⊞—Utilities

⊞—bvp.2

Boundary Values Problems for Beams

$lbvp_2$ $lbvp_2(f(x), x, \alpha\beta, N)$ solves the linear ODE $y'' + p \cdot y' + q \cdot y = r$

subject to the boundary conditions
$$\begin{cases} \alpha_1 \cdot y(a) + \beta_1 \cdot y'(a) = c_1 \\ \alpha_2 \cdot y(b) + \beta_2 \cdot y'(b) = c_2 \end{cases} \quad \text{in} \quad x = [a \;\; b]$$

f is such that $f(x) = [p(x) \;\; q(x) \;\; r(x)]$ and $\alpha\beta = \begin{bmatrix} \alpha 1 & \beta 1 & c1 \\ \alpha 2 & \beta 2 & c2 \end{bmatrix}$

bvp_2 $bvp_2(\varphi(x, y, y'), x, Yo, \alpha\beta, N, \varepsilon)$ solves the non linear ODE

$$y'' = \varphi(x, y, y') \quad \text{in} \quad x = [a \;\; b]$$

subject to the the same boundary conditions, with Yo as guess for the solution with dimension N+1.. If Yo=0, bvp.2 try with a line between f(a) and f(b) as guess.. ε is used as the tolerance for a Newton solver.

From the free body diagram, we try to write

$$y'' = \frac{M}{E \cdot I}$$

Defaults $[N \;\; \varepsilon] := [50 \;\; 10^{-9}]$

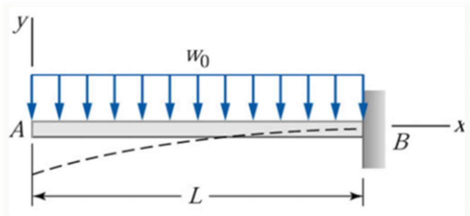
Short hands for plots

$$Plot(X, Y, Y', xm) := \left\| \begin{array}{l} y(x\,\mathrm{m}) \cdot (0 \le x\,\mathrm{m} \le L) \\ y'(x\,\mathrm{m}) \cdot (0 \le x\,\mathrm{m} \le L) \\ \text{augment}\left(\frac{X}{\mathrm{m}}, Y, ".", 6, "blue"\right) \\ \text{augment}\left(\frac{X}{\mathrm{m}}, Y', ".", 6, "red"\right) \\ \text{augment}\left(xm, \begin{bmatrix} y(xm) \\ y'(xm) \end{bmatrix}, "o"\right) \end{array} \right.$$

⊟—Example beam

Example http://web.ncyu.edu.tw/~lanjc/lesson/C3/class/Chap06-A.pdf

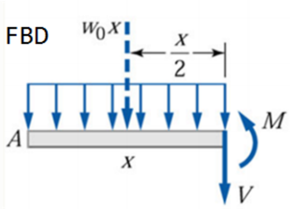
Cantilever beam with uniformly distributed load



$wo := 400 \frac{\mathrm{lbf}}{\mathrm{ft}}$ $L := 8\ \mathrm{ft}$ $E := 29 \cdot 10^6\ \mathrm{psi}$

$I := 285\ \mathrm{in}^4$ (W12×35 shape)

$$y''(x) := -\frac{wo}{2 \cdot E \cdot I} \cdot x^2$$



Using RK Method $D(x, y) := \begin{bmatrix} y_2 \\ y''(x) \end{bmatrix}$ IC: $y_0 := 0$ $y'_0 := 0$

$[\mathrm{m} \;\; \mathrm{kg} \;\; \mathrm{s}] := [1 \;\; 1 \;\; 1]$ Sterilize units

$$[X \;\; Y \;\; Y'] := Cols\left(\text{rkfixed}\left(\text{stack}\left(y_0, y'_0\right), L, 0, N-1, D\right)\right)$$

