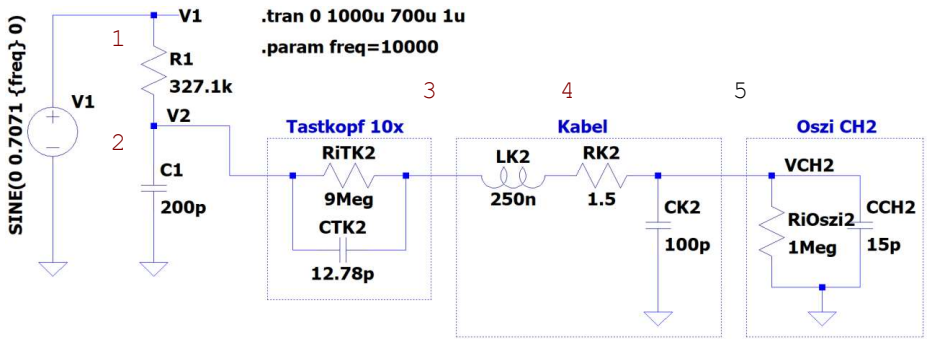
MNA & Spice#

Space#




```
V1 1 0
R1 1 2
C1 2 0
RT 2 3
CT 2 3
LK 3 4
RK 4 5
CK 5 0
RO 5 0
CO 5 0
```

$\chi s := "\chi s"$

```
V1 1 0 V1s
R1 1 2 327.1k
C1 2 0 200p
RT 2 3 9MEG
CT 2 3 12.78p
LK 3 4 250n
RK 4 5 1.5
CK 5 0 100p
RO 5 0 1MEG
CO 5 0 15p
```

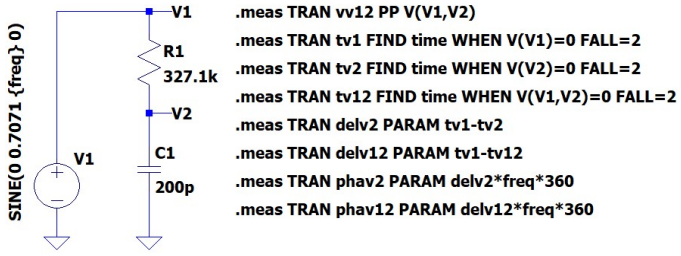
$\chi Ver := "\chi Ver"$

$CR := \text{num2str}(\text{str2num}(\text{concat}("\", "000D", "\")))$

$save := ".SAVE\ V(1)\ V(2)\ I(R1)"$

$mode := ".TRAN\ 0\ \{10/freq\}\ \{8/freq\}\ \{0.01/freq\}"$

$M(\varphi) :=$
$\quad param := \text{strrep}(".PARAM\ freq=@", "@", \text{num2str}(\text{eval}(\varphi\ s)))$
$\quad "\{CR\}\{circ\}\{CR\}\{mode\}\{CR\}\{param\}\{CR\}\{save\}\{CR\}.END", \_$



$PP(V) := \max(V) - \min(V)$

$find(T, V, fall) :=$
$\quad n := 2$
$\quad \text{for}\ count \in [1..fall]$
$\quad\quad \text{for}\ k \in [(n+1)..\text{length}(V)]$
$\quad\quad\quad \text{if}\ \text{sign}(V_k) \neq \text{sign}(V_n)$
$\quad\quad\quad\quad \textbf{break}$
$\quad\quad n := k$
$\quad T_n$

$f := 10000\ Hz$

$circ := \text{strrep}(\text{description}(\chi Ver), "V1s", "SINE(0\ 0.7071\ \{freq\}\ 0)")$

$M := M(f) \quad [T\ V_1\ V_2] := [\text{col}(M, 1)\ s\ \text{col}(M, 2)\ V\ \text{col}(M, 3)\ V] \quad V_{12} := V_1 - V_2 \quad \text{rows}(M) = 204$

