Strut-and-Tie Model for Deep Beam Design

A practical exercise using Appendix A of the 2002 ACI Building Code

BY JAMES K. WIGHT AND GUSTAVO J. PARRA-MONTESINOS

Although the Strut-and-Tie Method (STM) has been used for several years in Europe[1,2] and has been included in the Canadian Standard for the Design of Concrete Structures[3] since 1984 and the AASHTO LRFD Bridge Specifications[4] since 1994, it is a new concept for many structural engineers in the U.S. Procedures and recommendations for the use of STM to design reinforced concrete members were discussed in a State-of-the-Art Report from Joint ACI-ASCE Committee 445, Shear and Torsion,[5] but specific code requirements were not incorporated into the ACI Building Code until the 2002 edition,[6] as Appendix A. To help U.S. engineers improve their ability to use STM for analysis and design of concrete members, Joint ACI-ASCE Committee 445 and ACI Committee 318-E, Shear and Torsion, recently completed a publication that contains a variety of STM examples.[7] The STM model used here for the analysis and design of a deep beam is not unique. It should be noted that the STM procedure in Appendix A of the ACI Building Code (referred to as the Code) is a strength limit-state design approach. Serviceability limit-states (for example, deflections and reinforcement distribution) defined in the main body of the Code must also be checked.

THE PROBLEM

Figure 1 shows the beam to be analyzed and designed. The Code classifies the beam in Fig. 1 as a “deep beam” because the clear-span-to-total-depth ratio for this beam is less than 4.0. The member dimensions and loads are the same as those used for an example in the PCA Notes.[8] For this problem, however, the concentrated load is applied approximately at a third-point of the span, instead of at midspan. The concentrated load for this example was factored using the load factors specified in Chapter 9 of the Code. Thus, the appropriate strength reduction factor ϕ is 0.75 for the STM solution. The beam dead load, multiplied by the appropriate load factor, is assumed to be included in the concentrated load applied at the top of the beam. In practice, the columns supporting the beam would offer some resistance to beam end rotations and horizontal displacements. To be consistent with the example presented in the PCA Notes, however, we do not assume the column supports resist these deformations; instead, we assume they act as a combination of pin- and roller-type supports. The specified concrete compressive strength f'_c is 4000 psi (28 MPa) and the reinforcing steel is assumed to have a yield strength f_y of 60 ksi (410 MPa). The transverse dimension of the columns and deep beam is 20 in. (510 mm).

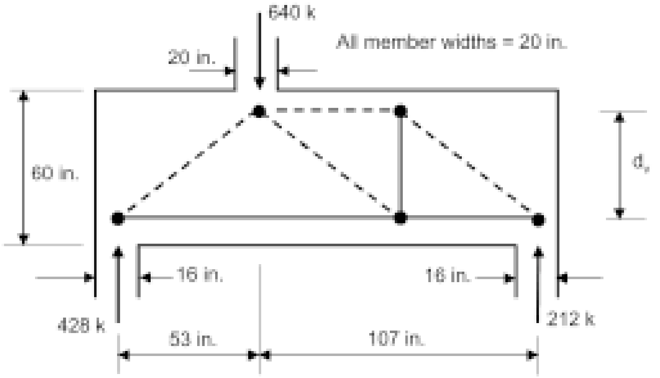


Fig. 1: The deep beam dimensions and general truss model assumed for the analysis and design in this article (1 in. = 25.4 mm, 1 k = 4.45 kN)

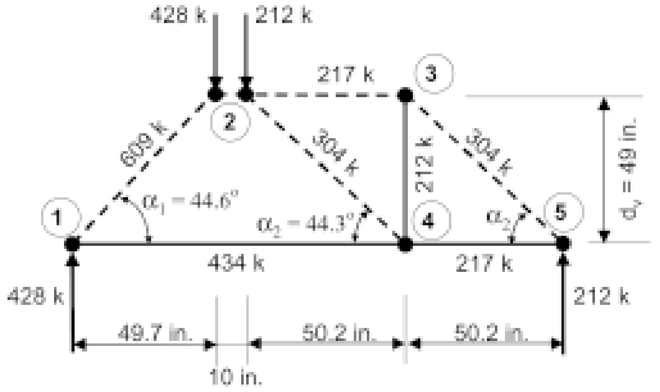


Fig. 2: Truss geometry and member forces after the second trial of analysis of the deep beam (1 in. = 25.4 mm, 1 k = 4.45 kN)

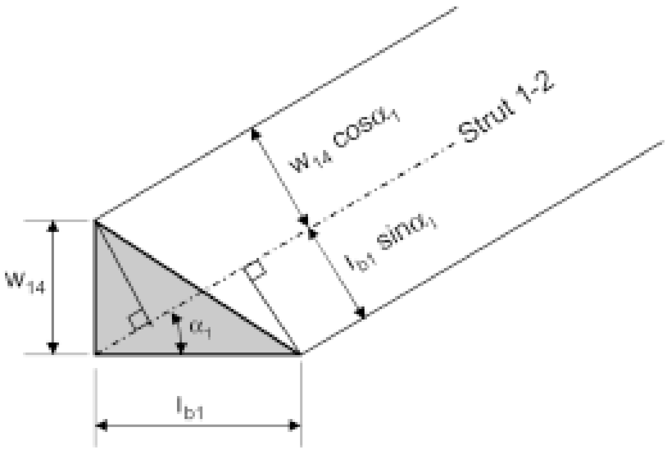


Fig. 3: Geometry and dimensions of Node 1 and Strut 1-2 and the procedure used to calculate the width of Strut 1-2