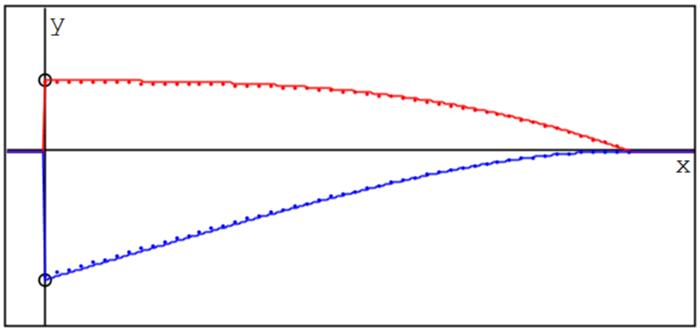
$\text{Clear}(\mathrm{m}, \mathrm{kg}, \mathrm{s}) = 1$ Restore units $[X \;\; Y \;\; Y'] := [X\,\mathrm{m} \;\; Y\,\mathrm{m} \;\; Y']$

Exact solution $y(x) := \frac{wo}{24 \cdot E \cdot I} \cdot \left(-x^4 + 4 \cdot L^3 \cdot x - 3 \cdot L^4\right)$ $y'(x) := \frac{d}{dx} y(x)$

Maximun deflection

$Y_N = -0.0417\ \text{in}$ or

$\boxed{y(0) = -0.0428\ \text{in}}$



Plot (X, Y, Y', 0)

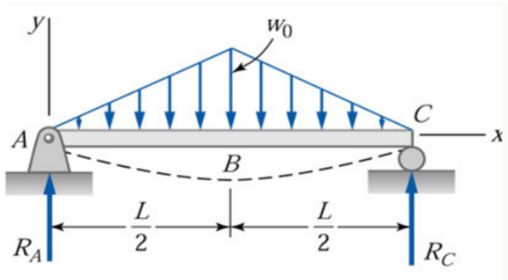
⊟— Example beam

Example http://web.ncyu.edu.tw/~lanjc/lesson/C3/class/Chap06-A.pdf

Simple supported beam with distributed load
distributed load of maximum intensity wo

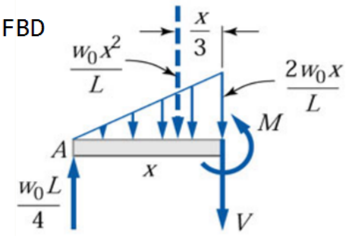
$wo := 400\ \dfrac{\text{lbf}}{\text{ft}}$ $L := 8\ \text{ft}$ $E := 29 \cdot 10^6\ \text{psi}$

$I := 285\ \text{in}^4$ (W12×35 shape)



$R_A := wo \cdot \dfrac{L}{4}$ $R_C := R_A$

FBD



$y''(x) := \dfrac{wo}{12 \cdot E \cdot I} \cdot \dfrac{3 \cdot L^2 \cdot x - 4 \cdot x^3}{L}$

$[\ m\ kg\ s\ ] := [\ 1\ 1\ 1\ ]$ Sterilize units

Using bvp.2 $\varphi(x, y, y') := y''(x)$ $\begin{cases} y(0) = 0 \\ y(L) = 0 \end{cases}$ -> $\alpha\beta := \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$[\ X\ Y\ Y'\ ] := Cols\left(bvp_2\left(\varphi(x, y, y'), [\ 0\ L\ ], 0, \alpha\beta, N-1, \varepsilon\right)\right)$ or (better):

Using lbvp.2 $f(x) := [\ 0\ 0\ y''(x)\ ]$ $[\ X\ Y\ Y'\ ] := Cols\left(lbvp_2\left(f(x), [\ 0\ L\ ], \alpha\beta, N-1\right)\right)$

$Clear(m, kg, s) = 1$ Restore units $[\ X\ Y\ Y'\ ] := [\ X\ m\ Y\ m\ Y'\ ]$

Exact solution $y(x) := -\dfrac{wo \cdot x}{960 \cdot L \cdot E \cdot I} \cdot \left(25 \cdot L^4 - 40 \cdot L^2 \cdot x^2 + 16 \cdot x^4\right)$ $y'(x) := \dfrac{d}{d\,x} y(x)$