$v_1 := PP(V_1) = 1.4135\ V \qquad t_1 := find(T, V_1, 2) = 900.5\ \mu s$

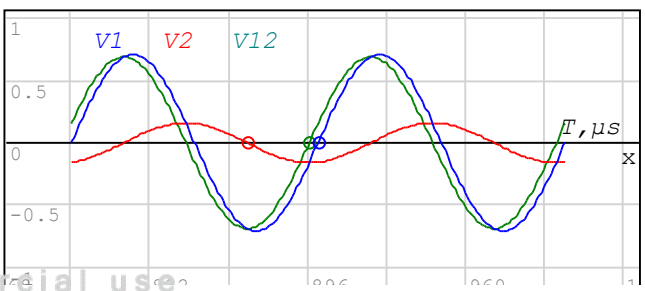
$v_2 := PP(V_2) = 0.3164\ V \qquad t_2 := find(T, V_2, 2) = 871.5\ \mu s$

$v_{12} := PP(V_{12}) = 1.3759\ V \qquad t_{12} := find(T, V_{12}, 2) = 896.5\ \mu s$

$\Delta t_2 := t_1 - t_2 = 29\ \mu s \qquad \Delta t_{12} := t_1 - t_{12} = 4\ \mu s$

$\varphi_2 := \Delta t_2 \cdot 2 \cdot \pi \cdot f = 104.4\ ^\circ \qquad \varphi_{12} := \Delta t_{12} \cdot 2 \cdot \pi \cdot f = 14.4\ ^\circ$

$\Pi :=$
- $\text{augment}(T\ MHz, V_1)$
- $\text{augment}(T\ MHz, V_2)$
- $\text{augment}(T\ MHz, V_{12})$
- $\text{augment}(t_1\ MHz, 0, "o", 10, "blue")$
- $\text{augment}(t_2\ MHz, 0, "o", 10, "red")$
- $\text{augment}(t_{12}\ MHz, 0, "o", 10, "green")$



$f := 2314\ \text{Hz}$

$circ := \text{strrep}(\text{description}(\chi Ver), \text{"V1s"}, \text{"PULSE(-0.8 0.8 0 1u 1u \{1/freq/2-1u\} \{1/freq\})"})$

$M := M(f)$ $\quad [T\ V_1\ V_2] := [\text{col}(M, 1)\ \text{s}\ \text{col}(M, 2)\ \text{V}\ \text{col}(M, 3)\ \text{V}]$ $\quad V_{12} := V_1 - V_2$ $\quad n := \text{rows}(M) = 239$

$v_1 := PP(V_1) = 1.6\ \text{V}$ $\quad t_1 := find(T, V_1, 2) = 3673.993\ \mu\text{s}$

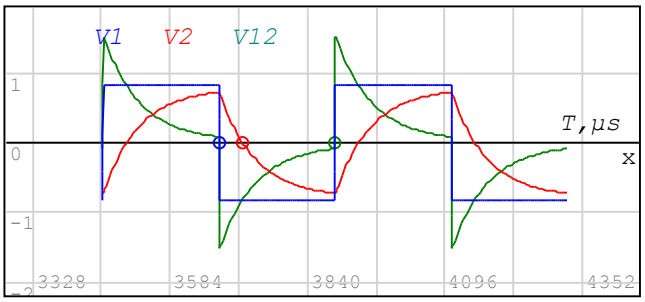
$v_2 := PP(V_2) = 1.4301\ \text{V}$ $\quad t_2 := find(T, V_2, 2) = 3719.008\ \mu\text{s}$

$v_{12} := PP(V_{12}) = 3.0087\ \text{V}$ $\quad t_{12} := find(T, V_{12}, 2) = 3889.4519\ \mu\text{s}$

$\Delta t_2 := t_1 - t_2 = -45.015\ \mu\text{s}$ $\quad \Delta t_{12} := t_1 - t_{12} = -215.4589\ \mu\text{s}$

