Arenstorf orbit

The Arenstorf orbits are closed trajectories of the restricted three-body problem. Two bodies of masses m and (1-m) moving in a circular rotation, and a third body of negligible mass moving in the same plane (such as a satellite-earth-moon system.)

Differential equations

Special periodic solutions (Arenstorf, 1963)

$y_1, y_2 = ?$

$$\mu = \frac{m_1}{m_1 + m_2}$$

$$D_1 = \left((y_1 + \mu)^2 + y_2^2\right)^{3/2}$$
$$D_2 = \left((y_1 - (1-\mu))^2 + y_2^2\right)^{3/2}$$

$$y_1'' = y_1 + 2y_2' - (1-\mu)\frac{y_1 + \mu}{D_1} - \mu\frac{y_1 - (1-\mu)}{D_2}$$
$$y_2'' = y_2 - 2y_1' - y_2\left(\frac{1-\mu}{D_1} + \frac{\mu}{D_2}\right)$$

```
D(t , y , μ) :=
    μp := 1 - μ
    y1 := y_3
    y2 := y_4
    r1 := (y_1 + μ)^2 + y_2^2
    r1 := r1 · √r1
    r2 := (y_1 - μp)^2 + y_2^2
    r2 := r2 · √r2
    y3 := y_1 + 2 · y_4 - (μp / r1) · (y_1 + μ) - (μ / r2) · (y_1 - μp)
    y4 := y_2 · (1 - μp / r1 - μ / r2) - 2 · y_3
    stack( y1 , y2 , y3 , y4 )
```

4 loops:
Mass parameter: m = 0.012277471

Coordinates:
y1(0) = 0.994
y2(0) = 0
y'1(0) = 0
y'2(0) = -2.00158510637908252240537862224

Period:
tper = 17.065216560157962558891720624 9

$$Y := \begin{pmatrix} 0.994 \\ 0 \\ 0 \\ -2.00158510637908252240537862224 \end{pmatrix} \qquad \mu := 0.012277471$$

