Sub Frame Analysis - Stiffness Method

Example 1 - Page 25
Worked Examples for the Design of Concrete Structures to Eurocode 2
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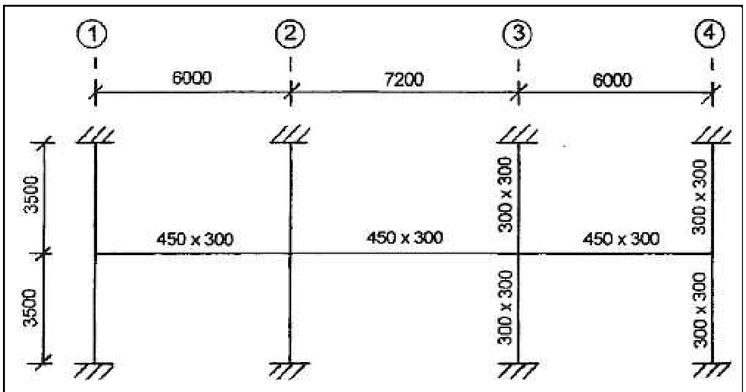

Dimensions of sub-frame

appVersion(− 4) = "0.99.7921.69"

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$t_0 := \text{time}(0)$

Define

$kNm := kN\ m$

Assume

$E := 40\ MPa$

Assume Young's Modulus for RC: This is not really required, but used only for consistent units in intermediate calculations. We can use any numerical value.
Ex E = 1 MPa OR E = 1

Loading

For the slabs, the maximum design load is 12.25 kN/m^2 and the minimum load is 1.25 × 5.0 = 6.25 kN/m^2. The loads on the first interior beam taking shear force coefficients for the slab of 0.6 for the end span and 0.5 for the interior span, are:

		(max)		(min)
Slab	1.1 × 4.8 × 12.25 =	64.7	1.1 × 4.8 × 6.25 =	33.0
Beam	1.25 × 0.3 × 0.3 × 25 =	2.8	=	2.8
		67.5 kN/m		35.8 kN/m

Joint Moment Sign Convention	
Clockwise	**+ ve**
Anticlockwise	**- ve**

PROGRAM 1: STIFFNESS METHOD: To obtain Moments of Beams, Uppe Columns & Lower Columns

Program 1: Support / Column Moments for All Cases Using Stiffness Method

$BMM(ff, LL, k) :=$

$$S_{BEAM} := \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$$ Beam Stiffness

$$\begin{bmatrix} FF1 := FF_1 \\ FF2 := FF_2 \\ FF3 := FF_3 \end{bmatrix}$$

$$fem := \frac{1}{12} \cdot \left[\overrightarrow{FF1 \cdot LL^2} \quad \overrightarrow{FF2 \cdot LL^2} \quad \overrightarrow{FF3 \cdot LL^2} \right]^T$$ FEM

$$j := [1 .. \text{rows}(fem)]$$

$$MM_j := \begin{bmatrix} -fem_{j_1} \\ fem_{j_1} - fem_{j_2} \\ fem_{j_2} - fem_{j_3} \\ fem_{j_3} \end{bmatrix}$$ Joint Moments

$$ST_j := -k^{-1} \cdot MM_j$$ Rotations at Joints

$$M1_j := \left(\begin{bmatrix} -fem_{j_1} \\ fem_{j_1} \end{bmatrix} + \left(\frac{E \cdot I_{beam}}{LL_1} \cdot S_{BEAM} \cdot \begin{bmatrix} ST_{j_1} \\ ST_{j_2} \end{bmatrix} \right) \right)$$ Beam 1 Support Moments at 1 & 2

$$M2_j := \left(\begin{bmatrix} -fem_{j_2} \\ fem_{j_2} \end{bmatrix} + \left(\frac{E \cdot I_{beam}}{LL_2} \cdot S_{BEAM} \cdot \begin{bmatrix} ST_{j_2} \\ ST_{j_3} \end{bmatrix} \right) \right)$$ Beam 2 Support Moments at 2 & 3

$$M3_j := \left(\begin{bmatrix} -fem_{j_3} \\ fem_{j_3} \end{bmatrix} + \left(\frac{E \cdot I_{beam}}{LL_3} \cdot S_{BEAM} \cdot \begin{bmatrix} ST_{j_3} \\ ST_{j_4} \end{bmatrix} \right) \right)$$