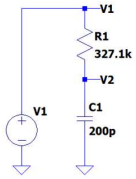
$\varphi_2 := \Delta t_2 \cdot 2 \cdot \pi \cdot f = -37.4993\ °$ $\quad \varphi_{12} := \Delta t_{12} \cdot 2 \cdot \pi \cdot f = -179.4858\ °$

$$\Pi := \begin{cases} \text{augment}(T\ \text{MHz}, V_1) \\ \text{augment}(T\ \text{MHz}, V_2) \\ \text{augment}(T\ \text{MHz}, V_{12}) \\ \text{augment}(t_1\ \text{MHz}, 0, \text{"o"}, 10, \text{"blue"}) \\ \text{augment}(t_2\ \text{MHz}, 0, \text{"o"}, 10, \text{"red"}) \\ \text{augment}(t_{12}\ \text{MHz}, 0, \text{"o"}, 10, \text{"green"}) \end{cases}$$



Symbolic MNA

Simple RC



V1	1	0
R1	1	2
C1	2	0

$\chi RC := \text{"}\chi\text{RC"}$

$[A\ B\ X\ V\ I] := MNA(\chi RC)$

$$A = \begin{bmatrix} \frac{1}{R1} & -\frac{1}{R1} & 1 \\ -\frac{1}{R1} & \frac{1 + s \cdot C1 \cdot R1}{R1} & 0 \\ 1 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ V1 \end{bmatrix} \qquad X = \begin{bmatrix} \text{V}(1) \\ \text{V}(2) \\ \text{I}(\text{V1}) \end{bmatrix}$$

$A \cdot B = X$

$$V = \begin{bmatrix} V1 \\ V1 \\ \frac{V1}{1 + s \cdot C1 \cdot R1} \end{bmatrix} \qquad I = \begin{bmatrix} -\frac{s \cdot C1 \cdot V1}{1 + s \cdot C1 \cdot R1} \end{bmatrix}$$

Versuch RC

$[A\ B\ X\ V\ I] := MNA(\chi s)$

$$V = \begin{bmatrix} -\frac{RT \cdot \left(s \cdot LK \cdot \left(\left(\left(RT \cdot \left(1 + s^2 \cdot CT \cdot LK\right) + s \cdot LK\right) \cdot \left(RT + R1 \cdot \left(1 + s \cdot RT \cdot (CT + C1)\right)\right) - \left(1 + s \cdot CT \cdot RT\right)^2 \cdot R1 \cdot s \cdot LK\right) \cdot RO + R1 \cdot \left(1 + s \cdot CT \cdot RT\right)^2 \cdot \left(-\left(RO + RK \cdot \right.\right.\right.\right.}{\left(RT + R1 \cdot \left(1 + s \cdot RT \cdot (CT + C1)\right)\right) \cdot \left(-\left(\left(RT \cdot \left(1 + s^2 \cdot CT \cdot LK\right) + s \cdot LK\right) \cdot \left(RT + R1 \cdot \left(1 + s \cdot RT \cdot (CT + C1)\right)\right) - \left(1 + s \cdot CT \cdot RT\right.\right.} \\ -\frac{}{-\left(\left(RT \cdot \left(1 + s^2 \cdot CT \cdot LK\right) + s \cdot LK\right) \cdot \left(RT + R1 \cdot \left(1 + s \cdot RT \cdot (CT + C1)\right)\right) - \left(1 + s \cdot CT \cdot RT\right)^2 \cdot R1 \cdot s \cdot LK\right) \cdot RO} \\ \frac{}{-\left(\left(RT \cdot \left(1 + s^2 \cdot CT \cdot LK\right) + s \cdot LK\right) \cdot \left(RT + R1 \cdot \left(1 + s \cdot RT \cdot (CT + C1)\right)\right) - \left(1 + s \cdot CT \cdot RT\right)^2 \cdot R1 \cdot s \cdot LK\right) \cdot RO} \\ \frac{}{\left(\left(RT \cdot \left(1 + s^2 \cdot CT \cdot LK\right) + s \cdot LK\right) \cdot \left(RT + R1 \cdot \left(1 + s \cdot RT \cdot (CT + C1)\right)\right) - \left(1 + s \cdot CT \cdot RT\right)^2 \cdot R1 \cdot s \cdot LK\right) \cdot RO} \end{bmatrix}$$