$t0 := 0 \quad n := 200 \qquad tmax := 17.065216560157962558891720624 9$

$res := mwode45(Y, t0, tmax, n, D(t, y, \mu))$

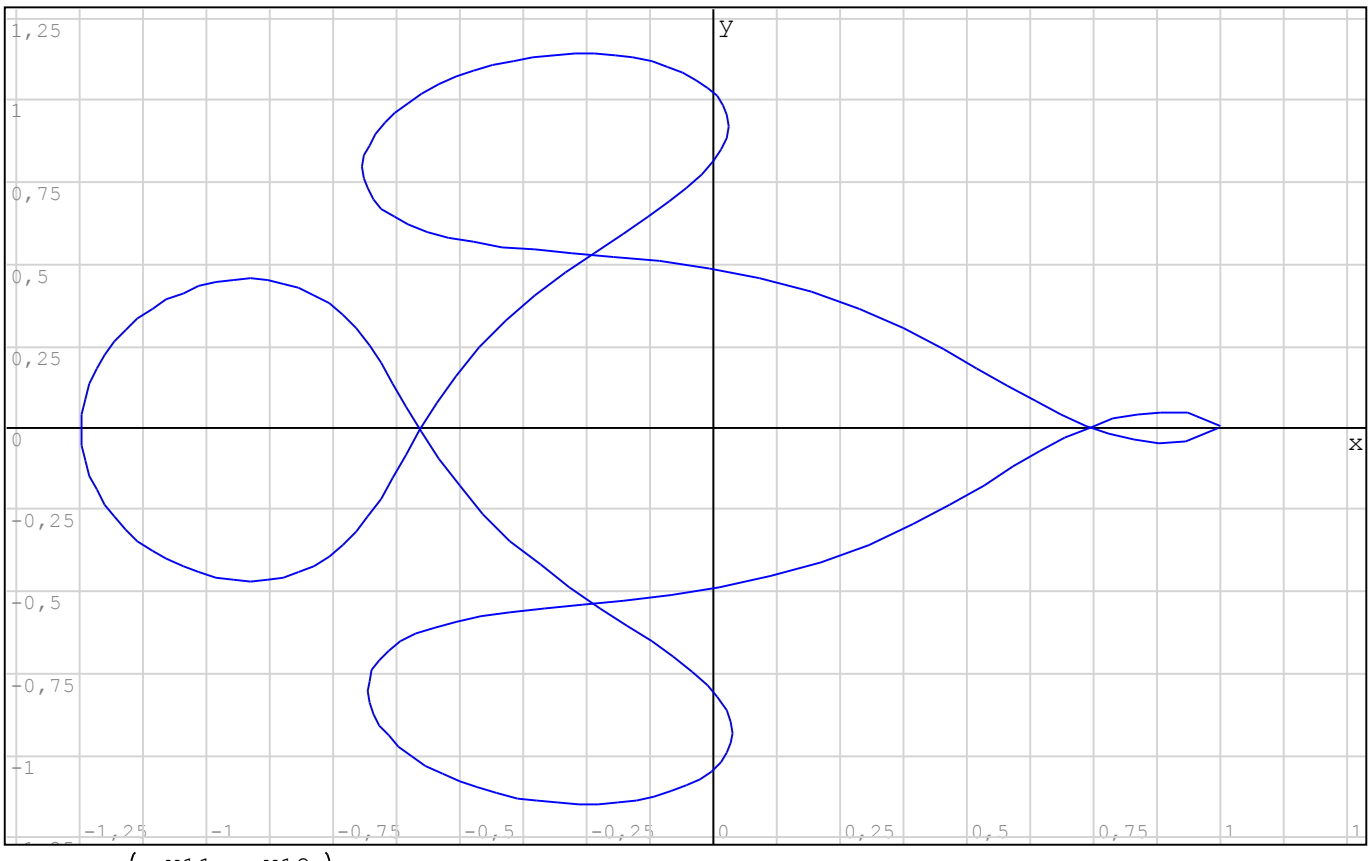
$res := mwode23(Y, t0, tmax, n, D(t, y, \mu))$

$res := mwode113(Y, t0, tmax, n, D(t, y, \mu))$

$res := mwode15s(Y, t0, tmax, n, D(t, y, \mu))$

$res := mwode23s(Y, t0, tmax, n, D(t, y, \mu))$

$T := col(res, 1) \quad Y11 := col(res, 2) \quad Y12 := col(res, 3)$



augment(Y11, Y12)

3 loops:
Mass parameter: m = 0.012277471

Coordinates:
y1(0) = 0.994
y2(0) = 0
y'1(0) = 0
y'2(0) = -2.031732629557336835730205792 4

Period:
tper = 11.124340337266085134999734047

$$Y := \begin{pmatrix} 0.994 \\ 0 \\ 0 \\ -2.031732629557336835730205792 4 \end{pmatrix} \qquad \mu := 0.012277471$$

$t0 := 0 \quad n := 200 \qquad tmax := 11.124340337266085134999734047$

res := mwode45(Y , t0 , tmax , n , D(t , y , μ))

