\ {x_2}^3 + 4\cdot x_3 - 4 \\ x_1 \cdot x_2 + x_3 - e^{x_3} \end{bmatrix}$$

$$X0 = \begin{bmatrix} 2 \\ 2.0564 \\ 1.594 \end{bmatrix} \qquad result = \begin{bmatrix} 1.4225 \\ 0.9754 \\ 0.768 \end{bmatrix} \qquad f(result) = \begin{bmatrix} -6.9325\cdot 10^{-15} \\ -1.1259\cdot 10^{-15} \\ -2.5088\cdot 10^{-15} \end{bmatrix}$$

2.The RSCR mechanism.A case where the axis of rotation are perpendicular

$L1 := 2 \qquad R := 0.9 \qquad A := \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T \qquad B := \begin{bmatrix} 0 & 3 & 0 \end{bmatrix}^T$

system of nonlinear equations

$$f(x) := \begin{bmatrix} x_2 \\ (x_1 - A_1)^2 + (x_2 - A_2)^2 + (x_3 - A_3)^2 - R^2 \\ (x_1 - x_4)^2 + (x_2 - x_5)^2 + (x_3 - x_6)^2 - L1^2 \\ (x_1 - x_4)\cdot(x_4 - x_7) + (x_2 - x_5)\cdot(x_5 - x_8) + (x_3 - x_6)\cdot(x_6 - x_9) \\ x_7 - 0.1 \\ (x_7 - B_1)^2 + (x_8 - B_2)^2 + (x_9 - B_3)^2 - L1^2 \\ (x_4 - x_7)\cdot(B_1 - x_7) + (x_5 - x_8)\cdot(B_2 - x_8) + (x_6 - x_9)\cdot(B_3 - x_9) \\ x_7 - x_4 \\ x_5 - 0.7 \end{bmatrix}$$

$$X0 = \begin{bmatrix} 2 \\ 2.1139 \\ 1.1673 \\ 1.7709 \\ 0.8615 \\ 1.2935 \\ 2.291 \\ 2.2349 \\ 1.8273 \end{bmatrix} \quad result = \begin{bmatrix} 1.9059 \\ -0.0091 \\ 0.0315 \\ 0.0623 \\ 0.7075 \\ -0.255 \\ 0.079 \\ 1.1686 \\ 0.7995 \end{bmatrix} \quad f(result) = \begin{bmatrix} -0.0091 \\ 0.0117 \\ -0.0057 \\ -0.0024 \\ -0.021 \\ -0.0005 \\ 5.0744 \cdot 10^{-5} \\ 0.0166 \\ 0.0075 \end{bmatrix}$$

3.Spatial four-bar (RSSR) mechanism

$CD1 := -1.74074$ $\quad CD2 := 0.51852$ $\quad CD3 := 0.81460$

$q1 := 5$ $\quad q2 := 3$ $\quad q3 := 2$ $\quad L1 := 0.9$ $\quad L2 := 7.2$ $\quad L3 := 5.4$

$L4 := 8$ $\quad L5 := 5$ $\quad L6 := 6$ $\quad a1 := 1$ $\quad b1 := 2$ $\quad c1 := 0.25$

system of nonlinear equations

$$f(x) := \begin{bmatrix} \left(-1 - x_4\right)^2 + \left(2 - x_5\right)^2 + \left(1 - x_6\right)^2 - 1.9^2 \\ x_1 - 5 \\ a1 \cdot x_4 + b1 \cdot x_5 + c1 \cdot x_6 + 0.5 \\ \left(q1 - x_1\right)^2 + \left(q2 - x_2\right)^2 + \left(q3 - x_3\right)^2 - 2.4^2 \\ \left(x_4 - x_1\right)^2 + \left(x_5 - x_2\right)^2 + \left(x_6 - x_3\right)^2 - 7.7^2 \\ x_5 - 0.1 \end{bmatrix}$$

$$X0 = \begin{bmatrix} 2 \\ 2.5022 \\ 2.1297 \\ 2.0615 \\ 1.5757 \\ 1.3705 \end{bmatrix} \quad result = \begin{bmatrix} 5 \\ 3.723 \\ 4.2885 \\ -0.9555 \\ 0.1006 \\ 1.0174 \end{bmatrix} \quad f(result) = \begin{bmatrix} 0.0001 \\ -1.2911 \cdot 10^{-8} \\ -1.2862 \cdot 10^{-5} \\ 1.0566 \cdot 10^{-7} \\ 1.2649 \cdot 10^{-6} \\ 0.0006 \end{bmatrix}$$