$$I = \left[-\frac{\left(-\left(\left(-\left(RO + RK\cdot\left(1 + s\cdot RO\cdot\left(CO + CK\right)\right)\right)\cdot\left(RK + s\cdot LK\right) + s\cdot LK\cdot RO\right)\cdot\left(\left(RT\cdot\left(1 + s^2\cdot CT\cdot LK\right) + s\cdot LK\right)\cdot\left(RT + R1\cdot\left(1 + s\cdot RT\cdot\left(CT + C1\right)\right)\right) - \left(1 + s\cdot CT\cdot RT\right)^2\cdot R1\cdot s\cdot LK\right) + \left(\right.\right.\right.}{\left(RT + R1\cdot\left(1 + s\cdot RT\cdot\left(CT + C1\right)\right)\right)\cdot\left(-\left(\left(RT\cdot\left(1 + s^2\cdot CT\cdot LK\right) + s\cdot LK\right)\cdot\left(RT + R1\cdot\left(1 + s\cdot RT\cdot\left(CT + C1\right)\right)\right) - \left(1 + s\cdot CT\cdot RT\right)^2\cdot R1\cdot s\cdot LK\right)\cdot RO\cdot s\cdot LK + \left(RO + RK\right.\right.\right.}$$

Impledance method

$\text{Clear}(f) = 1$ $\quad s := \sigma + i\cdot 2\cdot\pi\cdot f$ $\quad \sigma := 0$ $\quad V1 := V1(t, f)$

$ZC1 := \frac{1}{s\cdot C1}$ $\quad ZCK := \frac{1}{s\cdot CK}$ $\quad ZCT := \frac{1}{s\cdot CT}$ $\quad ZCO := \frac{1}{s\cdot CO}$ $\quad ZLK := s\cdot LK$

$$ZCH(f) := \frac{RO\cdot ZCO}{RO + ZCO} + \frac{(RK + ZLK)\cdot ZCK}{RK + ZLK + ZCK} + \frac{RT\cdot ZCT}{RT + ZCT}$$

$$V2zV1(f) := \text{eval}\left(\frac{ZC1\cdot ZCH(f)}{R1\cdot(ZC1 + ZCH(f)) + ZC1\cdot ZCH(f)}\right)$$

$V2(t, f) := V1\cdot V2zV1(f)$ Using MNA for comparing later

$V12(t, f) := V1 - V2(t, f)$ $\quad V2_m(t, f) := V_2$

$\text{Clear}(s) = 1$

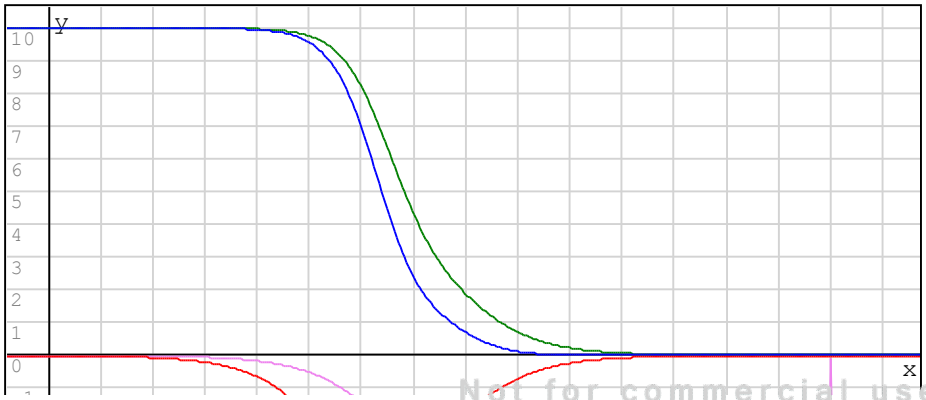
Values

Values

$R1 := 327.1\ k\Omega$	$C1 := 200\ pF$	$Vo := 0.5\sqrt{2}\ V$
$RT := 9\ M\Omega$	$CT := 12.78\ pF$	$\omega o := 2\cdot\pi\cdot f$
$RK := 1.5\ \Omega$	$CK := 100\ pF$	$LK := 250\ nH$
$RO := 1\ M\Omega$	$CO := 15\ pF$	$\varphi v := -90\ °$

freq analysis

$$\Pi(f(s), uy, ug) := \begin{cases} \text{Re}\left(f\left(10^{x}\ Hz\right)\right)\cdot uy \\ \text{Im}\left(f\left(10^{x}\ Hz\right)\right)\cdot uy \\ \left|f\left(10^{x}\ Hz\right)\right|\cdot uy \\ \arg\left(f\left(10^{x}\ Hz\right)\right)\cdot ug \end{cases}$$



Unstetigkeit bei x = 7,5 ?

