$n := 5$

$$X := \text{Random}(n) = \begin{bmatrix} 0.09113 \\ 0.02245 \\ 0.3336 \\ 0.1604 \\ 0.3713 \end{bmatrix} \qquad Y := \text{Random}(n) = \begin{bmatrix} 0.2285 \\ 0.6137 \\ 0.8952 \\ 0.3446 \\ 0.5624 \end{bmatrix}$$

$$f(x, y) := \left(a_{x2} \cdot x^2 + 2 \cdot a_{xy} \cdot x \cdot y + a_{y2} \cdot y^2 + 2 \cdot a_x \cdot x + 2 \cdot a_y \cdot y + a \right)$$

$a := 1$

$$M := \begin{bmatrix} X_1^2 & 2 \cdot X_1 \cdot Y_1 & Y_1^2 & 2 \cdot X_1 & 2 \cdot Y_1 \\ X_2^2 & 2 \cdot X_2 \cdot Y_2 & Y_2^2 & 2 \cdot X_2 & 2 \cdot Y_2 \\ X_3^2 & 2 \cdot X_3 \cdot Y_3 & Y_3^2 & 2 \cdot X_3 & 2 \cdot Y_3 \\ X_4^2 & 2 \cdot X_4 \cdot Y_4 & Y_4^2 & 2 \cdot X_4 & 2 \cdot Y_4 \\ X_5^2 & 2 \cdot X_5 \cdot Y_5 & Y_5^2 & 2 \cdot X_5 & 2 \cdot Y_5 \end{bmatrix}$$

$$v := \begin{bmatrix} -a \\ -a \\ -a \\ -a \\ -a \end{bmatrix} \qquad \begin{bmatrix} a_{x2} \\ a_{xy} \\ a_{y2} \\ a_x \\ a_y \end{bmatrix} := M^{-1} \cdot v = \begin{bmatrix} 12.84 \\ -32.24 \\ 27.94 \\ 16.55 \\ -9.276 \end{bmatrix}$$

$$I_1 := a_{x2} + a_{y2} = 40.7861$$

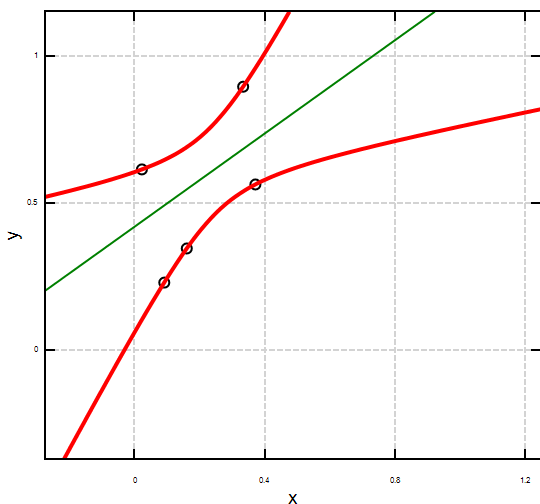$$I_2 := \left\| \begin{matrix} a_{x2} & a_{xy} \\ a_{xy} & a_{y2} \end{matrix} \right\| = -680.4 \qquad I_3 := \left\| \begin{matrix} a_{x2} & a_{xy} & a_x \\ a_{xy} & a_{y2} & a_y \\ a_x & a_y & a \end{matrix} \right\| = 459.5297$$

$$\varphi := -\text{atan}\left(a_{x2} - a_{y2} - \sigma, \; 2 \cdot a_{xy} \right) = 2.2412$$

$$\sigma := \frac{\sqrt{\left(a_{x2} - a_{y2} \right)^2 + \left(2 \cdot a_{xy} \right)^2}}{\text{sign}\left(I_3 \right)} = 66.2189$$

$$\Omega := \frac{\left[a_{xy} \cdot a_y - a_{y2} \cdot a_x \quad a_x \cdot a_{xy} - a_{x2} \cdot a_y \right]}{I_2} = \left[0.2401 \quad 0.609 \right]$$

$$vaxis(x, y) := \cos(\varphi) \cdot x + \sin(\varphi) \cdot y - \left(\cos(\varphi) \cdot \Omega_1 + \sin(\varphi) \cdot \Omega_2 \right)$$

$$\begin{cases} \text{"Hyperbola"} & \text{if} \quad I_2 < 0 \\ \text{"Ellipse"} & \text{if} \quad I_2 > 0 \\ \text{"Parabola"} & \text{otherwise} \end{cases} = \text{"Hyperbola"}$$

$$N := \sum \overrightarrow{vaxis(X, Y) > 0} = 2$$

Hyperbola ... or ellipse

$$\begin{cases} \text{"2+3 points"} & \text{if} \quad (N = 2) \vee (N = 3) \\ \text{"1+4 points"} & \text{if} \quad (N = 1) \vee (N = 4) \\ \text{"0+5 points"} & \text{otherwise} \end{cases} = \text{"2+3 points"}$$

$$\begin{cases} \text{augment}(X, Y) \\ f(x, y) \\ vaxis(x, y) \end{cases}$$