Internet: Example 3

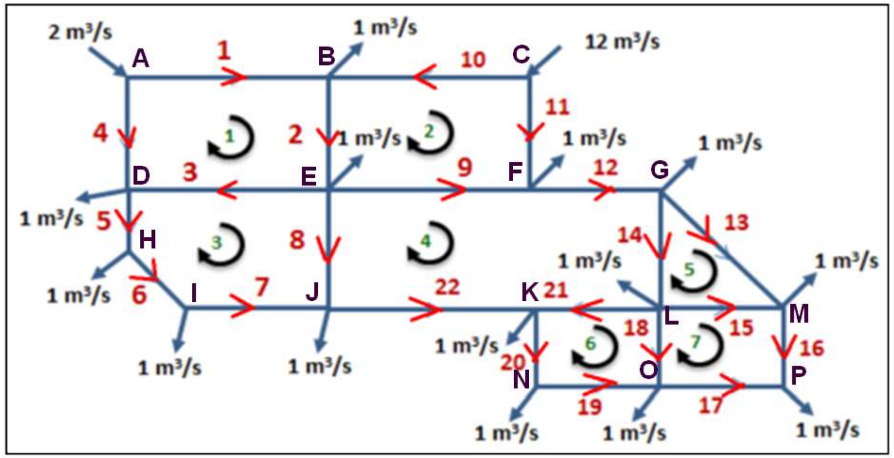
Chemical Engineer's Guide :

Solve for water piping network as shown in diagram. Hazen-Williams coefficient, C for each pipe is provided in table below.

Note: This is an example given in the link below, solved by Hardy Cross method using EXCEL.
Essentially it is an **overdetermined nonlinear** pipe network with **22 variables** {***pipe flows***) and **23 equations** (***16 Nodes + 7 Loops***).
Hence, the solution using SMath is based on a snippet by Viacheslav N. Mezentsev (uni) in SMath Forum.

https://cheguide.com/pipe_network.html

$t_0 := \text{time}(0)$



Pipe	Diameter mm	Length m	C
1	1000	1000	100
2	750	925	100
3	750	1000	100
4	750	925	100
5	500	350	100
6	500	671	100
7	500	400	100
8	500	650	100
9	750	1000	120
10	1000	1000	120
11	1000	925	120
12	1000	800	120
13	750	763	120
14	1000	650	120
15	750	400	120
16	750	125	120
17	750	400	120
18	500	125	120
19	500	800	120
20	500	125	120
21	500	800	120
22	500	1000	120

Head loss is calculated using **Hazen-Williams equation (SI units).**

$$H_L = 10.67 L Q^{1.85} / C^{1.85} D^{4.87}$$

$$\left(H_L = \frac{10.671 \cdot L \cdot |Q|^{0.85}}{C^{1.85} \cdot D^{4.87}} \cdot Q \right) = K \cdot (|Q|)^{0.85} \cdot Q$$

where, $$K = \frac{10.671 \cdot L}{C^{1.85} \cdot D^{4.87}}$$

Based on method discussed above 7 loop equations are formed and it takes 5 iterations to converge to final flow values. Refer attached excel spreadsheet for solution.

Define **K (Pipe Constant)** as given below

$$D := \begin{bmatrix} 1000 \\ 750 \\ 750 \\ 750 \\ 500 \\ 500 \\ 500 \\ 500 \\ 750 \\ 1000 \\ 1000 \\ 1000 \\ 750 \\ 1000 \\ 750 \\ 750 \\ 750 \\ 500 \\ 500 \\ 500 \\ 500 \\ 500 \end{bmatrix} \text{mm} \quad LL := \begin{bmatrix} 1000 \\ 925 \\ 1000 \\ 925 \\ 350 \\ 671 \\ 400 \\ 650 \\ 1000 \\ 1000 \\ 925 \\ 800 \\ 763 \\ 650 \\ 400 \\ 125 \\ 400 \\ 125 \\ 800 \\ 125 \\ 800 \\ 1000 \end{bmatrix} \text{m} \quad C := \begin{bmatrix} 100 \\ 100 \\ 100 \\ 100 \\ 100 \\ 100 \\ 100 \\ 100 \\ 120 \\ 120 \\ 120 \\ 120 \\ 120 \\ 120 \\ 120 \\ 120 \\ 120 \\ 120 \\ 120 \\ 120 \\ 120 \\ 120 \end{bmatrix}$$