Beam 3 Support Moments at 3 & 4

"--------------------------------"

$$M_{UC_j} := ST_j \cdot \frac{4 \cdot E \cdot I_{col}}{H_{UC}}$$

Upper Col Moments

$$M_{LC_j} := ST_j \cdot \frac{4 \cdot E \cdot I_{col}}{H_{LC}}$$

Lower Col Moments

augment $(M1, M2, M3, M_{UC}, M_{LC})$

PROGRAMS 2,3,4,5,6: To obtain SF, Max Span Moments, X_max

Program to find SF, Max Span Moment and Distance to Max Span Moment

PROGRAM 2: For any Loading Case Find SF, Max Span BM X_max

```
Param(M#, f, L#) := for j ∈ [1..rows(f)]
                        W_j := f_j · L#_j
                        V1_j := 0.5 · W_j − ( Σ M#_j / L#_j )
                        V2_j := W_j − V1_j
                        M_max_span_j := V1_j^2 / (2 · f_j) + M#_j1
                        X_max_j := V1_j / f_j
                    [ V1  V2  M_max_span  X_max ]
```

$$W = f \cdot L$$

$$V1 = \frac{W}{2} + \frac{(M_L + M_R)}{L}$$

$$V2 = W - V1$$

$$M_{max\_span} = \frac{V1^2}{2 \cdot f} + M_L$$

$$X_{max} = \frac{V1}{f}$$

PROGRAM 3: Calls PROGRAM 2 find SF, Max Span BM & X_max for ALL Loading Cases

```
Find_All(M#, ff, L#) := for j ∈ [1..rows(M#)]
                            M_case_j := M#_j[1..3]
                            B_j := Param(M_case_j, ff_j, L#)
                        M_case
                        B
```

PROGRAM 4: Extract All SFF

```
SF_All(res) := for j ∈ [1..rows(res)]
                   A_j := res_j1[1..2]
                   if j = 1
                       B := A_1
                   else
                       B := stack(B, A_j)
               B
```

PROGRAM 5: Extract ALL Max Span BMM

```
Span_M_All(res) := for j ∈ [1..rows(res)]
                       A_j := res_j13
                   A
```

PROGRAM 6: Extract All X-max

```
X_Max_All(res) := for j ∈ [1..rows(res)]
                      A_j := res_j14
                  A
```

DATA

Column dimensions

$b_{col} := 300\ mm$

$h_{col} := 300\ mm$

$H_{UC} := 3.5\ m$

$H_{LC} := 3.5\ m$

Beam dimendions

$L_{AB} := 6\ m$

$L_{BC} := 7.2\ m$

$L_{CD} := 6\ m$

$b_{beam} := 300\ mm$

$h_{beam} := 450\ mm$

Define

$$L := \begin{bmatrix} L_{AB} \\ L_{BC} \\ L_{CD} \end{bmatrix}$$

Calculation of Moments of Inertia : All Columns have same sizes & All Beams have same sizes

$$I_{col} := \frac{1}{12} \cdot b_{col} \cdot \left(h_{col}\right)^3 = 6.75 \cdot 10^8\ mm^4$$

$$I_{beam} := \frac{1}{12} \cdot b_{beam} \cdot \left(h_{beam}\right)^3 = 2.28 \cdot 10^9\ mm^4$$

Loading

Max

$$F1 := 67.5\ \frac{kN}{m}$$

Min

$$F2 := 35.8\ \frac{kN}{m}$$

Define Load Patterns for All 3 Cases

$$FF := \begin{bmatrix} \begin{bmatrix} F1 \\ F1 \\ F1 \end{bmatrix} & \begin{bmatrix} F1 \\ F2 \\ F1 \end{bmatrix} & \begin{bmatrix} F2 \\ F1 \\ F2 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 67.5 \\ 67.5 \\ 67.5 \end{bmatrix} & \begin{bmatrix} 67.5 \\ 35.8 \\ 67.5 \end{bmatrix} & \begin{bmatrix} 35.8 \\ 67.5 \\ 35.8 \end{bmatrix} \end{bmatrix} \frac{kN}{m}$$

