⊞—RK23

⊟—Non Lineal 2nd degree ODE

Values

$V := 20\ \frac{m}{s}$ $a := 50\ m$ $b := 100\ m$ $r := \sqrt{\frac{a^2 + b^2}{2}}$ $\omega := \frac{V}{r} = 0.253\ Hz$

$m := 189\ kg$ $h_G := 0.53\ m$ $b_G := 0.703\ m$ $g := 9.8\ \frac{m}{s^2}$

$I_{xG} := 6.73\ kg\ m^2$ $I_{zG} := 36.4\ kg\ m^2$

Equations

$$\begin{cases} x(t) := \left(a - a \cdot \sin\left(\omega \cdot t + \frac{\pi}{2}\right)\right) \\ y(t) := b \cdot \sin(\omega \cdot t) \end{cases}$$

$$c(t) := \frac{\left| \frac{d}{dt} x(t) \cdot \frac{d^2}{dt^2} y(t) - \frac{d}{dt} y(t) \cdot \frac{d^2}{dt^2} x(t) \right|}{\sqrt{\left(\left(\frac{d}{dt} x(t)\right)^2 + \left(\frac{d}{dt} y(t)\right)^2 \right)^3}}$$

$$R_c(t) := \frac{1}{c(t)}$$

ODE

$$m \cdot V^2 \cdot \frac{h_G}{R_c} \cdot \cos(\varphi) - m \cdot g \cdot h_G \cdot \sin(\varphi) - \left(I_{xG} + m \cdot h_G^2\right) \cdot \varphi'' = 0$$

For use the power of the linear algebra tools, numerical methods ask for convert the equation as a system of equations where each element is the derivative of the unknow function: $\varphi(t) = \varphi_1$, $\varphi'(t) = \varphi_2$ and $\varphi''(t) = \varphi_2$.

$$D(t, \varphi) := \begin{bmatrix} \varphi_2 \\ \dfrac{m \cdot V^2 \cdot \frac{h_G}{R_c(t)} \cdot \cos(\varphi_1) - m \cdot g \cdot h_G \cdot \sin(\varphi_1)}{I_{xG} + m \cdot h_G^2} \end{bmatrix}$$

$$\begin{bmatrix} t_o & t_{end} \end{bmatrix} := \begin{bmatrix} 0 & 6.187 \end{bmatrix} s$$

$$\begin{bmatrix} \varphi_o & \varphi_{end} \end{bmatrix} := \begin{bmatrix} 0 & 0.867 \end{bmatrix} rad$$

For an initual value (or Cauchy) problem, you solve the problem calling $RK23\left(D(t, \varphi), \begin{bmatrix} t_o & t_{end} \end{bmatrix}, \begin{bmatrix} \varphi_o & \varphi'_o \end{bmatrix}, N, \varepsilon x\right)$

But seems that you don't know φ'_o, but φ_{end} at t_{end} . You can try the "shooting method", implemented in mathcad as sbval.

Solve the boundary problem $f(t, \varphi, \varphi', \varphi'') = F(t)$ with $\varphi(t_o) = \varphi_o$ and $\varphi(t_{end}) = \varphi_{end}$

Newton Raphson Solver for the boundary problem

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sbval(φ'guess) := | [ f(ξ) := | (X := RK(ξ)) rows(X) 2 − φend   x := φ'guess ]
                  | for k ∈ [1..N]
                  |   | y := [ f(x)  f(x + h·UnitsOf(x)) ]
                  |   | if h·|y1| < εx·|y2 − y1|
                  |   |    break
                  |   | else
                  |   |    x := x − (h·UnitsOf(x)·y1) / (y2 − y1)
                  | if k ≤ N
                  |   x
                  | else
                  |   error("Max iters rached.")
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Runge Kutta Solver

