⊞—Numerical and Symbolic Inverse Laplace Transform

⊟—Maxima Laplace

Initial value problem

$$ode := \begin{bmatrix} \frac{d^2}{dt^2} f(t) + \frac{d}{dt} g(t) + 3 \cdot f(t) = 15 \cdot \exp(-t) \\ \frac{d^2}{dt^2} g(t) - 4 \cdot \frac{d}{dt} f(t) + 3 \cdot g(t) = 15 \cdot \sin(2 \cdot t) \end{bmatrix}$$

$$ic := \begin{bmatrix} 35 & -48 \\ 27 & -55 \end{bmatrix} \quad \begin{bmatrix} f(0) & f'(0) \\ g(0) & g'(0) \end{bmatrix}$$

$$fnc := \begin{bmatrix} f & F \\ g & G \end{bmatrix} \quad \begin{bmatrix} d\text{-}domain & s\text{-}domain \end{bmatrix}$$

Symbolic Solver call $ODE := LapOde(ode, ic, fnc, t, s)$

Laplace transform of the ode

$$ODE_1 = \begin{bmatrix} s \cdot (-35 + s \cdot F(s) + G(s)) + 3 \cdot (7 + F(s)) = \frac{15}{1+s} \\ 55 + \left(3 + s^2\right) \cdot G(s) - 4 \cdot (-35 + s \cdot F(s)) - 27 \cdot s = \frac{30}{4 + s^2} \end{bmatrix}$$

Freq-domain solution, used in numerical solution

$$ODE_2 = \begin{cases} F(s) = \dfrac{6 \cdot \left(2 \cdot \left(-6 + 85 \cdot s^2\right) + 153 \cdot s\right) + s^3 \cdot (185 + s \cdot (407 + s \cdot (-13 + 35 \cdot s)))}{2 \cdot (1+s) \cdot \left(18 + 7 \cdot s^4\right) + s^2 \cdot \left(49 + s \cdot \left(49 + s^3 \cdot (1+s)\right)\right)} \\ G(s) = \dfrac{3 \cdot \left(3 \cdot \left(-250 + 3 \cdot s^6\right) - 674 \cdot s\right) + s^2 \cdot (-787 + 2 \cdot s \cdot (-305 + (25 - 14 \cdot s) \cdot s))}{2 \cdot (1+s) \cdot \left(18 + 7 \cdot s^4\right) + s^2 \cdot \left(49 + s \cdot \left(49 + s^3 \cdot (1+s)\right)\right)} \end{cases}$$

Time-domain solution

$$ODE_3 = \begin{bmatrix} f(t) = \dfrac{3 \cdot \left(1 + 5 \cdot (2 \cdot \cos(t) - \sin(3 \cdot t)) \cdot e^t\right) + 2 \cdot \cos(2 \cdot t) \cdot e^t}{e^t} \\ g(t) = \dfrac{3 \cdot \left(-1 + 10 \cdot (-2 \cdot \sin(t) + \cos(3 \cdot t)) \cdot e^t\right) + \sin(2 \cdot t) \cdot e^t}{e^t} \end{bmatrix}$$

Plots: numerical and symbolic solutions

$Assign\left(ODE_2\right)$

$Assign\left(ODE_3\right)$

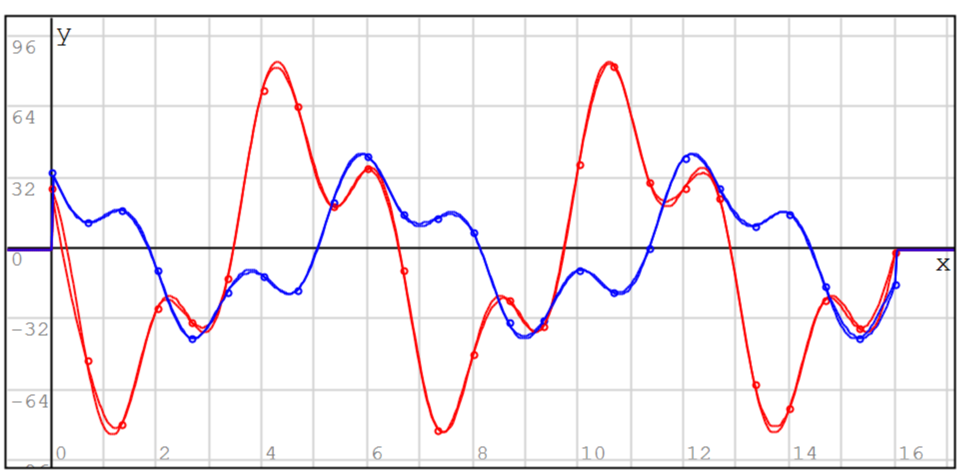
$[ti \; te \; N] := [0 \; 16 \; 25]$

$f := ILap(F(s), ti, te, N)$

$fn(t) := cinterp(f, t)$

$g := ILap(G(s), ti, te, N)$

$gn(t) := cinterp(g, t)$



Increase N for better numerical adjust.

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