## Plot

$$PPlot(xy(1), T, v) := \begin{array}{l} [\omega \ \omega_r \ r \ a \ b \ V \ m \ h_g \ I_w \ g] := v \\ [n := \text{length}(T) \ \ XY := \text{matrix}(n, 2) \ \ k := [1..n] \ \ c := [1..2]] \\ \text{eval}\left(\left(XY_{k\,c} := \text{try} \ \ xy\left(T_k\right)_c \ \ \text{on error} \ \ XY_{\max([1 \ k-1])\,c}\right)_{[2..(n-1)]\,c}\right) \end{array}$$

$$PPlot(x(1), y(1), T, v) := \begin{array}{l} xy(t\#) := [x(t\#) \ y(t\#)] \\ PPlot(xy(t\#), T, v) \end{array}$$

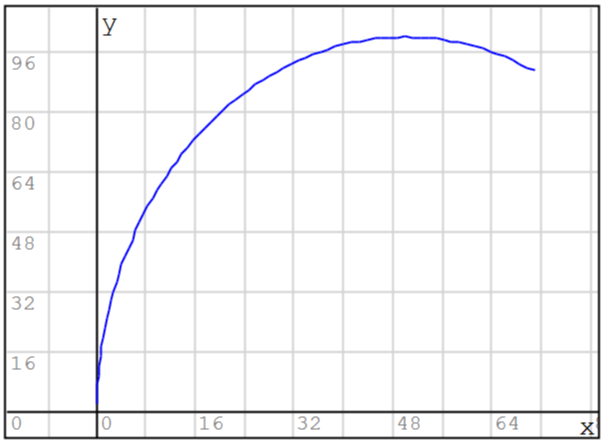
$$Id(t) := t$$

## k

$$T := [0, 0.1..8] \qquad \text{length}(T) = 81$$

$$values := \left[ \frac{V}{r} \ \ \frac{V}{0.32} \ \ \sqrt{\frac{a^2 + b^2}{2}} \ \ 50 \ 100 \ 20 \ 200 \ 0.6 \ 1.2 \ 9.806 \right] \qquad [\omega \ \omega_r \ r \ a \ b \ V \ m \ h_g \ I_w \ g]$$

$$\begin{cases} x(t) := a - a \cdot \sin\left(\omega \cdot t + \frac{\pi}{2}\right) \\ y(t) := b \cdot \sin(\omega \cdot t) \end{cases}$$




$$PPlot(x(t), y(t), T, values)$$

Derivatives & slopes

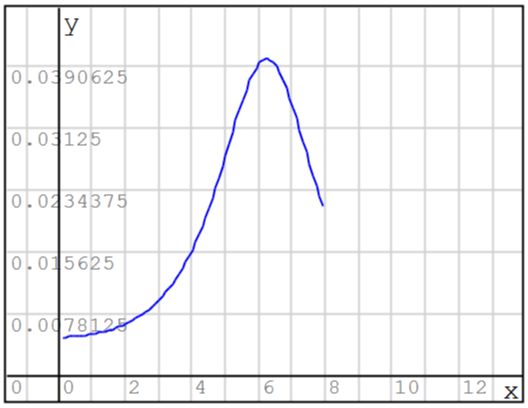
$$x'(t) := \frac{d}{d\,t} x(t) \qquad y'(t) := \frac{d}{d\,t} y(t) \qquad m(t) := \frac{y'(t)}{x'(t)}$$

$$x''(t) := \frac{d}{d\,t} x'(t) \qquad y''(t) := \frac{d}{d\,t} y'(t) \qquad slope(t) := \text{atan}(m(t))$$

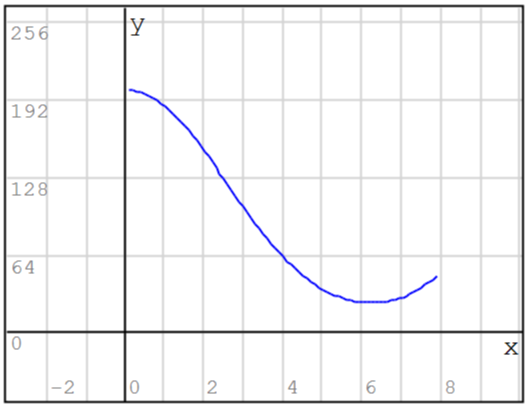
Curvature & Radious of curvature

$$c(t) := -\frac{x'(t) \cdot y''(t) - y'(t) \cdot x''(t)}{\sqrt{\left(x'(t)^2 + y'(t)^2\right)^3}} \qquad R_c(t) := \frac{1}{c(t)}$$

$$c(t) = -\frac{\omega^3 \cdot a \cdot b \cdot \left(\cos\left(\frac{\pi + 2 \cdot \omega \cdot t}{2}\right) \cdot \sin(\omega \cdot t) - \cos(\omega \cdot t) \cdot \sin\left(\frac{\pi + 2 \cdot \omega \cdot t}{2}\right)\right)}{\sqrt{\left(\omega^6 \cdot \left(\cos\left(\frac{\pi + 2 \cdot \omega \cdot t}{2}\right)^2 \cdot a^2 + \cos(\omega \cdot t)^2 \cdot b^2\right)\right)^3}}$$




