

## Steinmetz Solid Volume

DInt

Numerical  
procedure

```

DInt ( f ( 2 ), x1, x2, y1, y2, nx, ny ) :=
:= RK ( a, b, n ) := M := dn_AdamsMoulton ( [ 0 ], a, b, n, D ( x, y ) )
eval ( Mrows ( M ) 2 )
D ( y, u ) := D ( x, v ) := [ f ( x, y ) ]
RK ( x1, x2, nx )
RK ( y1, y2, ny )

```

Unit vectors of  
cylinders
$$c := \frac{1}{\sqrt{3}} \quad d := \frac{1}{\sqrt{2}}$$

$$U := \begin{bmatrix} 1 & 0 & 0 & c & c & c & -c & d & d & 0 & 0 & d & d \\ 0 & 1 & 0 & c & c & -c & c & d & -d & d & d & 0 & 0 \\ 0 & 0 & 1 & c & -c & c & c & 0 & 0 & d & -d & d & -d \end{bmatrix}$$

$$u(k) := \text{col}(U, k)$$
Normal in spherics &  
Position vector of  
the surface
$$n := \begin{bmatrix} \sin(\theta) \cdot \cos(\varphi) \\ \sin(\theta) \cdot \sin(\varphi) \\ \cos(\theta) \end{bmatrix}$$

$$r(k) := \frac{R}{\max(N_k := \text{norme}(n \times u(k)))}$$
Radius of the  
cylinders $R_4 := 4$   $R_1 := 1$ 

$$V(k, R) := \begin{cases} f(\theta, \varphi) := \text{eval} \left( \left| \frac{(r(k))^3}{3} \cdot \sin(\theta) \right| \right) \\ 8 \cdot \text{DInt} \left( f(\theta, \varphi), 0, \frac{\pi}{2}, 0, \frac{\pi}{2}, 20000, 20000 \right) \end{cases}$$

Volume

 $\text{err}(V_x) := \frac{|V - V_x|}{V_x}$ 
 $k_2 := [1 \ 2] \quad V := V(k_2, R_4) = 341.3544 \quad V_2 := \frac{16}{3} \cdot R_4^3 = 341.3333 \quad \text{err}(V_2) = 0.01 \%$ 
 $k_3 := [1 \ 2 \ 3] \quad V := V(k_3, R_4) = 300.0283 \quad V_3 := 8 \cdot (2 - \sqrt{2}) \cdot R_4^3 = 299.9227 \quad \text{err}(V_3) = 0.04 \%$ 
 $k_4 := [4 \ 5 \ 6 \ 7] \quad V := V(k_4, R_1) = 4.5466 \quad V_4 := 12 \cdot (2 \cdot \sqrt{2} - \sqrt{6}) = 4.5472 \quad \text{err}(V_4) = 0.01 \%$ 
 $k_6 := [8 \dots 13] \quad V := V(k_6, R_1) = 4.3041 \quad V_6 := \frac{16}{3} \cdot (3 + 2 \cdot \sqrt{3} - 4 \cdot \sqrt{2}) = 4.3053 \quad \text{err}(V_6) = 0.03 \%$ 
 $k_7 := [1 \dots 7] \quad V := V(k_7, R_1) = 4.2803$ 

## Maple code

```

restart: with(linalg):
c := 1 / sqrt(3): d := 1 / sqrt(2):
U := matrix([
  [1, 0, 0, c, c, c, -c, d, d, 0, 0, d, d],
  [0, 1, 0, c, c, -c, c, d, -d, d, d, 0, 0],
  [0, 0, 1, c, -c, c, c, 0, 0, d, -d, d, -d]
]):
n := vector([sin(theta)*cos(phi), sin(theta)*sin(phi),
cos(theta)]):
V := proc(K, R)
  local N, r, f;
  N := [seq(norm(crossprod(n, col(U, i[k])), 2), i[k] in

```

Maple 11 - [Untitled (1) - [Server 1]]

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V2 := V([1, 2], R\_4);  
V3 := V([1, 2, 3], R\_4);  
V4 := V([4, 5, 6, 7], R\_1);

```

K)];
  r := R / max(op(N));
  f := abs(r^3 * sin(theta) / 3);
  return 8 * evalf(Int(Int(f, theta = 0 .. Pi/2), phi
= 0 .. Pi/2, epsilon = 1e-3));
end proc:

```

```

R_4 := 4: R_1 := 1:

```

```

V2 := V([1, 2], R_4);
V3 := V([1, 2, 3], R_4);
V4 := V([4, 5, 6, 7], R_1);
V6 := V([8, 9, 10, 11, 12, 13], R_1);
V7 := V([1, 2, 3, 4, 5, 6, 7], R_1);

```

```

V6 := V([8, 9, 10, 11, 12, 13], R_1);
V7 := V([1, 2, 3, 4, 5, 6, 7], R_1);

```

```
V2 := 341.3333344
```

```
V3 := 299.9231153
```

```
V4 := 4.547515887
```

```
V6 := 4.305210600
```

```
V7 := 4.280148573
```

```
[> |
```

Alvaro

```
appVersion(4) = "1.75.9678.0"
```