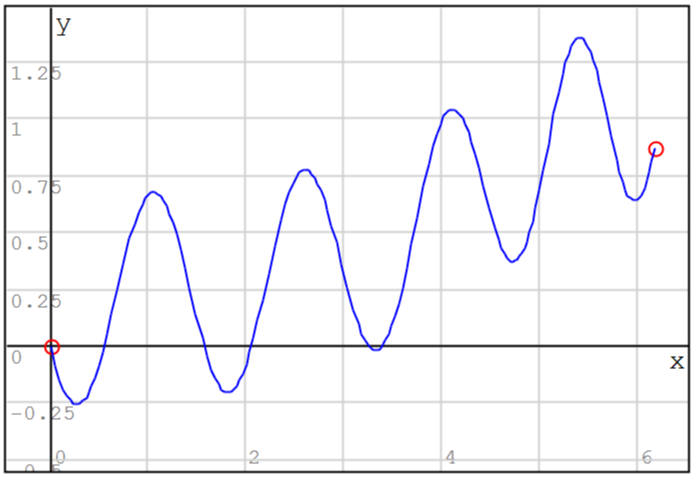
$$RK(\varphi'_o) := \left| RK23\left(D(t, \varphi), \begin{bmatrix} t_o & t_{end} \end{bmatrix}, \begin{bmatrix} \varphi_o & \varphi'_o \end{bmatrix}, N, \varepsilon x\right)\right.$$

Values $N := 25$ $h := 10^{-5}$ $\varepsilon x := 10^{-3}$

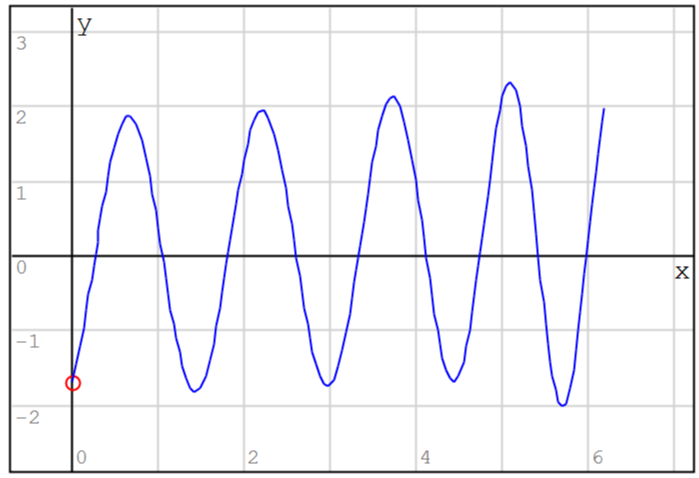
Solution $\varphi'_{guess} := 1$ Hz $\varphi' := sbval(\varphi'_{guess}) = -1.6658$ Hz $\varphi sol := RK(\varphi')$

$\mathrm{rows}(\varphi sol) = 178$

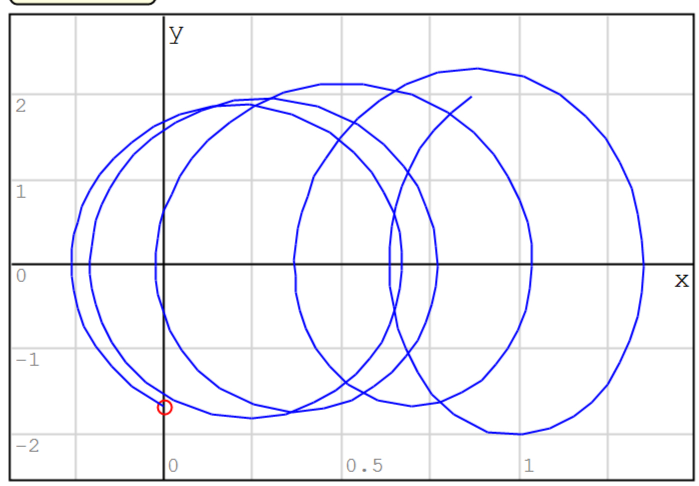
φ(t)



φ'(t)



State space



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