PPlot ( Id ( t ), c ( t ), T, values )

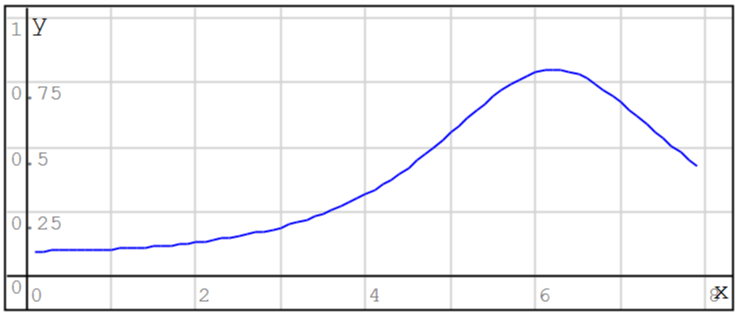



PPlot ( Id ( t ), $R_c$ ( t ), T, values )

$$\psi'(t) := \frac{V}{R_c(t)}$$

in rad/sec per calcoli successivi

velocità di imbardata




PPlot ( Id ( t ), ψ' ( t ), T, values )

$$\varphi_{id}(t) := \operatorname{atan}\left(\frac{V^2}{g \cdot R_c(t)}\right)$$

$$\varphi_{id}(t) = -\operatorname{atan}\left(\frac{V^2 \cdot \omega^3 \cdot a \cdot b \cdot \left(\cos\left(\frac{\pi + 2 \cdot \omega \cdot t}{2}\right) \cdot \sin(\omega \cdot t) - \cos(\omega \cdot t) \cdot \sin\left(\frac{\pi + 2 \cdot \omega \cdot t}{2}\right)\right)}{g \cdot \sqrt{\omega^6 \cdot \left(\cos\left(\frac{\pi + 2 \cdot \omega \cdot t}{2}\right)^2 \cdot a^2 + \cos(\omega \cdot t)^2 \cdot b^2\right)^3}}\right)$$

$$\varphi_{Iw}(t) := \frac{I_w \cdot \omega_r}{h_g} \cdot \frac{\cos\left(\varphi_{id}(t)\right) \cdot \psi'(t)}{\sqrt{(m \cdot g)^2 + \left(m \cdot R_c(t) \cdot (\psi'(t))^2\right)^2}}$$

$$\varphi_{Iw}(t) = -\frac{\cdots}{h_g \cdot \sqrt{\frac{g^2 \cdot \sqrt{\omega^6 \cdot \left(\cos\left(\frac{\pi + 2 \cdot \omega \cdot t}{2}\right)^2 \cdot a^2 + \cos(\omega \cdot t)^2 \cdot b^2\right)^3}^2 + V^4 \cdot \omega^6 \cdot a^2 \cdot b^2 \cdot \left(\cos\left(\frac{\pi + 2 \cdot \omega \cdot t}{2}\right) \cdot \sin(\omega \cdot t) - \cos \cdots\right.}{g^2 \cdot \sqrt{\omega^6 \cdot \left(\cos\left(\frac{\pi + 2 \cdot \omega \cdot t}{2}\right)^2 \cdot a^2 + \cos(\omega \cdot t)^2 \cdot b^2\right)^3}^2}}}$$