INITIAL STRUT-AND-TIE MODEL

Figure 1 shows the initial strut-and-tie model (or truss) assumed for analysis and design of this deep beam. The broken lines represent compression members (struts) and the solid lines represent tension members (ties). For simplicity, the nodes (nodal zones, or intersection of the struts and ties) are shown as dimensionless points. A trial value must be selected for the depth of the truss d_v to solve for the truss member forces. With these forces, the dimensions of the struts, ties, and nodal zones can be established, and the value for d_v can be verified or modified with a second iteration. Because of the small span-to-depth ratio (approximately 1) for the left portion of the beam, only a single strut is used between the concentrated load and the support. The right portion of the beam requires more truss members because a single strut acting at a shallow angle would not be safe or practical. To control the use of shallow angle struts, the Code requires a minimum angle of 25 degrees between struts and ties.

SOLUTION OF LEFT PORTION OF BEAM

It is convenient for the analysis of Node 2 to break the concentrated load at the top of the beam into two parts and solve the left portion of the beam to establish values for d_v and α_1 (Fig. 2). For equilibrium, 2/3 of the concentrated load and thus 2/3 of the node dimension (that is, the top column dimension) will be assigned to the left part of Node 2. Similarly, 1/3 of the load and node dimension are assigned to the right part of Node 2. The two node "points" shown for Node 2 are both part of a single "nodal zone." The following gives an iterative procedure to find d_v, α_1, and the truss member forces, while Fig. 2 gives the second iteration values.

Step 1: Establish truss geometry and truss member forces

Assume

$$d_v = 60 \text{ in.} - 2(7.5 \text{ in.}) = 45 \text{ in.} \ (1140 \text{ mm})$$

$\tan \alpha_1 = (45 \text{ in.}/49.7 \text{ in.})$ implies that $\alpha_1 = 42.2$ degrees

From equilibrium at Node 1

$$\Sigma(F_y) = 428\text{ k} - F_{12}(\sin\alpha_1) = 0 \text{ implies } F_{12} = 637\text{ k (2830 kN)}$$

$$\Sigma(F_x) = F_{14} - F_{12}(\cos\alpha_1) = 0 \text{ implies } F_{14} = 472\text{ k (2100 kN)}$$

Figure 3 gives the geometry of Node 1. Because different effective compressive strengths will be used in Node 1 and Strut 1-2, and because the strut's stress must be checked on a plane perpendicular to its axis, the geometry of the node is a little complicated. Figure 3 shows the procedure to calculate the width of Strut 1-2.

The effective compressive strength for a node is defined as

$$f_{cu} = (0.85)\,\beta_n\,f'_c$$

Node 1 is a compression-compression-tension (CCT) node, so $\beta_n = 0.8$. Thus, the effective compressive strength for Node 1 at nominal conditions is

$$f_{cu}(1) = (0.85)\,\beta_n\,f'_c = (0.85)(0.80)(4.0\text{ ksi}) = 2.72\text{ ksi (18.7 MPa)}$$

Use this nominal strength and $\phi = 0.75$ to check stress at the base of the node

$$f(\text{base}) = \frac{R_1}{(b_w)(l_{b1})} = \frac{428\text{ k}}{(20\text{ in.})(16\text{ in.})}$$

$$f(\text{base}) = 1.34\text{ ksi (9.2 MPa)} < \phi(2.72\text{ ksi}) = 2.04\text{ ksi (14.1 MPa) (o.k.)}$$

Also, find the width of Tie 1-4, which defines the height of Node 1

$$w_{14} = \frac{F_{14}}{\phi(b_w)f_{cu}(1)} = \frac{472\text{ k}}{0.75(20\text{ in.})(2.72\text{ ksi})} = 11.6\text{ in. (295 mm)}$$

To ease calculations, assume the height of Node 1 (w_{14}) is equal to 12 in. (305 mm).

The effective compressive strength for Strut 1-2, $f_{cu}(1\text{-}2)$, probably controls the stress on the inclined face of Node 1. This value is determined using the same expression as given previously for a node, with β_s substituted for β_n. For Strut 1-2, use $\beta_s = 0.75$, which assumes that a minimum amount of reinforcement will be provided across the strut as required in Section A.3.3 of the Code

$$f_{cu}(1\text{-}2) = 0.85\,\beta_s f'_c = (0.85)(0.75)(4.0\text{ ksi}) = 2.55\text{ ksi (17.6 MPa)}$$

Now, use the geometry of Node 1 shown in Fig. 3 to determine the width of Strut 1-2

$$w_s(1\text{-}2) = w_{14}(\cos\alpha_1) + l_{b1}(\sin\alpha_1) = (12\text{ in.})(0.741) + (16\text{ in.})(0.672)$$
$$= 8.89\text{ in.} + 10.8\text{ in.} = 19.7\text{ in. (500 mm)}$$

Now check the strut capacity

$$\phi\,F_{ns}(1\text{-}2) = \phi f_{cu} w_s(1\text{-}2) b_w = (0.75)(2.55\text{ ksi})(19.7\text{ in.})(20\text{ in.})$$
$$= 754\text{ k (3360 kN)} > 637\text{ k (2830 kN) (o.k.)}$$

Based on the analysis of Node 1, assume that the height of Node 2, which is a compression-compression-compression (CCC) node and thus, $\beta_n = 1.0$, will be equal to 10 in. (250 mm). Then, for a second trial, assume that $d_v = 60\text{ in.} - (12\text{ in.} + 10\text{ in.})/2 = 49\text{ in.}$ (1240 mm). Reevaluating the truss with this value leads to

$$\alpha_1 = 44.6\text{ degrees; } F_{12} = 609\text{ k (2710 kN);}$$
$$\text{and } F_{14} = 434\text{ k (1930 kN).}$$

Because the forces in Strut 1-2 and Tie 1-4 are lower than in the first trial, there is no need to make further checks at Node 1.