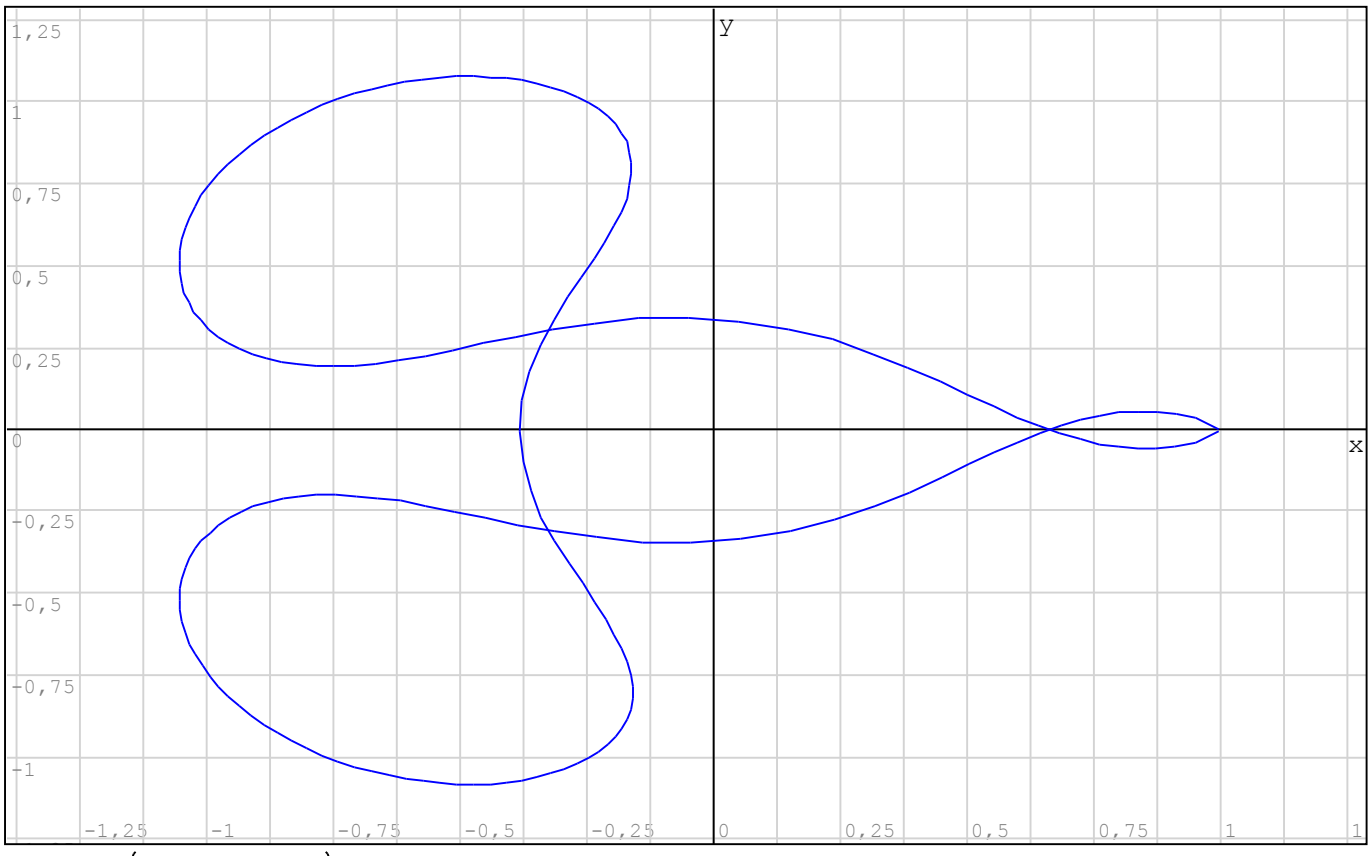
res := mwode23(Y , t0 , tmax , n , D(t , y , μ))

res := mwode113(Y , t0 , tmax , n , D(t , y , μ))

res := mwode15s(Y , t0 , tmax , n , D(t , y , μ))

res := mwode23s(Y , t0 , tmax , n , D(t , y , μ))

T := col(res , 1) Y11 := col(res , 2) Y12 := col(res , 3)



augment(Y11 , Y12)

```
2 loops:
Mass parameter:   m = 0.012277471

Coordinates:
y1(0)  = 1.2
y2(0)  = 0
y'1(0)  = 0
y'2(0)  = -1.049357510
Period:
tper  = 6.192169331
```

$$Y := \begin{pmatrix} 1.2 \\ 0 \\ 0 \\ -1.049357510 \end{pmatrix} \qquad t0 := 0 \quad n := 200 \quad tmax := 6.192169331 \qquad \mu := 0.012277471$$

res := mwode45(Y , t0 , tmax , n , D(t , y , μ))