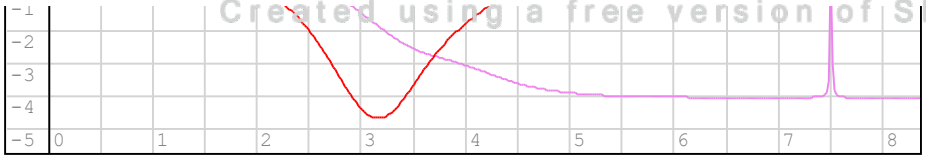
$\arg\left(ZCH\left(10^{7}\ Hz\right)\right) = -1.5694$

$\arg\left(ZCH\left(10^{7.5}\ Hz\right)\right) = -0.1121$

$\arg\left(ZCH\left(10^{8}\ Hz\right)\right) = -1.5707$

Bei welcher Frequenz ist ZCH = 5 ?

$\text{Re}\left(ZCH\left(1500\ Hz\right)\right) = 5118.0751\ k\Omega$

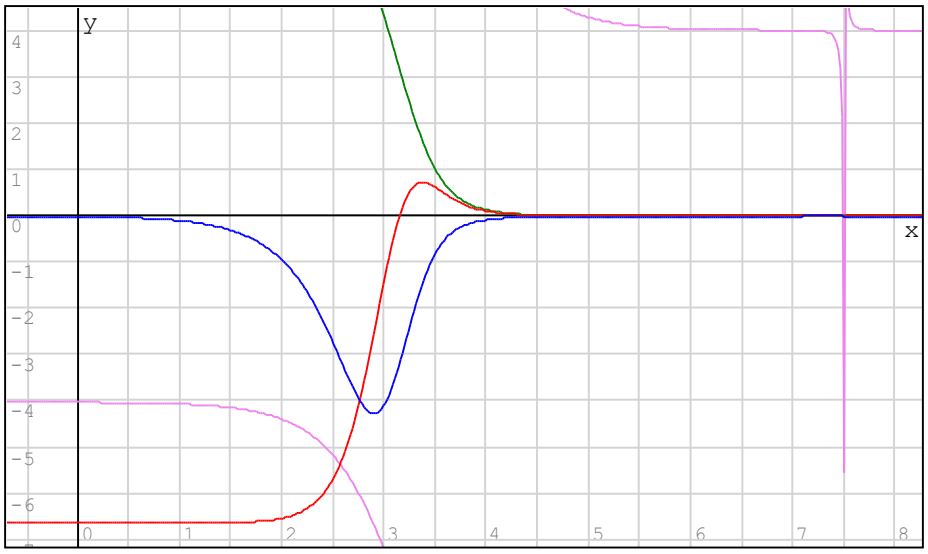


$\Pi\left(ZCH(\varphi),\ M\Omega^{-1},\ 8\cdot\pi^{-1}\right)$

$\mathrm{Re}\left(ZCH(1640\ Hz)\right)=4999.6537\ k\Omega$

$\mathrm{Re}\left(ZCH(1600\ Hz)\right)=4828.7825\ k\Omega$

$dZCH(f):=\frac{d}{d\,f}ZCH(f)$



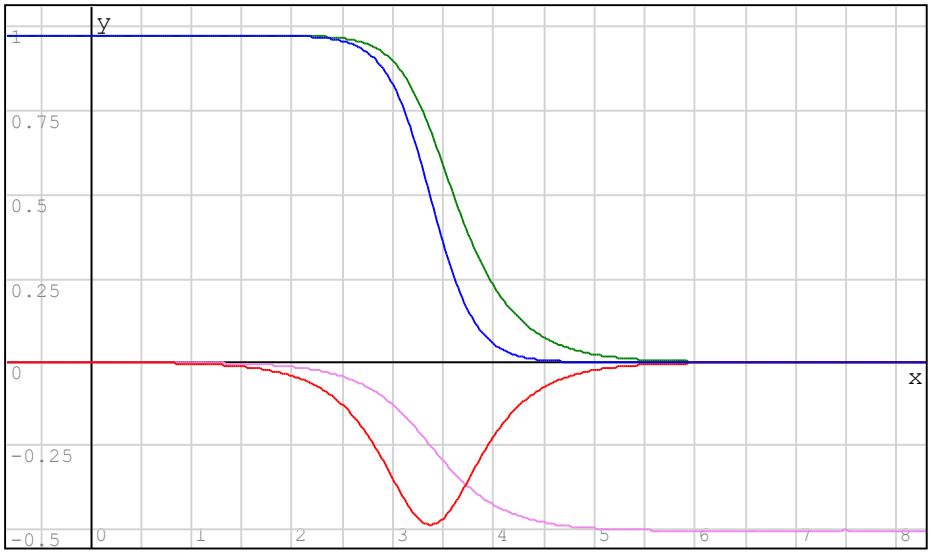
$\Pi\left(dZCH(\varphi),\ k\Omega^{-1},\ 8\cdot\pi^{-1}\right)$

Nullstelle von Im(ZCH(f))

$\mathrm{Im}\left(dZCH(1422\ Hz)\right)=-3.0036\ \Omega\ s$

$\mathrm{Im}\left(dZCH(1423\ Hz)\right)=-0.8387\ \Omega\ s$

$\mathrm{Im}\left(dZCH(1424\ Hz)\right)=1.3214\ \Omega\ s$



$\Pi\left(V2zV1(\varphi),\ 1,\ 1\cdot\pi^{-1}\right)$

Wo ist die Phase -45° ?

$\arg\left(V2zV1(2.4\ kHz)\right)=-44.9317\ °$

$\arg\left(V2zV1(2.4058\ kHz)\right)=-44.9999\ °$

$\arg\left(V2zV1(2.5\ kHz)\right)=-46.0845\ °$

