d

equrep

isol

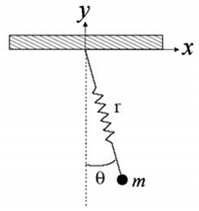
Simple utilities

dsol

Spring Pendulum

Spring Pendulum

Constants $\partial m := 0$ $\partial a := 0$ $\partial g := 0$



Gen Coords $\partial\theta := \theta' \cdot \partial t$ $\partial\theta' := \theta'' \cdot \partial t$

$\partial r := r' \cdot \partial t$ $\partial r' := r'' \cdot \partial t$

Position $x := r \cdot \sin(\theta)$ $y := -r \cdot \cos(\theta)$

$x' := simp\left(\frac{d(x)}{\partial t}\right)$ $y' := simp\left(\frac{d(y)}{\partial t}\right)$

Lagrangian $ke := \frac{1}{2} \cdot m \cdot \left(x'^{2} + y'^{2}\right)$ $pe := m \cdot g \cdot y$ $se := \frac{1}{2} \cdot k \cdot (r - a)^{2}$

$L := ke - pe + se$

Equations of motion $\theta'' := isol\left(\frac{1}{\partial t} \cdot d\left(\frac{\partial(L, \theta')}{\partial\theta'}\right) - \frac{\partial(L, \theta)}{\partial\theta}, \theta''\right)$

$r'' := isol\left(\frac{1}{\partial t} \cdot d\left(\frac{\partial(L, r')}{\partial r'}\right) - \frac{\partial(L, r)}{\partial r}, r''\right)$

$$\theta'' = -\frac{\left(\cos(\theta)\cdot\left(\sin(\theta)\cdot r' + r\cdot\cos(\theta)\cdot\theta'\right) + \sin(\theta)\cdot\left(-\cos(\theta)\cdot r' + r\cdot\sin(\theta)\cdot\theta'\right) + \theta'\cdot r\right)\cdot r' + r\cdot g\cdot\sin(\theta)}{r^{2}}$$

$$r'' = \frac{-(r-a)\cdot k + m\cdot\left(\theta'\cdot\left(\cos(\theta)\cdot\left(\sin(\theta)\cdot r' + r\cdot\cos(\theta)\cdot\theta'\right) + \sin(\theta)\cdot\left(-\cos(\theta)\cdot r' + r\cdot\sin(\theta)\cdot\theta'\right)\right) + g\cdot\cos(\theta)\right)}{m}$$

Numeric example

$\theta_0 := 60°$ $\theta'_0 := 0$ $a := 2$ $m := 1$ $g := 9.81$ $t_{end} := 6$

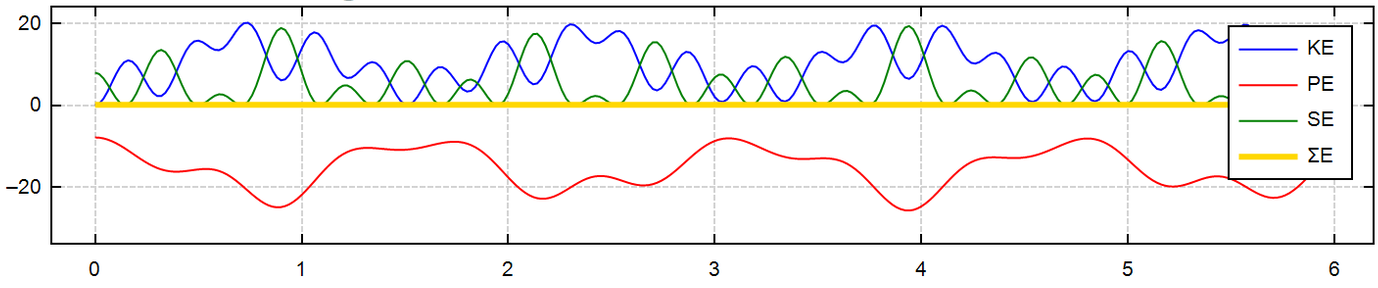
$r_0 := 0.8 \cdot a$ $r'_0 := 0$ $k := 100$ $N := 300$ $n := 1 + [0..N]$