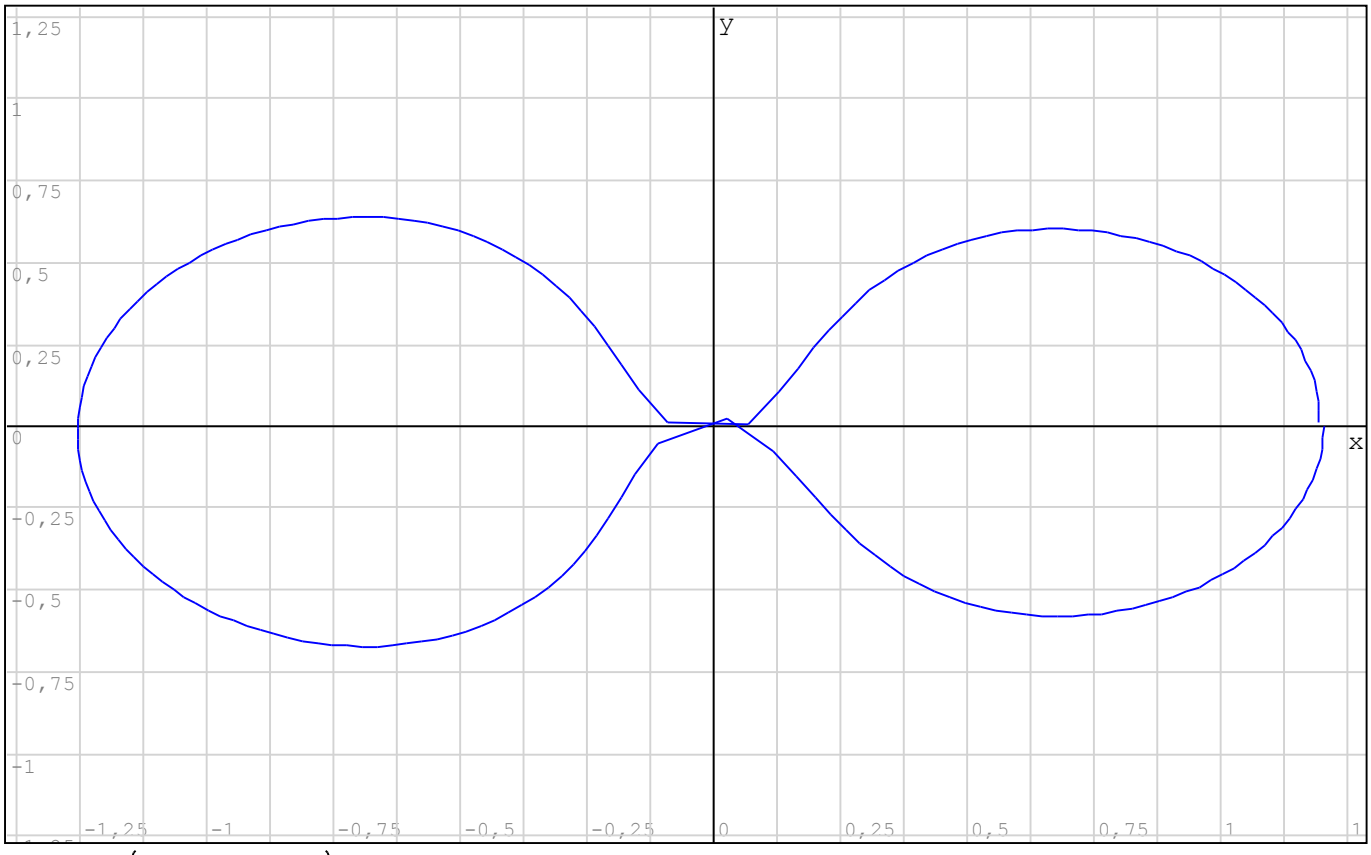
res := mwode23(Y , t0 , tmax , n , D(t , y , μ))

res := mwode113(Y , t0 , tmax , n , D(t , y , μ))

res := mwode15s(Y , t0 , tmax , n , D(t , y , μ))

res := mwode23s(Y , t0 , tmax , n , D(t , y , μ))

T := col(res , 1) Y11 := col(res , 2) Y12 := col(res , 3)



augment(Y11 , Y12)