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sine

Sine wave $f:=2314\ Hz$ $V1(t,f):=Vo\cdot e^{i\cdot(2\cdot\pi\cdot f\cdot t+\varphi v)}$

Discretizing for plots $T:=\frac{[1..60]-1}{60-1}\cdot\frac{2.5}{f}$

Using spice# $mode:=".TRAN\ 0\ \{4/freq\}\ \{0/freq\}\ \{0.01/freq\}"$

$circ := \text{strrep}(\text{description}(\chi Ver), \text{"V1s"}, \text{"SINE(0 1 1k)"})$

$M := M(f)$ $\text{rows}(M) = 430$

Using MNA with numerical ILT $V1 := \dfrac{Vo \cdot 2 \cdot \pi \cdot f}{s^2 + (2 \cdot \pi \cdot f)^2}$ LT of V1 $V_{2\mathcal{L}} := \mathcal{IL}\left(V_2, T_{[1..20]}\right)$ Inverse LT of V2. Very slow, use some few points

Usint MNA with phasors $V2_m(t, f)$ from above

Comparing V2 with the 4 methods

$$\Pi := \begin{cases} \text{augment}\left(T \cdot f, \overrightarrow{\text{Re}(V1(T, f))}\right) \\ \text{augment}\left(T \cdot f, \overrightarrow{\text{Re}(V2(T, f))}\right) \\ \text{augment}\left(T \cdot f, \overrightarrow{\text{Re}(V2_m(T, f))}, \text{"o"}, 8, \text{"red"}\right) \\ \text{augment}\left(\text{col}(M, 1) \cdot f, \text{col}(M, 3)\right) \\ \text{augment}\left(T_{[1..20]} \cdot f, V_{2\mathcal{L}}, \text{"o"}, 8\right) \end{cases}$$