Step 2: Check maximum shear force permitted in a deep beam

Code Section 11.8.3 defines an upper limit for the shear force permitted in a deep beam. With the centroid of Tie 1-4 established, the effective flexural depth of the beam d is $h - (w_{14}/2) = 54$ in. (1370 mm). Thus, the check of Code Section 11.8.3 requires

$$V_u \le \phi V_n\,(\text{max}) = \phi(10)\sqrt{f'_c}\,b_w d = (0.75)(10)\sqrt{4000\text{ psi}}\,(20\text{ in.})(54\text{ in.})$$

$$V_u = 428\text{ k (1900 kN)} \le \phi V_n\,(\text{max}) = 512\text{ k (2280 kN) (o.k.)}$$

Step 3: Make checks at Node 2

A sketch of the left side of Node 2 is given in Fig. 4. The top dimension is set equal to 2/3 of the column dimension, that is, 13.3 in. (340 mm). The vertical dimension of the node was assumed in Step 1 to be 10 in. (250 mm). Check the stress on the top face of Node 2 (CCC node) using $\beta_n = 1.0$

$$f_{cu}(2) = 0.85\,\beta_n f'_c = 3.40\text{ ksi (23.4 MPa), and } \phi f_{cu}(2) = 2.55\text{ ksi (17.6 MPa).}$$

$$f(\text{top}) = \frac{428\text{ k}}{(13.3\text{ in.})b_w} = \frac{428\text{ k}}{(13.3\text{ in.})(20\text{ in.})}$$

$$f(\text{top}) = 1.61\text{ ksi (11.1 MPa)} \le 2.55\text{ ksi (17.6 MPa) (o.k.)}$$

Check stress on vertical face of left part of Node 2

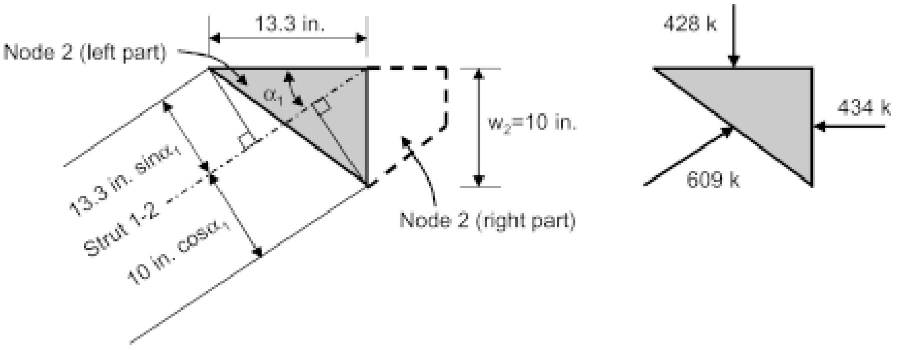


Fig. 4: Geometry, forces, and dimensions for the left part of Node 2 (1 in. = 25.4 mm, 1 k = 4.45 kN)

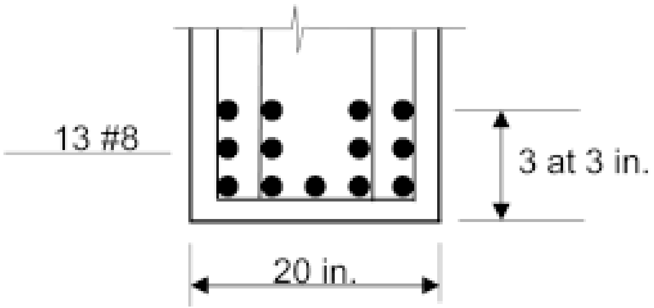


Fig. 5: Reinforcement required in Tie 1-4. Thirteen No. 8 bars were arranged in three rows (1 in. = 25.4 mm)

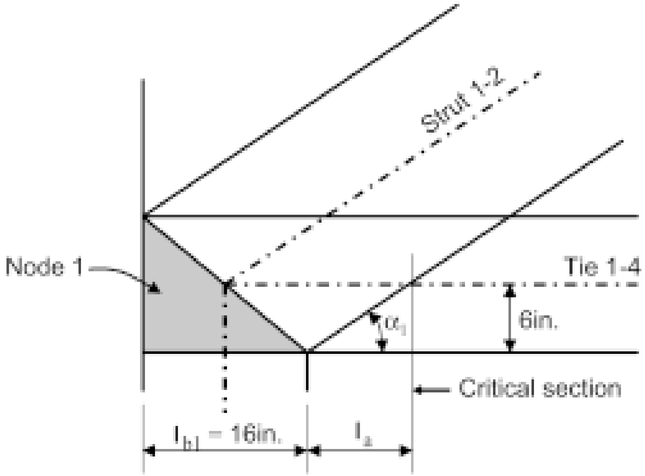


Fig. 6: Location of critical section for anchorage of reinforcement in Node 1 (1 in. = 25.4 mm)

$$f(\text{vert. face}) = \frac{434\text{k}}{(10\text{ in.})b_w} = 2.17\text{ ksi (15.0 MPa)}$$

$$< 2.55\text{ ksi (17.6 MPa) (o.k.)}$$

Determine width of Strut 1-2 at Node 2

$$w_s(1\text{-}2) = w_2\,(\cos\alpha_1) + 13.3\text{ in. }(\sin\alpha_1) = 7.12\text{ in.} + 9.36\text{ in.} = 16.5\text{ in. (420 mm)}$$