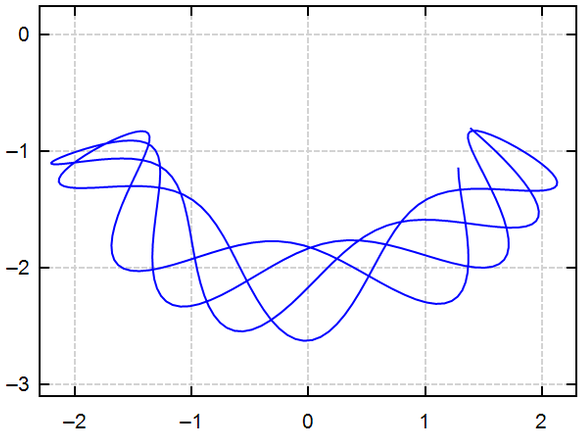
$$U := \text{Rkadapt}\left(\text{stack}\left(\theta_0, \theta'_0, r_0, r'_0\right), 0, t_{end}, N, D(t, u) := \left|\begin{matrix}[\theta\ \theta'\ r\ r'] := \left[u_1\ u_2\ u_3\ u_4\right] \\ \text{stack}(\theta', \theta'', r', r'')\end{matrix}\right.\right)$$

$[T\ \theta\ \theta'\ r\ r'] := MCols(U)$ $[X\ Y\ KE\ PE\ SE] := \left[\vec{x}\ \vec{y}\ \overrightarrow{ke}\ \overrightarrow{pe}\ \overrightarrow{se}\right]$

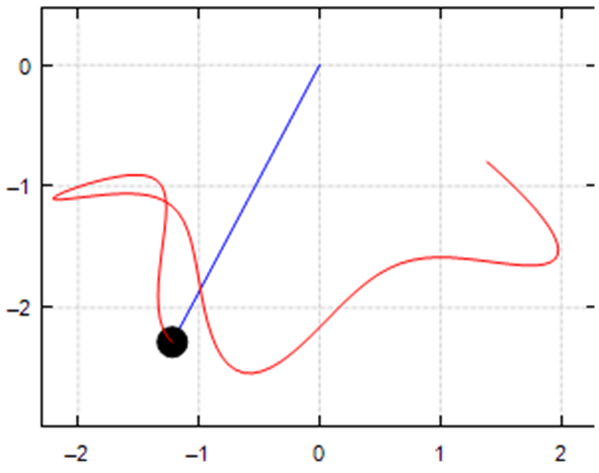
$$\Pi(n) := \begin{cases} \left[\begin{matrix} 0 & 0 \\ X_n & Y_n \end{matrix}\right] \\ \left[X_n\ Y_n\ "."\ 15\ "black"\right] \\ \text{augment}\left(X_{[1..n]}, Y_{[1..n]}\right) \end{cases}$$

$$\Pi := \begin{cases} \text{augment}(T, KE) \\ \text{augment}(T, PE) \\ \text{augment}(T, SE) \\ \text{augment}(T, KE + PE + SE) \end{cases}$$





augment(X, Y)



Example using RKA From Mathcad toolbox

Example $Options(dsol, dsolver = \text{"Rkadapt"}) = \text{"Rkadapt"}$

$$\begin{cases} m \cdot r(t) \cdot \theta''(t) + 2 \cdot m \cdot r'(t) \cdot \theta'(t) + m \cdot g \cdot \sin(\theta(t)) = 0 & \theta(0) = \theta_0 \quad \theta'(0) = \theta'_0 \\ m \cdot r''(t) - m \cdot r(t) \cdot \theta'(t)^2 + k \cdot (r(t) - a) - m \cdot g \cdot \cos(\theta(t)) = 0 & r(0) = r_0 \quad r'(0) = r'_0 \end{cases}$$