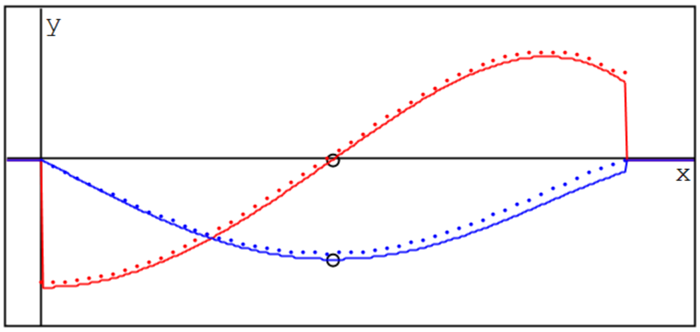
Maximun deflection

$CInterp(X, Y, 0.5 \cdot L) = -0.068\ \text{mm}$

or

$\boxed{y(0.5 \cdot L) = -0.0725\ \text{mm}}$

Only an approximate result.



Plot (X, Y, Y', 0.5·L)

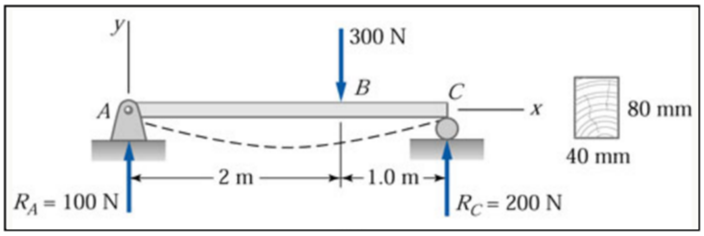
Example beam

Example http://web.ncyu.edu.tw/~lanjc/lesson/C3/class/Chap06-A.pdf

$E := 12\ \text{GPa}$ $Q := 300\ \text{N}$

$x_B := 2\ \text{m}$ $L := 3\ \text{m}$

$$I := \frac{40\ \text{mm} \cdot (80\ \text{mm})^3}{12}$$



$$y''(x) := \begin{cases} \frac{1}{3} \cdot \frac{Q}{E \cdot I} \cdot x & \text{if } x \le x_B \\ \frac{1}{3} \cdot \frac{Q}{E \cdot I} \cdot x - \frac{Q}{E \cdot I} \cdot (x - x_B) & \text{otherwise} \end{cases}$$

$[\text{m}\ \text{kg}\ \text{s}] := [1\ 1\ 1]$ Sterilize units

Using bvp.2 $\varphi(x, y, y') := y''(x)$ $\begin{cases} y(0) = 0 \\ y(L) = 0 \end{cases}$ -> $\alpha\beta := \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$$[X\ Y\ Y'] := Cols(bvp_2(\varphi(x, y, y'), [0\ L], 0, \alpha\beta, N-1, \varepsilon))$$

Using shooting method

$D(x, y) := \begin{bmatrix} y_2 \\ y''(x) \end{bmatrix}$ IC: $y_0 := 0$ $y'_0 := -0.1$ Guess

BC: $y_L := 0$

$sol(y'_0) := \text{rkfixed}(\text{stack}(y_0, y'_0), 0, L, N-1, D)$ $Eq(y'_0) := sol(y'_0)_{N\,2} - y_L$

$y'_0 := S_{NR}(Eq, y'_0)$ $[\Xi\ \Psi\ \Psi'] := Cols(sol(y'_0))$

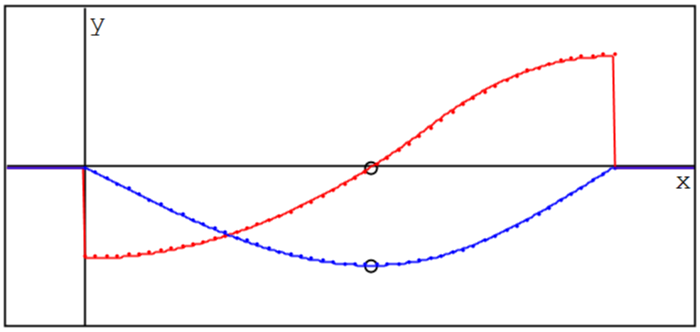
$\text{Clear}(\text{m}, \text{kg}, \text{s}) = 1$ Restore units $[\Xi\ \Psi\ \Psi'] := [\Xi\ \text{m}\ \Psi\ \text{m}\ \Psi']$