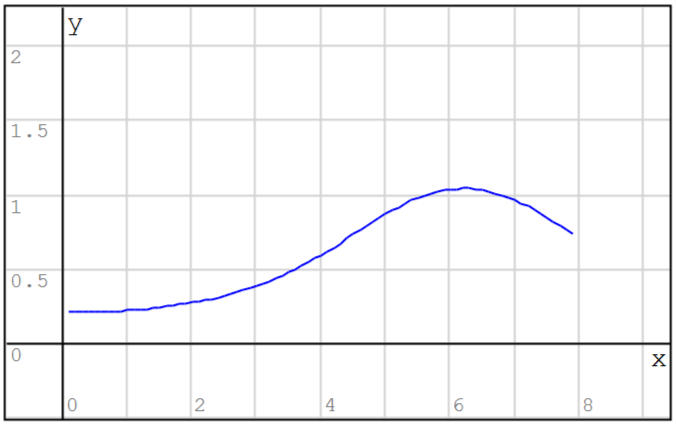
$$\varphi(t) := \varphi_{id}(t) + \varphi_{Iw}(t)$$

$$\varphi'(t) := \frac{d}{d\,t}\varphi(t)$$

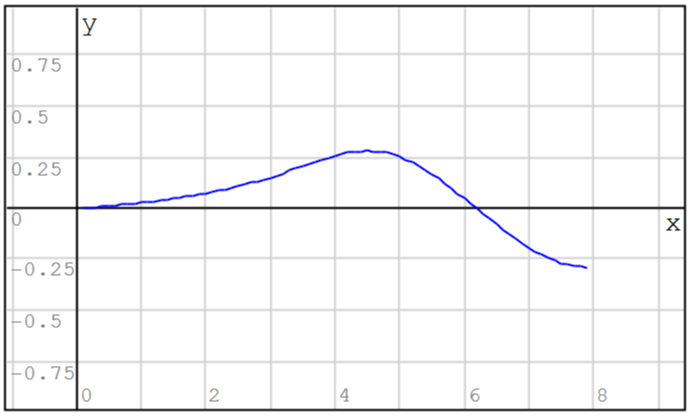
Now diff works for numerics

$$\varphi'(t) = -\frac{3 \cdot \omega \cdot \left(\sin\left(\frac{\pi + 2 \cdot \omega \cdot t}{2}\right) \cdot \cos\left(\frac{\pi + 2 \cdot \omega \cdot t}{2}\right) \cdot a^2 + \sin(\omega \cdot t) \cdot \cos(\omega \cdot t) \cdot b^2\right) \cdot \left(\sqrt{\omega^6 \cdot \left(\cos\left(\frac{\pi + 2 \cdot \omega \cdot t}{2}\right)^2 \cdot a^2 + \cos(\omega \cdot t \cdots\right.}\right.}{\cdots}$$

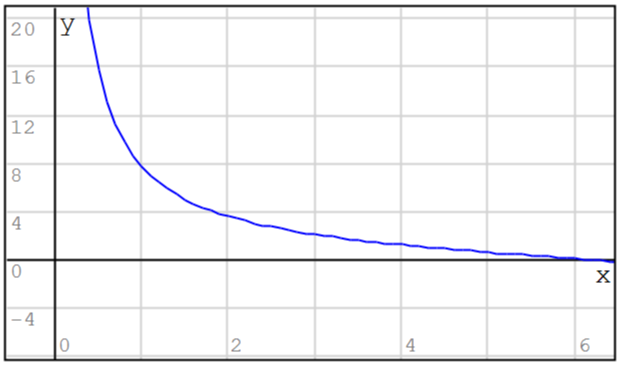
velocità di rollio



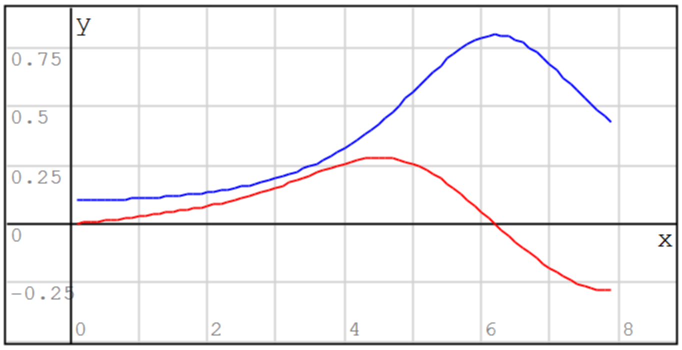

PPlot ( Id ( t ), φ ( t ), T, values )




PPlot ( Id ( t ), φ' ( t ), T, values )




PPlot ( Id ( t ), m ( t ), T, values )



