Integrating and differentiating piecewise functions with Maxima

Unit step function

$$\Phi(x\#) := \begin{cases} \text{if } \Phi s = \text{"sign"} \\ \quad \dfrac{1+\operatorname{sign}(x\#)}{2} \\ \text{else} \\ \quad \dfrac{1}{2} + \dfrac{x\#}{2 \cdot |x\#|} \end{cases}$$

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$$\int f\# \, d\, x\# := \text{ℳ}\left(\int f\# \, d\, x\#\right)$$

if c1 greathar than c2 then a else b.

$$\Phi\_GT(c1\#, c2\#, a\#, b\#) := a\# \cdot \Phi(c1\# - c2\#) - b\# \cdot \Phi(c1\# - c2\#) + b\#$$

Max of two expressions

$$\Phi\_MAX(u\#, v\#) := \Phi\_GT(u\#, v\#, u\#, v\#)$$

Min of two expressions

$$If\_MIN(u\#, v\#) := \Phi\_GT(v\#, u\#, u\#, v\#)$$

if c equals zero then a else zero

$$\Phi\_EQ(c\#, a\#) := a\# \cdot (\Phi(c\# + \Phi\varepsilon) - \Phi(c\# - \Phi\varepsilon))$$

if c1 equals c2 then a else b

$$\Phi\_EQ(c1\#, c2\#, a\#, b\#) := b\# + \Phi\_EQ(c1\# - c2\#, a\#) - \Phi\_EQ(c1\# - c2\#, b\#)$$

$\Phi s := \text{"sign"}$ $\quad \Phi\varepsilon := 0.000000001$ $\quad h := 0.0000001$

$$I(t) := \int g(t) \, d\, t$$

$$In(t) := I(t)\Big|_{t=0} + \int_0^t g(\tau) \, d\, \tau$$

Numerical and symbolic derivatives.

$$D(t) := \begin{cases} \Phi s := \text{"abs"} \\ \dfrac{d}{d\,t} g(t) \end{cases}$$

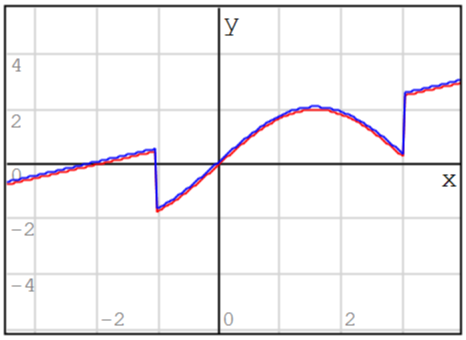
$$Dn(t) := \frac{g(t+h) - g(t-h)}{2\,h}$$

Inequalities

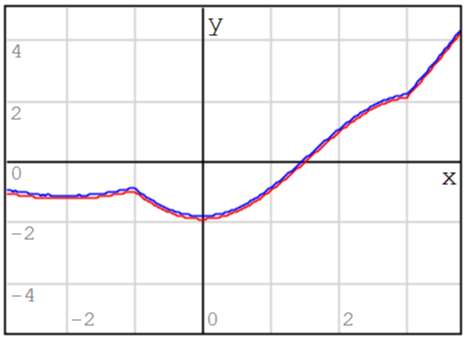
$c1 := t^2 - 2 \cdot t$ $\quad c2 := 3$ $\quad a := 0.5 \cdot t + 1$ $\quad b := 2 \cdot \sin(t)$

$$f(t) := \begin{cases} \text{if } c1 > c2 \\ \quad a \\ \text{else} \\ \quad b \end{cases}$$

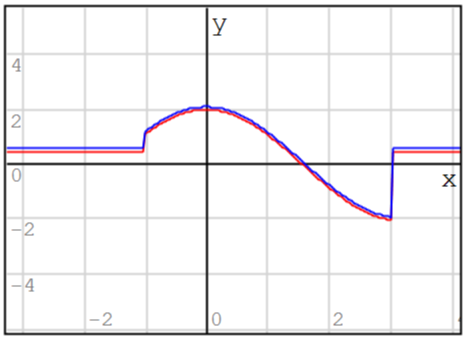
$$g(t) := \Phi\_GT(c1, c2, a, b)$$



$\begin{cases} f(x) + 0.1 \\ g(x) \end{cases}$



$\begin{cases} In(x) + 0.1 \\ I(x) \end{cases}$



$\begin{cases} D(x) + 0.1 \\ Dn(x) \end{cases}$

Equalities

$\text{Clear}(\Phi\varepsilon) = 1$

$$f(x) := \begin{cases} 1 & \text{if } x = 0 \\ 2 & \text{if } x = 1 \\ 3 & \text{if } x = 2 \\ 0 & \text{otherwise} \end{cases}$$

Nuerically this have a poor representation and significance.