$y(x) := CInterp(\Xi, \Psi, x)$ $y'(x) := CInterp(\Xi, \Psi', x)$

Maximun deflection

$x_{max} := S_{NR}(y', x_B) = 1.621\ \text{m}$

$y(x_{max}) = -7.1271\ \text{mm}$

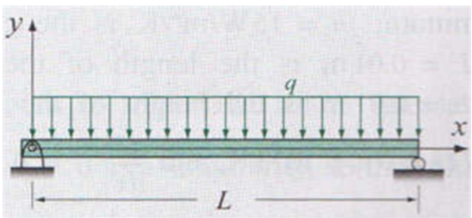


$Plot(X, Y, Y', x_{max})$

Example

Example https://edu.epito.bme.hu/local/coursepublicity/mod/resource/view.php?id=59032

$L := 4\ \text{m}$ $EI := 1.4 \cdot 10^7\ \frac{\text{N}}{\text{m}^2}$ $q := 10 \cdot 10^3\ \frac{\text{N}}{\text{m}}$



$$y''(x) := \frac{1}{2 \cdot EI} \cdot \sqrt{\left(1 + y'^2\right)^3} \cdot q \cdot \left(L \cdot x - x^2\right)$$

$[m\ kg\ s] := [1\ 1\ 1]$ Sterilize units

Using bvp.2

$\varphi(x, y, y') := y''(x)$ $\begin{cases} y(0) = 0 \\ y'(L) = 0 \end{cases}$ $M := \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$[X\ Y\ Y'] := Cols\left(bvp_2\left(\varphi(x, y, y'), [0\ L], 0, M, N-1, \varepsilon\right)\right)$

Using shooting method

$D(x, y) := \begin{bmatrix} y_2 \\ \varphi(x, y_1, y_2) \end{bmatrix}$ $[y_0\ y_L] := [0\ 0]$ $y'_0 := 1$ Guess

$sol(y'_0) := \text{rkfixed}\left(\begin{bmatrix} y_0 \\ y'_0 \end{bmatrix}, 0, L, N-1, D\right)$ $Eq(y'_0) := sol(y'_0)_{N\,2} - y_L$

$y'_0 := S_{NR}(Eq, y'_0)$ $[\Xi\ \Psi\ \Psi'] := Cols(sol(y'_0))$

Clear(m, kg, s) = 1 Restore units $[\Xi\ \Psi\ \Psi'] := [\Xi\ m\ \Psi\ m\ \Psi']$

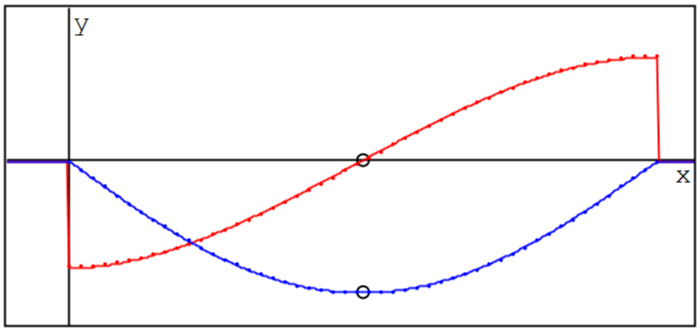
$y(x) := CInterp(\Xi, \Psi, x)$ $y'(x) := CInterp(\Xi, \Psi', x)$

Maximun deflection

$x_{max} := S_{NR}(y', 0.4 \cdot L) = 1.9864\ m$

$y(x_{max}) = -2.3795\ mm$

Clear(m, kg, s) = 1 Restore units



$Plot(X, Y, Y', 0.5 \cdot L)$

Alvaro