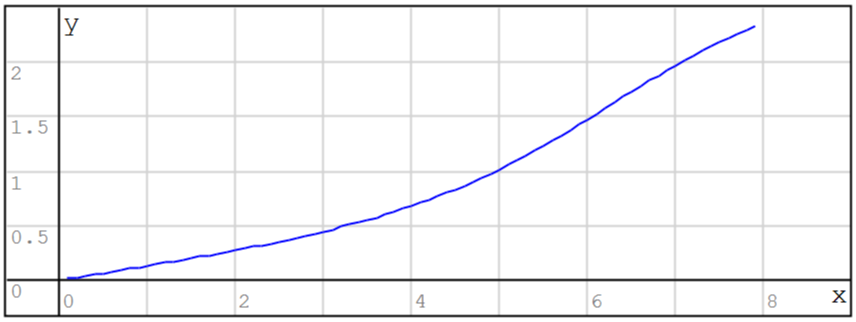

PPlot ( Id ( t ), ψ' ( t ), T, values )
PPlot ( Id ( t ), φ' ( t ), T, values )

in rad/sec per calcoli successivi

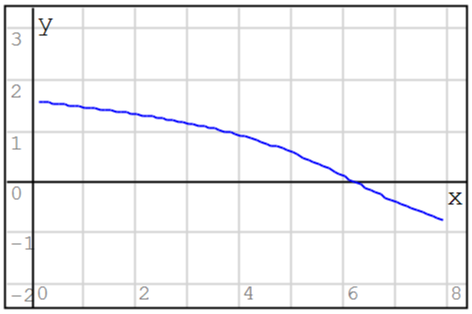
$$\psi(t) := \frac{\pi}{2} - \operatorname{atan}(m(t))$$

andamento dell'angolo tra i due sistemi di riferimento (fisso e mobile) rispetto a t

$$\psi(t) = \frac{\pi + 2 \cdot \operatorname{atan}\left(\dfrac{\cos(\omega \cdot t) \cdot b}{\cos\left(\dfrac{\pi + 2 \cdot \omega \cdot t}{2}\right) \cdot a}\right)}{2}$$



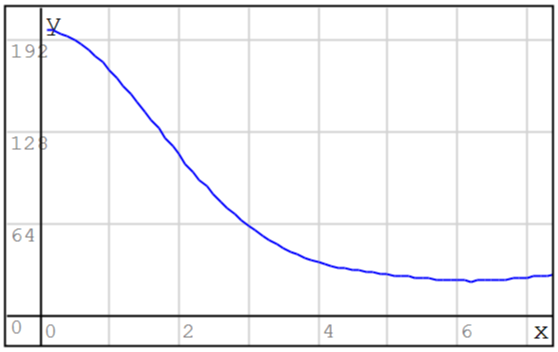

PPlot ( Id ( t ), ψ ( t ), T, values )




PPlot ( Id ( t ), slope ( t ), T, values )

## coordinate traccia di mozzi nel sistema solidalle alla motocicletta

$$\begin{cases} x_m(t) := 0 \\ y_m(t) := \dfrac{\psi'(t) \cdot V}{(\psi'(t))^2 + (\varphi'(t))^2} \end{cases}$$

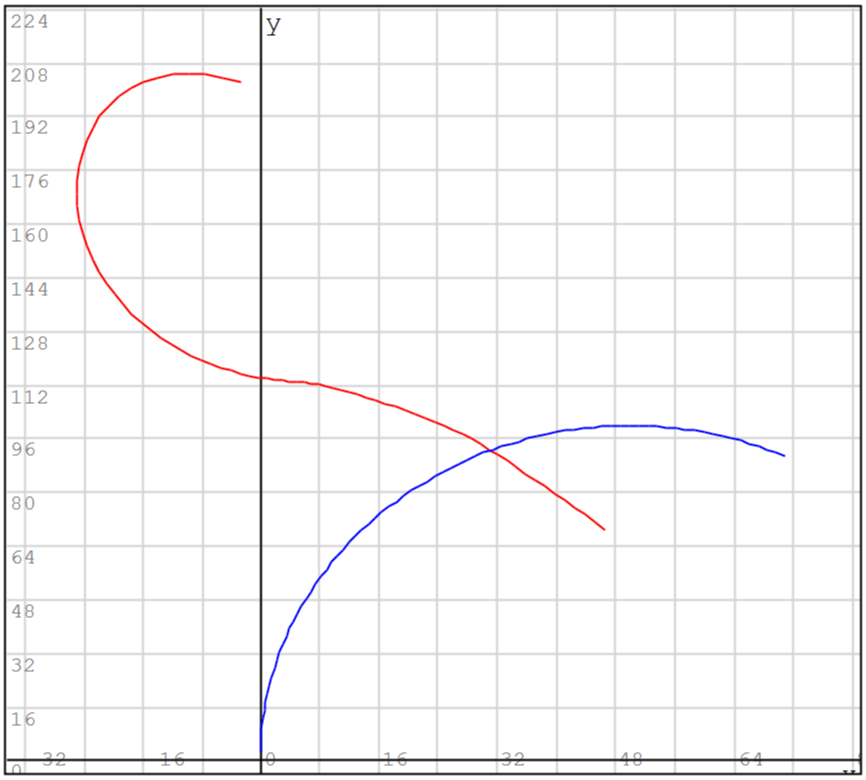



PPlot ( Id(t), $y_m(t)$, T, values )