$\arg(V1(0\,s, f)) = -90\,°$

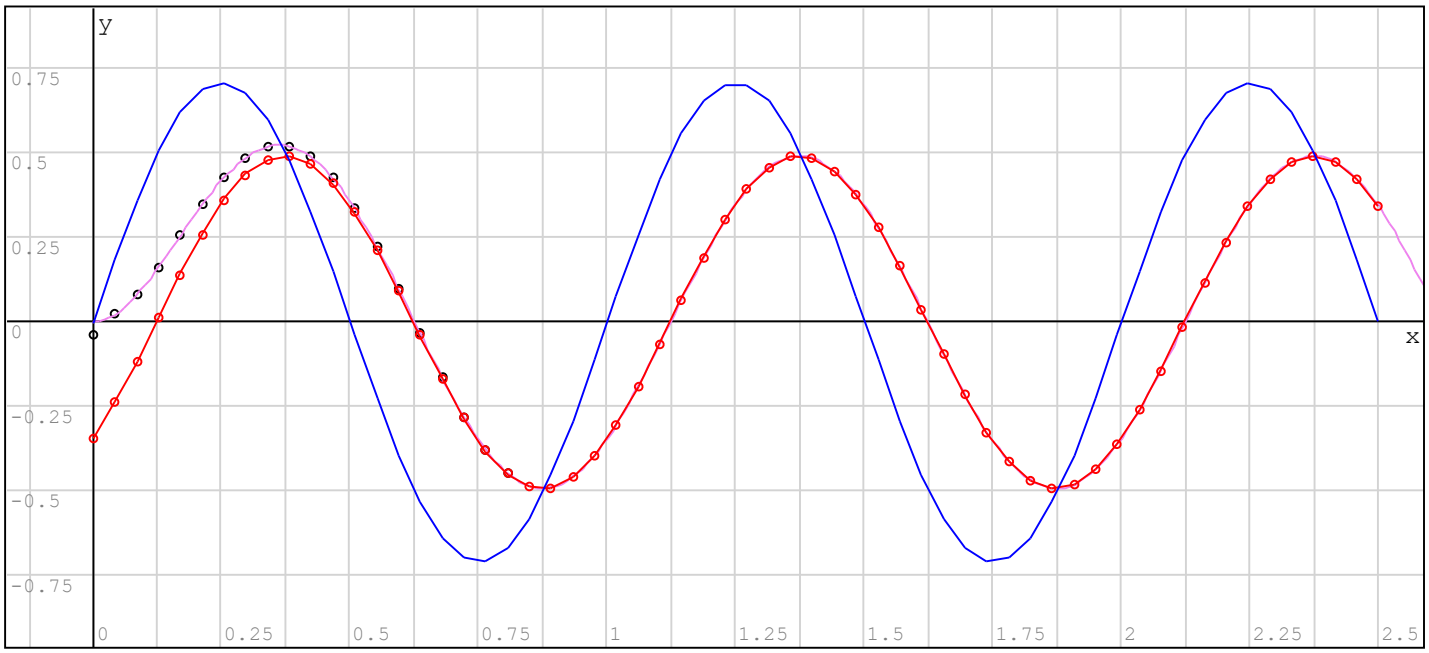
$\arg(V2(0\,s, f)) = -133.9004\,°$

$\arg(V2(0\,s, f)) - \arg(V1(0\,s, f)) = -43.9004\,°$

$\arg(V12(0\,s, f)) = -46.0905\,°$

$\arg(V12(0\,s, f)) - \arg(V1(0\,s, f)) = 43.9095\,°$

Note: the phasors method returns only the stationary solution



pulse

Pulse wave $f := 2314\ \text{Hz}$ $V1(t, f) := Vo \cdot \text{sign}(\sin(2 \cdot \pi \cdot f \cdot t))$

Discretizing for plots $T := \dfrac{[1..200] - 1}{200 - 1} \cdot \dfrac{2.5}{f} + \dfrac{4}{f}$ $T_{\mathcal{L}} := \dfrac{[1..15] - 1}{15 - 1} \cdot \dfrac{1}{f} + \dfrac{4}{f}$