Check capacity of Strut 1-2 at Node 2 (critical end)

$$\phi F_{ns}(1\text{-}2) = \phi f_{cu}(1\text{-}2)w_s(1\text{-}2)\ (b_w) = (0.75)(2.55\text{ ksi})(16.5\text{ in.})(20\text{ in.}) = 631\text{ k (2810 kN)} > 609\text{ k (2710 kN)}$$

Step 4: Select reinforcement for Tie 1-4

Determine required area of reinforcing steel:

$$A_s(\text{req'd.}) = \frac{F_{14}}{\phi f_y} = \frac{434\text{ k}}{(0.75)(60\text{ ksi})} = 9.64\text{ in.}^2(6220\text{ mm}^2)$$

Select 13 No. 8 bars [A_s = 10.3 in.^{2} (6630 mm^2)], arranged in three rows (Fig. 5).

Check anchorage at Node 1:

From Fig. 6, l_a = (6 in.)/(tan α_1) = (6 in.)/(0.986) = 6.08 in. (155 mm)

Thus, available anchorage length = $l_a + l_{b1}$ – 1.5 in. (cover) = 20.6 in. (520 mm)

Development length for a hooked No. 8 bar (Code section 12.5)

$$l_{dh} = (0.02\beta\lambda f_y/\sqrt{f'_c})d_b = [(0.02)(1)(1)(60{,}000\text{ psi})/\sqrt{4000\text{ psi}}](1.0\text{ in.})$$

$$l_{dh} = 19.0\text{ in.(480 mm)}$$
$$[> 8d_b\text{ and } > 6\text{ in. (150 mm)}]$$

Although l_{dh} is less than the available length, it would be a tight fit if only 90 degree hooks were used for all the bars in each of the three rows. In-plane, 180 degree hooks could be used for some of the bars to partially relieve the reinforcement congestion. The use of mechanical anchorage devices, which have been used successfully in tests of deep beams,[9] could also be considered.

Step 5: Provide minimum reinforcement in Strut 1-2

Code Section A.3.3 requires that a minimum percentage of reinforcement be distributed across bottle-shaped struts to control cracking along the axis of the strut. This reinforcement can be provided either in an orthogonal mesh or as vertical-only or horizontal-only reinforcement. For this beam, both vertical and horizontal reinforcement will be used to satisfy the minimum reinforcement requirement. Because of the depth of the beam, the horizontal reinforcement in the lower portion of the beam must also satisfy the requirements for skin reinforcement in Code Section 10.6.7. Finally, we believe it is good detailing practice to have the horizontal and vertical steel also satisfy the minimum percentage and maximum spacing requirements in Code Sections 11.8.4 and 11.8.5, respectively. The angle between the axis of Strut 1-2 and the vertical reinforcement is

$$\gamma_1(\text{vertical steel}) = 90 - 44.6 = 45.4 \text{ degrees}$$

For vertical steel, use No. 4 ties, with four legs, at a spacing of 10 in. (250 mm)(< $d/5$)

$$\rho_v = \frac{4(0.20 \text{ in.}^2)}{(10 \text{ in.})(20 \text{ in.})} = 0.00400 > 0.0025$$

$$\rho_v(\sin \gamma_1) = (0.00400)(0.712) = 0.00285$$

The angle between the axis of Strut 1-2 and the horizontal reinforcement is

$$\gamma_2(\text{horizontal steel}) = 44.6 \text{ degrees}$$

For horizontal steel, use two No. 4 bars per layer, with a spacing of 8 in. (200 mm) between layers, to satisfy the skin reinforcement requirement (Fig. 7). Checking the percentage of horizontal reinforcement

$$\rho_h = \frac{2(0.20 \text{ in.}^2)}{(8 \text{ in.})(20 \text{ in.})} = 0.00250 > 0.0015$$

$$\rho_h(\sin \gamma_2) = (0.00250)(0.702) = 0.00176$$

Finally, checking the requirements of Code Section A.3.3

$$\Sigma(\rho_i)(\sin \gamma_i) = 0.00461 > 0.003 \text{ (o.k.)}$$

SOLUTION OF RIGHT PORTION OF BEAM

Figure 2 shows the truss geometry and forces for the right portion of the deep beam.

$$\tan \alpha_2 = (49 \text{ in.})/(50.2 \text{ in.}) \text{ implies that } \alpha_2 = 44.3 \text{ degrees}$$

Step 6: Evaluate right side of Node 2

For consistency, the acting compressive stress on the interior vertical face of Node 2 must be the same for both the left side and right side of the node (Fig. 8). In Step 3, the acting compressive stress on the vertical face was calculated to be 2.17 ksi (15.0 MPa). To clarify vector equilibrium for this part of Node 2, the horizontal force acting on the interior vertical face is shown as two components. Strut 2-3 will behave like the compression zone in a flexural member, so we assume it is a prismatic strut with $\beta_s = 1.0$. Then, the effective compressive strength for Strut 2-3 multiplied by ϕ is

$$\phi f_{cu}(2\text{-}3) = \phi(0.85)\,\beta_s\, f'_c = (0.75)(0.85)(1.0)(4.0 \text{ ksi}) = 2.55 \text{ ksi (17.6 MPa)}$$

