⊞—RK23

⊟—Non Lineal 2nd degree ODE

Values

$V := 20\ \frac{m}{s}$ $\quad a := 50\ m$ $\quad b := 100\ m$ $\quad r := \sqrt{\frac{a^2 + b^2}{2}}$ $\quad \omega := \frac{V}{r} = 0.253\ Hz$

$m := 189\ kg$ $\quad h_G := 0.53\ m$ $\quad b_G := 0.703\ m$ $\quad g := 9.8\ \frac{m}{s^2}$

$I_{xG} := 6.73\ kg\ m^2$ $\quad I_{zG} := 36.4\ kg\ m^2$

Equations

$$\begin{cases} x(t) := \left(a - a \cdot \sin\left(\omega \cdot t + \frac{\pi}{2}\right)\right) \\ y(t) := b \cdot \sin(\omega \cdot t) \end{cases}$$

$$c(t) := \frac{\left| \frac{d}{dt} x(t) \cdot \frac{d^2}{dt^2} y(t) - \frac{d}{dt} y(t) \cdot \frac{d^2}{dt^2} x(t) \right|}{\sqrt{\left(\left(\frac{d}{dt} x(t)\right)^2 + \left(\frac{d}{dt} y(t)\right)^2 \right)^3}}$$

$$R_c(t) := \frac{1}{c(t)}$$

ODE

$$m \cdot V^2 \cdot \frac{h_G}{R_c} \cdot \cos(\varphi) - m \cdot g \cdot h_G \cdot \sin(\varphi) - \left(I_{xG} + m \cdot h_G^2\right) \cdot \varphi'' = 0$$

For use the power of the linear algebra tools, numerical methods ask for convert the equation as a system of equations where each element is the derivative of the unknow function: $\varphi(t) = \varphi_1$, $\varphi'(t) = \varphi_2$ and $\varphi''(t) = \varphi_2$.

$$D(t, \varphi) := \begin{bmatrix} \varphi_2 \\ \dfrac{m \cdot V^2 \cdot \frac{h_G}{R_c(t)} \cdot \cos\left(\varphi_1\right) - m \cdot g \cdot h_G \cdot \sin\left(\varphi_1\right)}{I_{xG} + m \cdot h_G^2} \end{bmatrix}$$

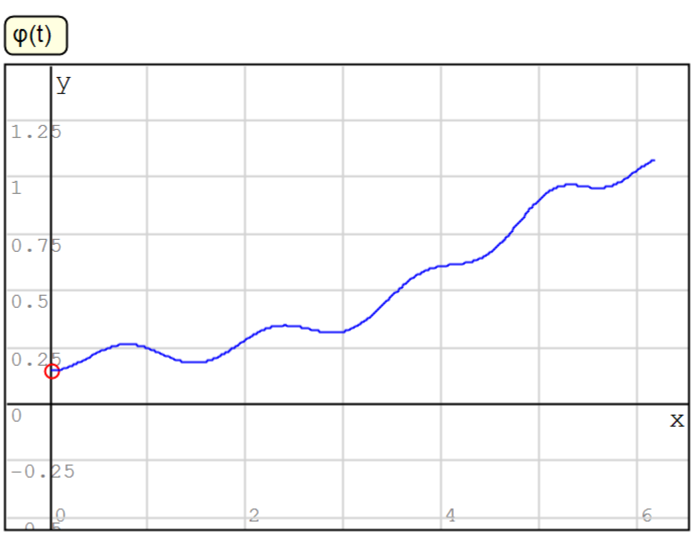
$$\begin{bmatrix} t_o & t_{end} \end{bmatrix} := \begin{bmatrix} 0 & 6.187 \end{bmatrix} s$$

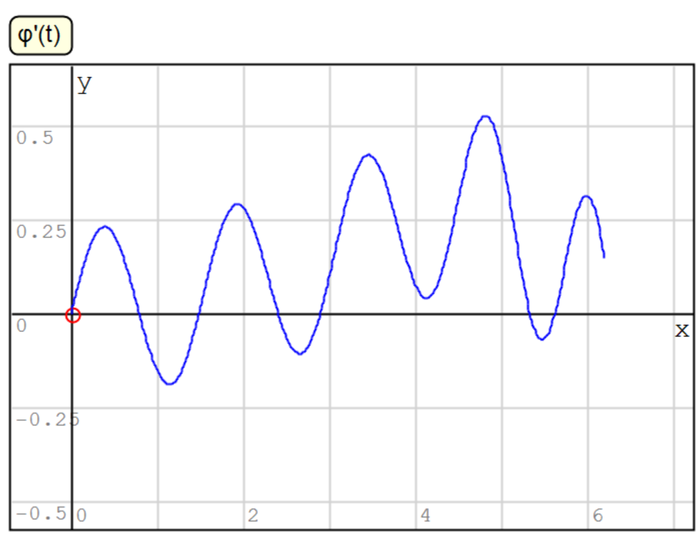
$$\begin{bmatrix} \varphi_o & \varphi'_o \end{bmatrix} := \begin{bmatrix} 0.146\ rad & 0\ \frac{rad}{s} \end{bmatrix}$$

For an initual value (or Cauchy) problem, you solve the problem calling

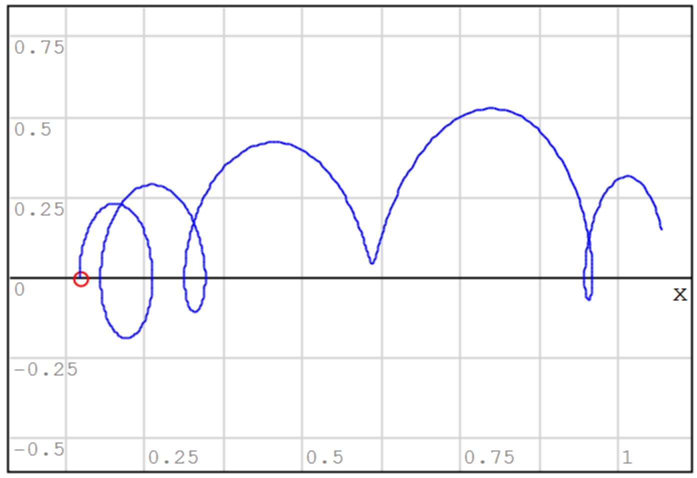
$$\varphi sol := RK23\left(D(t, \varphi), \begin{bmatrix} t_o & t_{end} \end{bmatrix}, \begin{bmatrix} \varphi_o & \varphi'_o \end{bmatrix}, 50, 10^{-5}\right)$$

$$\text{rows}(\varphi sol) = 118$$





State space



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