4.System of 12 equations

$D1 := 3$ $\quad D2 := 2$ $\quad D3 := 2$ $\quad G1 := 2$ $\quad G2 := 2$ $\quad G3 := -1$

$A1 := 3.6$ $\quad A2 := 1.2$ $\quad A3 := 1.2$ $\quad L1 := 0.9$ $\quad L2 := 0.5$ $\quad L3 := 2.2$ $\quad L4 := 2.7$ $\quad L5 := 1$

system of nonlinear equations

$$f(x) := \begin{bmatrix} (D1 - x_4)^2 + (D2 - x_5)^2 + (D3 - x_6)^2 - L3^2 \\ (D1 - x_7)^2 + (D2 - x_8)^2 + (D3 - x_9)^2 - L2^2 \\ x_4 - x_5 + 0.5 \cdot x_6 - 2 \\ x_7 - x_8 + 0.5 \cdot x_9 - 2 \\ (x_4 - x_7)^2 + (x_5 - x_8)^2 + (x_6 - x_9)^2 - L2^2 - L3^2 \\ x_1 + x_2 - 0.5 \cdot x_3 - 4 \\ (G1 - x_1)^2 + (G2 - x_2)^2 + (G3 - x_3)^2 - L5^2 \\ (x_4 - x_1)^2 + (x_5 - x_2)^2 + (x_6 - x_3)^2 - L4^2 \\ (x_7 - x_{10})^2 + (x_8 - x_{11})^2 + (x_9 - x_{12})^2 - L1^2 \\ x_{12} - 1 \\ x_{10} - 3.5 \\ x_1 + x_3 \end{bmatrix}$$

$$X0 = \begin{bmatrix} 2 \\ 2.2052 \\ 1.3639 \\ 1.9442 \\ 2.4686 \\ 2.5264 \\ 2.5195 \\ 1.7707 \\ 1.9824 \\ 2.5246 \\ 3.4337 \\ 4.3964 \end{bmatrix} \quad result = \begin{bmatrix} 1.079 \\ 2.3815 \\ -1.079 \\ 2.3201 \\ 0.5608 \\ 0.4813 \\ 3.3391 \\ 2.1786 \\ 1.6789 \\ 3.5 \\ 1.6101 \\ 1 \end{bmatrix} \quad f(result) = \begin{bmatrix} -2.3768 \cdot 10^{-15} \\ 3.1691 \cdot 10^{-15} \\ 4.9518 \cdot 10^{-16} \\ -5.4402 \cdot 10^{-15} \\ -9.9035 \cdot 10^{-15} \\ 4.6423 \cdot 10^{-15} \\ 9.5327 \cdot 10^{-15} \\ 2.7854 \cdot 10^{-14} \\ -2.7584 \cdot 10^{-15} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

5. Cascade of Two Continuous Stirred-Tank Reactors with Recycle

Consider a cascade of two continuous stirred-tank reactors with recycle undergoing exothermic first-order chemical reactions. The steady state is described by four nonlinear equations for the reaction conversions $x1, x2$, dimensionless temperature $T1, T2$, and parameter α (Damköhler number).

$\alpha := 0.00271$

$$f(x) := \begin{bmatrix} \alpha \cdot \mathbf{e}^{\frac{x_3}{1+0.001 \cdot x_3}} \cdot (1 - x_1) - x_1 \\ x_1 + \alpha \cdot \mathbf{e}^{\frac{x_4}{1+0.001 \cdot x_4}} \cdot (1 - x_2) - x_2 \\ \alpha \cdot \mathbf{e}^{\frac{x_3}{1+0.001 \cdot x_3}} \cdot (1 - x_1) - 3 \cdot x_3 \\ 22 \cdot \alpha \cdot \mathbf{e}^{\frac{x_4}{1+0.001 \cdot x_4}} \cdot (1 - x_2) + x_3 - 3 \cdot x_4 \end{bmatrix}$$

$$X0 = \begin{bmatrix} 2 \\ 1.4478 \\ 2.2037 \\ 2.1095 \end{bmatrix} \qquad result = \begin{bmatrix} 0.0027 \\ 0.0055 \\ 0.0009 \\ 0.0205 \end{bmatrix} \qquad f(result) = \begin{bmatrix} 5.0098 \cdot 10^{-16} \\ -2.1409 \cdot 10^{-17} \\ -4.9882 \cdot 10^{-16} \\ -4.8526 \cdot 10^{-16} \end{bmatrix}$$