Therefore, the acting compressive stress on the interior vertical face of Node 2 governs for calculating the width of Strut 2-3

$$w_{23} = \frac{F_{23}}{(2.17 \text{ ksi})b_w} = \frac{217 \text{ k}}{(2.17 \text{ ksi})(20 \text{ in.})} = 5.00 \text{ in. (125 mm)}$$

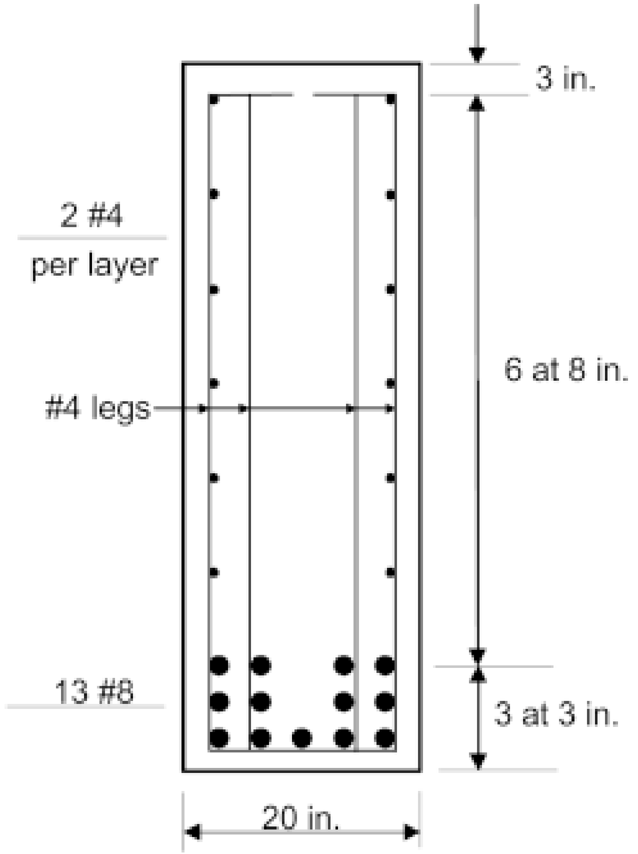


Fig. 7: Beam section showing the horizontal steel to satisfy the skin reinforcement requirement (1 in. = 25.4 mm)

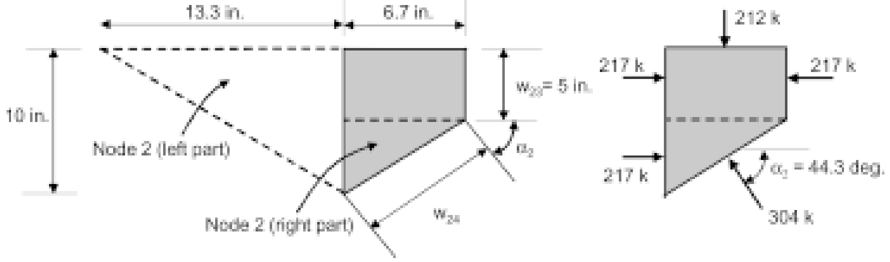


Fig. 8: Geometry, forces, and dimensions for the right part of Node 2. For consistency, the compressive stress on the vertical faces of Node 2 must be equal (1 in. = 25.4 mm, 1 k = 4.45 kN)

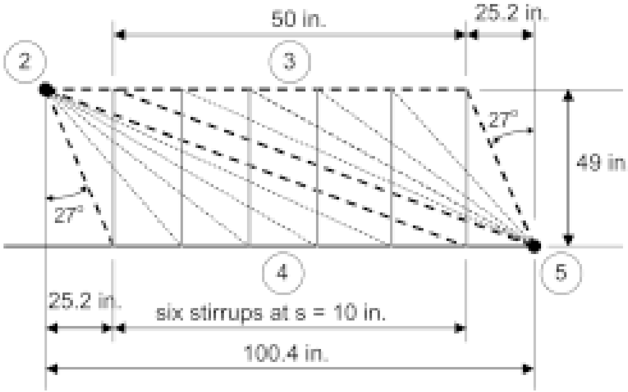


Fig. 9: Fan-shaped Struts 2-4 and 3-5 in the right span of the beam engage several stirrups (1 in. = 25.4 mm)

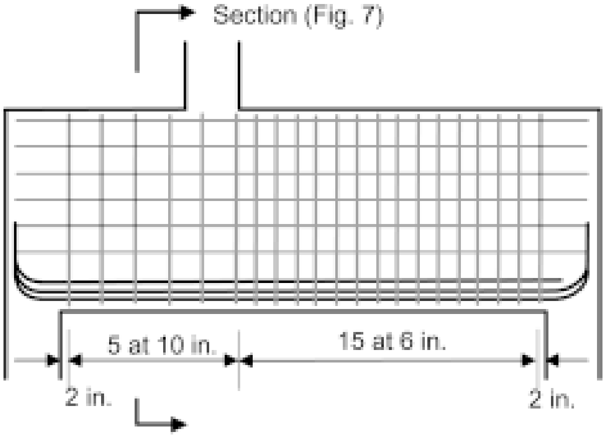


Fig. 10: Longitudinal reinforcement and stirrup spacing for the right portion of the deep beam. A total of 14 stirrups are provided at a uniform spacing of 6 in. (1 in. = 25.4 mm)