$$K := \overrightarrow{\frac{10.671 \cdot \frac{LL}{\text{m}}}{C^{1.85} \cdot \left(\frac{D}{\text{m}}\right)^{4.87}}} = \begin{bmatrix} 2.1291 \\ 7.9946 \\ 8.6428 \\ 7.9946 \\ 21.7916 \\ 41.7776 \\ 24.9047 \\ 40.4701 \\ 6.1684 \\ 1.5196 \\ 1.4056 \\ 1.2157 \\ 4.7065 \\ 0.9877 \\ 2.4674 \\ 0.771 \\ 2.4674 \\ 5.5545 \\ 35.5489 \\ 5.5545 \\ 35.5489 \\ 44.4361 \end{bmatrix}$$

$$\overrightarrow{\text{rows}\left(\begin{bmatrix} D \\ LL \\ C \\ K \end{bmatrix}\right)} = \begin{bmatrix} 22 \\ 22 \\ 22 \\ 22 \end{bmatrix}$$

$$\begin{bmatrix} Q_A \\ Q_B \\ Q_C \\ Q_D \\ Q_E \\ Q_F \\ Q_G \\ Q_H \\ Q_I \\ Q_J \\ Q_K \\ Q_L \\ Q_M \\ Q_N \\ Q_O \\ Q_P \end{bmatrix} := \begin{bmatrix} 2 \\ -1 \\ 12 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \end{bmatrix} \qquad Q_{issues} := \begin{bmatrix} Q_A \\ Q_B \\ Q_C \\ Q_D \\ Q_E \\ Q_F \\ Q_G \\ Q_H \\ Q_I \\ Q_J \\ Q_K \\ Q_L \\ Q_M \\ Q_N \\ Q_O \\ Q_P \end{bmatrix}$$

$\sum Q_{issues} = 0$ Sanity check

Number of Issue points

$Points := \mathrm{rows}\left(Q_{issues}\right) = 16$

Number of Loops

$Loops := 7$

Number of Pipes

$Pipes := 22$

$Tot_{eqns} := Points + Loops = 23$

$Tot_{eqns} > Pipes = 1$

Hence, overdetermined system

$guess_{[1..Pipes]} := 0.1$

$$guess = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$$

$\mathrm{rows}(guess) = 22$

AlgLib Solver for a system of nonlinear equations(Uni)

```
result := for  k ∈ [1..rows(f(Q))]
              u_k := Q_k
          Jac(Q) := Jacobian(f(u), u)
          StepMax := 0
          Eps := 10^(−14)
          u := al_nleqsolve(guess, StepMax, Eps, f(Q), Jac(Q))
```

Define the system of 23 equations (16 Nodes + 7 Loops) ; 22 Variables (pipe flows) : Hence, Overdetermimed Nonlinear System