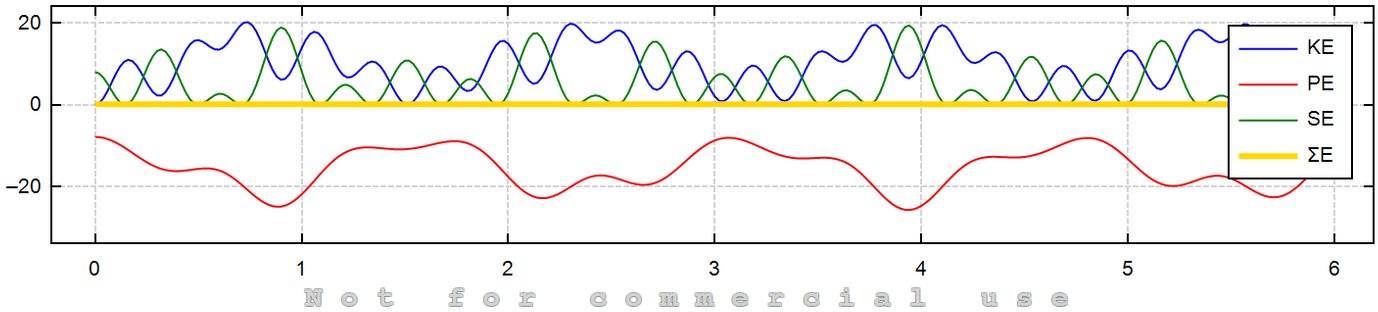
$$U := dsol\left(\begin{pmatrix} \theta(t) \\ r(t) \end{pmatrix}, t_{end}, N\right)$$

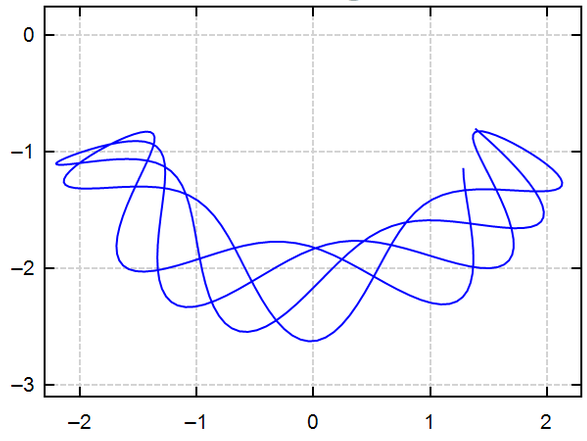
$$[T \ \theta \ \theta' \ R \ R'] := MCols(U) \qquad X := \overrightarrow{R \cdot \sin(\theta)} \qquad Y := \overrightarrow{-R \cdot \cos(\theta)}$$

$$KE := \overrightarrow{\frac{1}{2} \cdot m \cdot \left(R'^2 + (R \cdot \theta')^2\right)} \qquad PE := \overrightarrow{m \cdot g \cdot Y} \qquad SE := \overrightarrow{\frac{k \cdot (R - a)^2}{2}}$$

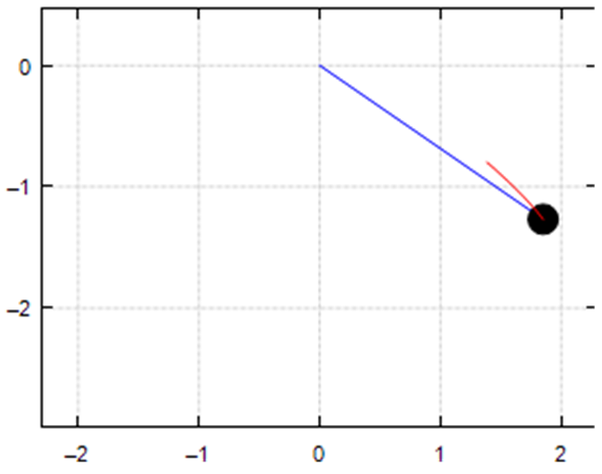
$$\Pi(n) := \begin{cases} \begin{bmatrix} 0 & 0 \\ X_n & Y_n \end{bmatrix} \\ \begin{bmatrix} X_n & Y_n & \text{"."} & 15 & \text{"black"} \end{bmatrix} \\ augment\left(X_{[1..n]}, Y_{[1..n]}\right) \end{cases}$$

$$\Pi := \begin{cases} augment(T, KE) \\ augment(T, PE) \\ augment(T, SE) \\ augment(T, KE + PE + SE) \end{cases}$$





augment(X, Y)



Alvaro

appVersion(4) = "1.73.9126.0"