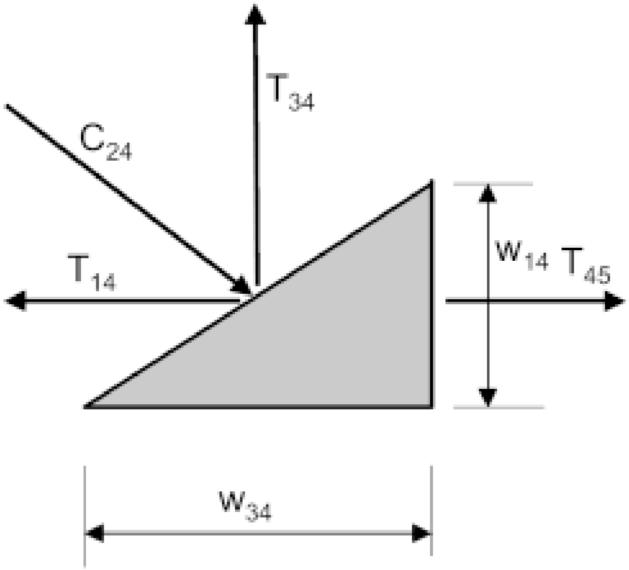


Fig. 11: General sketch of Node 4 geometry, which is considered a CTT node

Now use geometry to determine width of Strut 2-4 at Node 2

$$w_{24} = 5\text{ in. }(\cos \alpha_2) + 6.7\text{ in. }(\sin \alpha_2) = 3.58\text{ in.} + 4.66\text{ in.} = 8.24\text{ in. } (210\text{ mm})$$

Check strength of Strut 2-4 at Node 2, use $f_{cu}(2\text{-}4) = f_{cu}(1\text{-}2) = 2.55$ ksi (17.6 MPa)

$$\phi F_{ns}(2\text{-}4) = \phi f_{cu}(2\text{-}4) w_{24} b_w = 0.75(2.55\text{ ksi})(8.24\text{ in.})(20\text{ in.}) = 315\text{ k } (1400\text{ kN}) > 304\text{ k } (1350\text{ kN})\text{ (o.k.)}$$

Step 7: Evaluate reinforcement required in Tie 3-4

$$\phi F_{nt}(3\text{-}4) = \phi A_s(3\text{-}4) f_y \geq 212\text{ k } (945\text{ kN})$$

Thus,

$$A_s \geq \frac{212\text{ k}}{(0.75)(60\text{ ksi})} = 4.71\text{ in.}^2\ (3040\text{ mm}^2)$$

Select No. 4 bars with four legs per stirrup set, A_v (per set) = 0.80 in.^{2} (520 mm^2). Thus, we need to use at least six sets of No. 4 stirrups with four legs for $A_S(3\text{-}4) = 4.80$ in.^{2} (3100 mm^2).

It is reasonable to assume that the compression Strut 2-4 will "fan out" and engage several stirrups, as shown in Fig. 9. Strut 3-5 will also fan out (Fig. 9). The limiting dimension for the wide portion of the fan-shaped strut can be determined by using the minimum angle required between a strut and tie, defined as 25 degrees in Appendix A of the Code. The dimensions shown in Fig. 9 satisfy this minimum angle requirement. Some designers might prefer to concentrate the stirrup reinforcement at approximately the location of Tie 3-4 in the truss model (Fig. 2). The writers, however, believe that a uniform spacing of the transverse reinforcement within the dimensions of the fan-shaped struts is a more reasonable solution.

The six stirrup sets required for Tie 3-4 will need to be combined with the vertical reinforcement (stirrups) required as crack-control reinforcement crossing the inclined struts, as calculated in Step 5. If that reinforcement is provided at a spacing of 12 in. (305 mm), all the minimum reinforcement requirements are satisfied. Thus, a total for 14 stirrups, six for Tie 3-4 and eight for crack control, are provided at a uniform spacing of 6 in. (150 mm) in the right portion of deep beam (Fig. 10).

Step 8: Check of Node 4

Figure 11 is a general sketch of Node 4. This can be considered a compression-tension-tension (CTT) node because there is a force transfer between Tie 1-4 and Tie 4-5 within the node. The distribution of longitudinal and vertical reinforcement already determines the vertical and horizontal dimensions of the node. As determined by the distribution of longitudinal reinforcement at Node 1 (Fig. 5), the vertical dimension of Node 4 is 12 in. (305 mm). Then, conservatively assuming that the node extends horizontally over the spread between the closest six stirrups required for Tie 3-4 (placed at a 6 in. [150 mm] spacing), the horizontal dimension of Node 4 is 30 in. (760 mm).