$$f(Q) := \begin{bmatrix} Q_A - Q_1 - Q_4 \\ Q_B + Q_1 + Q_{10} - Q_2 \\ Q_C - Q_{10} - Q_{11} \\ Q_D + Q_4 + Q_3 - Q_5 \\ Q_E + Q_2 - Q_9 - Q_8 - Q_3 \\ Q_F + Q_{11} - Q_{12} + Q_9 \\ Q_G + Q_{12} - Q_{13} - Q_{14} \\ Q_H + Q_5 - Q_6 \\ Q_I + Q_6 - Q_7 \\ Q_J + Q_8 + Q_7 - Q_{22} \\ Q_K + Q_{22} + Q_{21} - Q_{20} \\ Q_L - Q_{21} + Q_{14} - Q_{15} - Q_{18} \\ Q_M + Q_{13} + Q_{15} - Q_{16} \\ Q_N + Q_{20} - Q_{19} \\ Q_O + Q_{18} + Q_{19} - Q_{17} \\ Q_P + Q_{16} + Q_{17} \\ K_1 \cdot \left(\left|Q_1\right|\right)^{0.85} \cdot Q_1 + K_2 \cdot \left(\left|Q_2\right|\right)^{0.85} \cdot Q_2 + K_3 \cdot \left(\left|Q_3\right|\right)^{0.85} \cdot Q_3 - K_4 \cdot \left(\left|Q_4\right|\right)^{0.85} \cdot Q_4 \\ -K_{10} \cdot \left(\left|Q_{10}\right|\right)^{0.85} \cdot Q_{10} + K_{11} \cdot \left(\left|Q_{11}\right|\right)^{0.85} \cdot Q_{11} - K_9 \cdot \left(\left|Q_9\right|\right)^{0.85} \cdot Q_9 - K_2 \cdot \left(\left|Q_2\right|\right)^{0.85} \cdot Q_2 \\ -K_3 \cdot \left(\left|Q_3\right|\right)^{0.85} \cdot Q_3 + K_8 \cdot \left(\left|Q_8\right|\right)^{0.85} \cdot Q_8 - K_7 \cdot \left(\left|Q_7\right|\right)^{0.85} \cdot Q_7 - K_6 \cdot \left(\left|Q_6\right|\right)^{0.85} \cdot Q_6 - K_5 \cdot \left(\left|Q_5\right|\right)^{0.85} \cdot Q_5 \\ K_9 \cdot \left(\left|Q_9\right|\right)^{0.85} \cdot Q_9 + K_{12} \cdot \left(\left|Q_{12}\right|\right)^{0.85} \cdot Q_{12} + K_{14} \cdot \left(\left|Q_{14}\right|\right)^{0.85} \cdot Q_{14} + K_{21} \cdot \left(\left|Q_{21}\right|\right)^{0.85} \cdot Q_{21} - K_{22} \cdot \left(\left|Q_{22}\right|\right)^{0.85} \cdot Q_{22} - K_8 \cdot \left(\left|Q_8\right|\right)^{0.85} \cdot Q_8 \\ K_{13} \cdot \left(\left|Q_{13}\right|\right)^{0.85} \cdot Q_{13} - K_{15} \cdot \left(\left|Q_{15}\right|\right)^{0.85} \cdot Q_{15} - K_{14} \cdot \left(\left|Q_{14}\right|\right)^{0.85} \cdot Q_{14} \\ -K_{21} \cdot \left(\left|Q_{21}\right|\right)^{0.85} \cdot Q_{21} + K_{18} \cdot \left(\left|Q_{18}\right|\right)^{0.85} \cdot Q_{18} - K_{19} \cdot \left(\left|Q_{19}\right|\right)^{0.85} \cdot Q_{19} - K_{20} \cdot \left(\left|Q_{20}\right|\right)^{0.85} \cdot Q_{20} \\ K_{15} \cdot \left(\left|Q_{15}\right|\right)^{0.85} \cdot Q_{15} + K_{16} \cdot \left(\left|Q_{16}\right|\right)^{0.85} \cdot Q_{16} - K_{17} \cdot \left(\left|Q_{17}\right|\right)^{0.85} \cdot Q_{17} - K_{18} \cdot \left(\left|Q_{18}\right|\right)^{0.85} \cdot Q_{18} \end{bmatrix}$$

$\text{rows}(f(Q)) = 23$

Results

$$Q_{final} := result = \begin{bmatrix} -0.53 \\ 2.535 \\ 0.211 \\ 2.53 \\ 1.741 \\ 0.741 \\ -0.259 \\ 1.477 \\ -0.153 \\ 4.066 \\ 7.934 \\ 6.782 \\ 1.848 \\ 3.934 \\ 0.943 \\ 1.791 \\ -0.791 \\ 1.05 \\ -0.841 \\ 0.159 \\ 0.94 \\ 0.218 \end{bmatrix}$$

Internet Example Result using in EXCEL

	A	B	C
138			
139	**Iteration**		
140	**5**	Pipe	Flow
141			m3/s
142		1	-0.530
143		2	2.535
144		3	0.211
145		4	2.530
146		5	1.741
147		6	0.741
148		7	-0.259
149		8	1.477
150		9	-0.153
151		10	4.066
152		11	7.934
153		12	6.782
154		13	1.848
155		14	3.934
156		15	0.943
157		16	1.791
158		17	-0.791
159		18	1.050
160		19	-0.841
161		20	0.159
162		21	0.940
163		22	0.218

$$f\left(Q_{final}\right) = \begin{bmatrix} 1.7764 \cdot 10^{-15} \\ -2.2204 \cdot 10^{-15} \\ 0 \\ -6.8834 \cdot 10^{-15} \\ -1.9984 \cdot 10^{-15} \\ -7.2164 \cdot 10^{-16} \\ 1.0214 \cdot 10^{-14} \\ 5.9952 \cdot 10^{-15} \\ -9.992 \cdot 10^{-16} \\ 0 \\ 9.992 \cdot 10^{-16} \\ -2.2204 \cdot 10^{-15} \\ 1.1102 \cdot 10^{-15} \\ 0 \\ -1.1102 \cdot 10^{-16} \\ -5.107 \cdot 10^{-15} \\ 8.218 \cdot 10^{-15} \\ 1.5081 \cdot 10^{-13} \\ -2.6136 \cdot 10^{-13} \\ 7.4561 \cdot 10^{-14} \\ -3.952 \cdot 10^{-14} \\ 3.9784 \cdot 10^{-14} \\ -1.3478 \cdot 10^{-14} \end{bmatrix}$$

$time(0) - t_0 = 1.8\ s$