This other is better

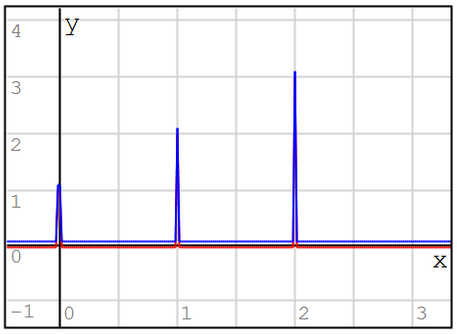
$$f(x) := \begin{cases} 1 & \text{if } |x - 0| < \Phi\varepsilon \\ 2 & \text{if } |x - 1| < \Phi\varepsilon \\ 3 & \text{if } |x - 2| < \Phi\varepsilon \\ 0 & \text{otherwise} \end{cases}$$

Same using unit step Φ

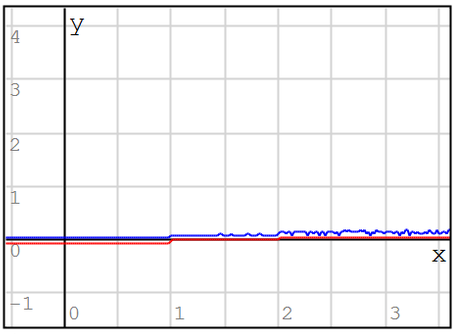
$g(x) := |\Phi\_EQ(x, 0, 1, 0) + \Phi\_EQ(x, 1, 2, 0) + \Phi\_EQ(x, 2, 3, 0)$

Use a big epsilon for this example

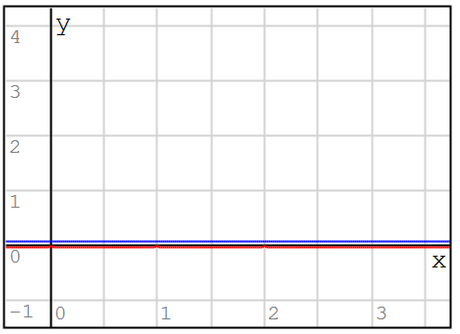
$\Phi\varepsilon := 0.01$



$\begin{cases} f(x) + 0.1 \\ g(x) \end{cases}$



$\begin{cases} In(x) + 0.1 \\ I(x) \end{cases}$



$\begin{cases} Dn(x) + 0.1 \\ D(x) \end{cases}$

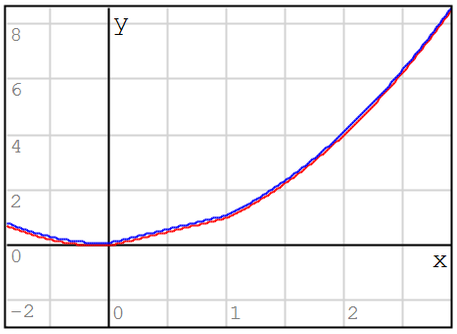
Expected value for the integral is zero, but for derivatives is infinity at the discontinuities.

Max and Min

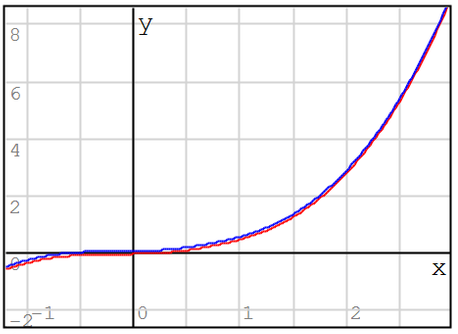
$a := t \qquad b := t^2$

$f(t) := |\max([a\ b])$

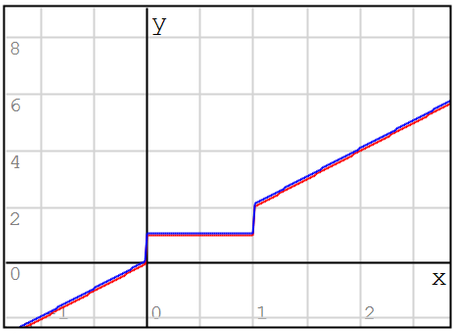
$g(t) := |\Phi\_MAX(a, b)$



$\begin{cases} f(x) + 0.1 \\ g(x) \end{cases}$



$\begin{cases} In(x) + 0.1 \\ I(x) \end{cases}$



$\begin{cases} Dn(x) + 0.1 \\ D(x) \end{cases}$

Alvaro