For this CTT node, β_n = 0.6 and the effective compressive strength is

$$f_{cu}(4) = (0.85)(\beta_n)(f'_c) = (0.85)(0.6)(4 \text{ ksi}) = 2.04 \text{ ksi } (14.1 \text{ MPa})$$

The force transfer at the level of the longitudinal steel is 217 kips (965 kN), so the minimum height of the Node w_{14} is

$$w_{14} \geq \frac{F_{14} - F_{45}}{\phi f_{cu}(4) b_w} = \frac{217 \text{ k}}{(0.75)(2.04 \text{ ksi})(20 \text{ in.})} =$$

$$7.09 \text{ in. } (180 \text{ mm}) \leq 12 \text{ in. } (305 \text{ mm})$$

A similar check can be made for the horizontal dimension, but because the vertical force in Tie 3-4 is 212 kip (945 kN), the required horizontal dimension will be close to 7 in. (180 mm), which is significantly less than 30 in. (760 mm). Because Strut 2-4 will be very wide at this node, there is no need to check the effective compressive strength of the strut. Thus, the dimensions of Node 4 are acceptable. Further, because Node 3 is a CCT node that carries similar loads and has a similar geometry to Node 4, there is no need to check Node 3.

Step 9: Confirm that vertical and horizontal steel satisfies Code Section A.3.3

The same horizontal reinforcement is provided here as was used in the left portion of the beam (Step 5), but the spacing for the vertical reinforcement used for crack control has effectively been increased to 12 in. (305 mm), as stated in Step 7. Combining this change with the small change in the angle of inclination for the strut leads to the following results.

ρ_v = 0.00333, and $\Sigma(\rho_i)(\sin \gamma_i) = 0.00413 > 0.003$ (o.k.).

Step 10: Check dimensions and anchorage at Node 5

Set the width of Tie 4-5 equal to 9 in. (230 mm), so the centroid of the tie will correspond to the centroid of the bottom two layers of longitudinal reinforcement, not including the center bar of the bottom layer (Fig. 12). The effective compressive strength of Node 5 (CCT node) will be the same as that for Node 1, that is $f_{cu}(5)$ = 2.72 ksi (18.7 MPa). Using this value to check the minimum width of Tie 4-5 shows

$$w_{45}(\min) = \frac{F_{45}}{\phi f_{cu}(4) b_w} = \frac{217 \text{ k}}{(0.75)(2.72 \text{ ksi})(20 \text{ in.})} =$$

$$(5.32 \text{ in. } (135 \text{ mm}) < 9 \text{ in. } (230 \text{ mm}) \text{ (o.k.)}$$

Based on the check of Node 1 (Step 1), the stress at the base of Node 5 is acceptable because the reaction force is approximately half of that at Node 1 while the base dimensions and the effective compressive strength of this node are the same.

Reinforcement required in Tie 4-5

$$A_s(\text{req'd.}) = \frac{F_{45}}{\phi f_y} = \frac{217 \text{ k}}{(0.75)(60 \text{ ksi})} = 4.82 \text{ in.}^2 (3110 \text{ mm}^2)$$

To provide the required area of reinforcement at Node 5, assume eight bars from the bottom two layers of No. 8 bars will be fully anchored at Node 5 (center bar in lowest layer will not be hooked). Thus

$$A_s(\text{provided}) = 8(0.79 \text{ in.}^2) = 6.32 \text{ in.}^2 (4080 \text{ mm}^2) > 4.82 \text{ in.}^2 (3110 \text{ mm}^2) \text{ (o.k.)}$$

The critical section for anchorage of this reinforcement is at a distance l_a from face of support (Fig. 12)

$$l_a = \frac{w_{45}/2}{\tan \alpha_2} = \frac{4.5 \text{ in.}}{0.977} = 4.61 \text{ in. } (120 \text{ mm})$$

Available anchorage distance = $l_a + l_{b5} - 1.5$ in. = 19.1 in. (485 mm).

From Step 4, $l_{dh}(1)$ = 19.0 in. (480 mm), but this can be reduced here because significantly more steel is provided than required

$$l_{dh}(5) = l_{dh}(1) \frac{A_s(\text{req'd.})}{A_s(\text{provided})}$$

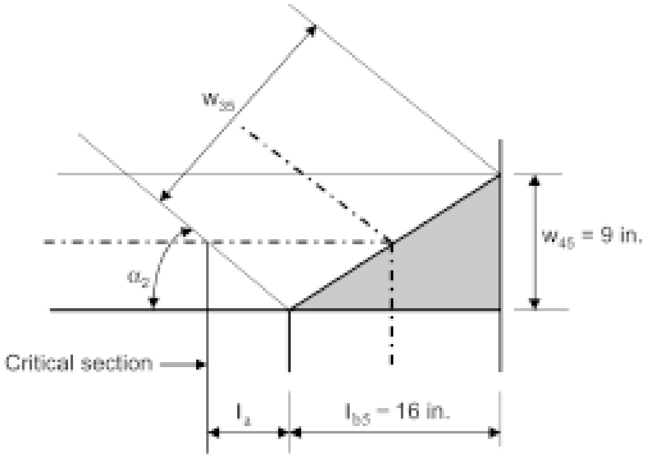


Fig. 12: Geometry and dimensions of Node 5. The critical section for anchorage of the reinforcement is at a distance l_a from the face of the support (1 in. = 25.4 mm)

$$l_{dh}(5) = 19.0 \text{ in.}\left(\frac{4.82 \text{ in.}^2}{6.32 \text{ in.}^2}\right) = 14.5 \text{ in. (370 mm)} \leq$$

$$19.1 \text{ in. (485 mm) (o.k.)}$$

Step 11: Check strength of Strut 3-5 at Node 5

Because the minimum crack-control reinforcement required by Code Section A.3.3 (Step 9) crosses this strut, the effective compressive strength, $f_{cu}(3\text{-}5)$, is the same as that used for Strut 1-2 (2.55 ksi [17.6 MPa]). Strut 3-5 must be checked at its narrower end, which occurs at Node 5. From Fig. 12, this width is

$$w_{35} = l_{b5} \sin \alpha_2 + w_{45} \cos \alpha_2 = 17.6 \text{ in. (447 mm)}$$