Using spice# $mode := \text{".TRAN 0 \{8/freq\} \{4/freq\} \{0.01/freq\}"}$

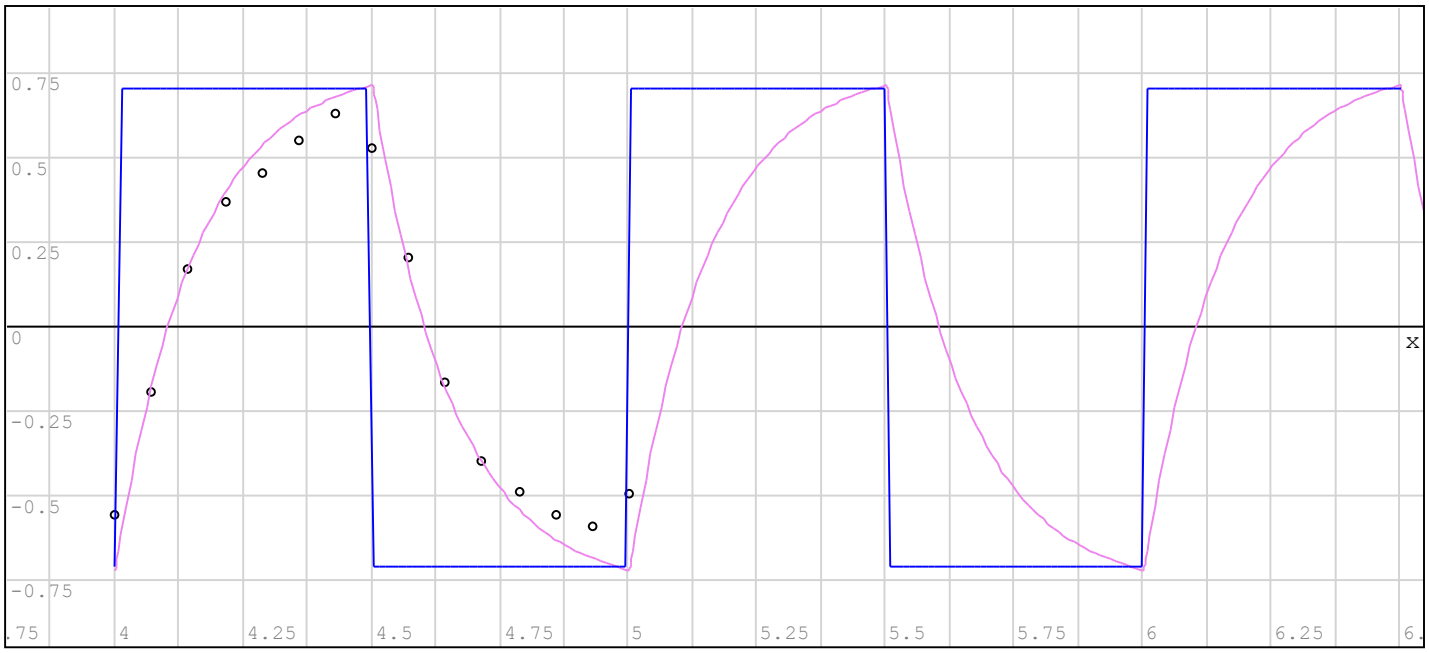
$circ := \text{strrep}(\text{description}(\chi Ver), \text{"V1s"}, \text{"PULSE(-0.8 0.8 0 1u 1u \{1/freq/2-1u\} \{1/freq\})"})$

$M := M(f)$ $\text{rows}(M) = 477$

Inverting the LT Vo ( ) $V_{\mathcal{L}} := \mathcal{IL}(V, T_{\mathcal{L}})$ Very slow and not very accurate

$$V1(T,f) := \frac{1}{s}\cdot\tanh\left(\frac{1}{4\cdot f}\right)$$

$$\Pi := \begin{cases} \text{augment}\left(T\cdot f, \overrightarrow{V1(T,f)}\right) \\ \text{"to do"} \\ \text{"to do"} \\ \text{augment}\left(\text{col}(M,1)\cdot f, \text{col}(M,3)\right) \\ \text{augment}\left(T_{2\mathcal{L}}\cdot f, V_{2\mathcal{L}}, \text{"o"}, 8\right) \end{cases}$$



Alvaro

$$\text{appVersion}(4) = \text{"1.73.9126.0"}$$