$$y_m(t) = -\frac{}{\sqrt{\omega^6 \cdot \left(\cos\left(\frac{\pi + 2 \cdot \omega \cdot t}{2}\right)^2 \cdot a^2 + \cos(\omega \cdot t)^2 \cdot b^2\right)^3} \cdot \left( V^2 \cdot \omega^4 \cdot a^2 \cdot b^2 \cdot \left(\cos\left(\frac{\pi + 2 \cdot \omega \cdot t}{2}\right) \cdot \sin(\omega \cdot t) - \cos(\omega \cdot t) \cdot \sin\left(\frac{\pi + 2 \cdot \omega}{2}\right.\right.\right.}$$

## coordinate centro di curvatura nel sistema solidalle alla motocicletta

$$\begin{cases} x_t(t) := 0 \\ y_t(t) := \dfrac{V}{\psi'(t)} \end{cases} \qquad R(\theta) := \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \qquad xy_M(t) := \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} + R(\psi(t)) \cdot \begin{bmatrix} x_m(t) \\ y_m(t) \end{bmatrix}$$




PPlot ( x(t), y(t), T, values )
PPlot ( $xy_M(t)$, T, values )

$$y_t(t) = -\frac{\sqrt{\omega^6 \cdot \left(\cos\left(\frac{\pi + 2 \cdot \omega \cdot t}{2}\right)^2 \cdot a^2 + \cos(\omega \cdot t)^2 \cdot b^2\right)^3}}{\omega^3 \cdot a \cdot b \cdot \left(\cos\left(\frac{\pi + 2 \cdot \omega \cdot t}{2}\right) \cdot \sin(\omega \cdot t) - \cos(\omega \cdot t) \cdot \sin\left(\frac{\pi + 2 \cdot \omega \cdot t}{2}\right)\right)}$$

$$xy_M(t) = \begin{bmatrix} a \cdot \left( \left(1 - \sin\left(\frac{\pi + 2 \cdot \omega \cdot t}{2}\right)\right) \cdot \sqrt{\omega^6 \cdot \left(\cos\left(\frac{\pi + 2 \cdot \omega \cdot t}{2}\right)^2 \cdot a^2 + \cos(\omega \cdot t)^2 \cdot b^2\right)^3} \cdot \left( V^2 \cdot \omega^4 \cdot a^2 \cdot b^2 \cdot \left(\cos\left(\frac{\pi + 2 \cdot \omega \cdot t}{2}\right) \cdot \mathrm{si} \right.\right.\right. \\ \hline \\ b \cdot \left( \sin(\omega \cdot t) \cdot \sqrt{\omega^6 \cdot \left(\cos\left(\frac{\pi + 2 \cdot \omega \cdot t}{2}\right)^2 \cdot a^2 + \cos(\omega \cdot t)^2 \cdot b^2\right)^3} \cdot \left( V^2 \cdot \omega^4 \cdot a^2 \cdot b^2 \cdot \left(\cos\left(\frac{\pi + 2 \cdot \omega \cdot t}{2}\right) \cdot \sin(\omega \cdot \right.\right.\right. \\ \hline \end{bmatrix}$$

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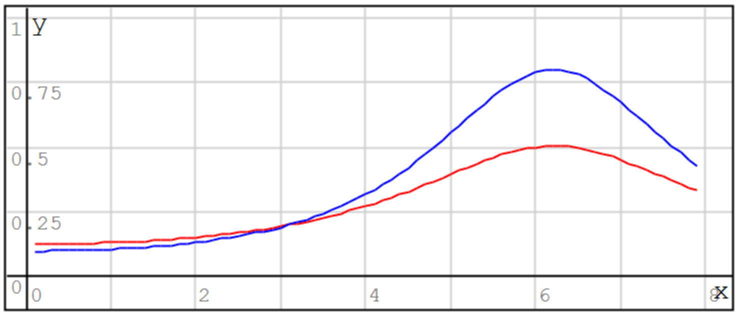
Notes

1. You define twice ψ(t), check your integral

$$\psi(t) := \left(\frac{\pi}{2} - \mathrm{atan}(m(t))\right)$$

$$yy'(t) := \frac{d}{dt}\psi(t)$$




$$\begin{cases} PPlot(Id(t), \psi'(t), T, values) \\ PPlot(Id(t), yy'(t), T, values) \end{cases}$$

*Alvaro*