Thus, the strength of this strut at Node 5 is

$$\phi F_{ns}(3\text{-}5) = \phi f_{cu}(3\text{-}5) w_{35}\, b_w = (0.75)(2.55 \text{ ksi})(17.6 \text{ in.})(20 \text{ in.}) = 673 \text{ k (3000 kN)} > 304 \text{ k (1350 kN) (o.k.)}$$

Step 12: Check straight anchorage of No. 8 bars from Node 4 to Node 5

Assume the top layer of bars and the middle bar in the bottom row are developed as straight bars from Node 4 to Node 5 (Fig. 2). The available anchorage to the center of support at Node 5 is at least 50.2 in. (1280 mm). The required development length from Section 12.2.2 of the Code is

$$l_d = \frac{f_y \alpha \beta \lambda}{20\sqrt{f'_c}} d_b = \frac{(60{,}000 \text{ psi})(1)(1)(1)}{20\sqrt{4000 \text{ psi}}}(1.0 \text{ in.}) =$$

$$47.4 \text{ in. (1200 mm)} < 50.2 \text{ in. (1280 mm)}$$

FINAL DETAILS

Figure 7 and 10 show the final reinforcement details. The anchorage of three layers of No. 8 bars with 90 degree hooks will cause some detailing and construction problems at Node 1. As discussed earlier, the use of mechanical anchorage devices or 180 degree hooks in the plane of the reinforcement layers may be required.

The spacing of stirrups in the right span of the beam is smaller than that in the left span. Most designers would expect a wider spacing in the right span because the design shear force is lower than in the left span. This apparent anomaly occurs because there is essentially no concrete contribution (former V_c term) in the right span, that is, all the design shear is assigned to the web reinforcement. In the left span, the entire design shear is essentially carried by the concrete strut, with the addition of minimum specified amounts of vertical and horizontal steel to control cracking in the bottle-shaped Strut 1-2.

As stated in the introduction, this is not a unique solution and other designs could be developed using STM. Comments on this design and suggested modifications are welcome.

References

1. Schlaich, J.; Schäfer, K.; and Jennewein, M., "Toward a Consistent Design of Structural Concrete," *PCI Journal*, V. 32, No. 3, 1987, pp. 74-150.

2. FIP Commission 3, "FIP Recommendations, Practical Design of Structural Concrete," *FIP Congress*, SETO, London, England, 1996.

3. Canadian Standards Association, "Design of Concrete Structures, CSA Standard A23.3-94," Canadian Standards Association, Ottawa, Canada, 1994.

4. AASHTO, "AASHTO LRFD Bridge Specifications for Highway Bridges" (2001 Interim Revisions), American Association of Highway and Transportation Officials, Washington, D.C., 1998.

5. ACI-ASCE Committee 445, "Recent Approaches to Shear Design of Structural Concrete," *ASCE Journal of Structural Engineering*, V. 124, No. 12, 1998, pp. 1375-1417.

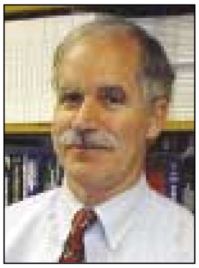
6. ACI Committee 318, "Building Code Requirements for Structural Concrete (ACI 318-02)," American Concrete Institute, Farmington Hills, MI, 2002, 443 pp.

7. Reineck, K.-H., ed., *Examples for the Design of Structural Concrete with Strut-and-Tie Models*, SP-208, American Concrete Institute, Farmington Hills, MI, 2002, 242 pp.

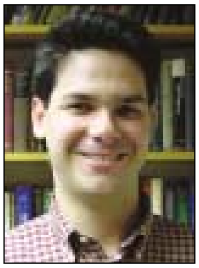
8. Fanella, D., and Rabbat, B., "Notes on ACI 318-02 Building Code Requirements for Structural Concrete," Portland Cement Association, 2002, Skokie, IL.

9. Aguilar, G.; Matamoros, A.; Parra-Montesinos, G.; Ramirez, J.; and Wight, J. K., "Experimental Evaluation of Design Procedures for Shear Strength of Deep Reinforced Concrete Beams," *ACI Structural Journal*, V. 99, No. 4, July-Aug. 2002, pp. 539-548.

Received and reviewed under Institute publication policies.



James K. Wight, FACI, is a professor of civil engineering at the University of Michigan, Ann Arbor. He is Chair of ACI Committee 318, Structural Concrete Building Code, and former Chair of ACI Committee 318-F, New Materials, Products, Ideas. Wight is also a member of ACI Committee 369, Seismic Repair and Rehabilitation, and Joint ACI-ASCE Committees 352, Joints and Connections in Monolithic Concrete Structures, and 445, Shear and Torsion. His primary research interest is earthquake resistant design of reinforced concrete structures.



ACI member **Gustavo J. Parra-Montesinos** is an assistant professor at the University of Michigan, Ann Arbor, where he obtained his PhD in 2000. He is member of Joint ACI-ASCE Committee 352, Joints and Connections in Monolithic Concrete Structures, and Secretary of ACI Committee 335, Composite and Hybrid Structures. His research interests include the seismic behavior and design of reinforced concrete, hybrid steel-concrete, and fiber-reinforced concrete structures.