1. Stifness Coeff due to unit rotation at A (Node 1)

$$K_{11} := \frac{4 \cdot E \cdot I_{col}}{H_{UC}} + \frac{4 \cdot E \cdot I_{col}}{H_{LC}} + \frac{4 \cdot E \cdot I_{beam}}{L_{AB}} = 122.4643\ kNm$$

$$K_{21} := \frac{2 \cdot E \cdot I_{beam}}{L_{AB}} = 30.37\ kNm$$

$K_{31} := 0$ $\qquad$ $K_{41} := 0$

2. Stifness Coeff due to unit rotation at B (Node 2)

$$K_{12} := \frac{2 \cdot E \cdot I_{beam}}{L_{AB}} = 30.37\ kNm$$

$$K_{22} := \frac{4 \cdot E \cdot I_{col}}{H_{UC}} + \frac{4 \cdot E \cdot I_{col}}{H_{LC}} + \frac{4 \cdot E \cdot I_{beam}}{L_{AB}} + \frac{4 \cdot E \cdot I_{beam}}{L_{BC}} = 173.0893\ kNm$$

$$K_{32} := \frac{2 \cdot E \cdot I_{beam}}{L_{BC}} = 25.31\ kNm$$

$K_{42} := 0$

3. Stifness Coeff due to unit rotation at C (Node 3)

$K_{13} := 0$

$$K_{23} := \frac{2 \cdot E \cdot I_{beam}}{L_{BC}} = 25.31\ kNm$$

$$K_{33} := \frac{4 \cdot E \cdot I_{col}}{H_{UC}} + \frac{4 \cdot E \cdot I_{col}}{H_{LC}} + \frac{4 \cdot E \cdot I_{beam}}{L_{BC}} + \frac{4 \cdot E \cdot I_{beam}}{L_{AB}} = 173.09\ kNm$$

$$K_{43} := \frac{2 \cdot E \cdot I_{beam}}{L_{CD}} = 30.37\ kNm$$

4. Stifness Coeff due to unit rotation at D (Node 4)

$$K_{14} := 0$$

$$K_{24} := 0$$

$$K_{34} := \frac{2 \cdot E \cdot I_{beam}}{L_{CD}}$$

$$K_{44} := \frac{4 \cdot E \cdot I_{col}}{H_{UC}} + \frac{4 \cdot E \cdot I_{col}}{H_{LC}} + \frac{4 \cdot E \cdot I_{beam}}{L_{CD}} = 122.46\ kNm$$

Global Stiffness Matrix

$$K := \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{21} & K_{22} & K_{23} & K_{24} \\ K_{31} & K_{32} & K_{33} & K_{34} \\ K_{41} & K_{42} & K_{43} & K_{44} \end{bmatrix} = \begin{bmatrix} 122.46 & 30.37 & 0 & 0 \\ 30.37 & 173.09 & 25.31 & 0 \\ 0 & 25.31 & 173.09 & 30.37 \\ 0 & 0 & 30.37 & 122.46 \end{bmatrix} kNm$$

Find Support / Column Moments for all 3 Cases. Using PROGRAM 1

Call PROGRAM 1

$$M_{all} := BMM(FF, L, K)$$

UC End | Beam | LC

48.9	−97.8	48.9	8.5	267.6	−284.6	8.5

	AB BA	BC CB	CD DC	Upper Col	Lower Col	

$$M_{all} = \begin{bmatrix} \begin{bmatrix} -97.8 \\ 267.5 \end{bmatrix} & \begin{bmatrix} -284.6 \\ 284.6 \end{bmatrix} & \begin{bmatrix} -267.5 \\ 97.8 \end{bmatrix} & \begin{bmatrix} 48.9 \\ 8.6 \\ -8.6 \\ -48.9 \end{bmatrix} & \begin{bmatrix} 48.9 \\ 8.6 \\ -8.6 \\ -48.9 \end{bmatrix} \\ \begin{bmatrix} -112.8 \\ 215.5 \end{bmatrix} & \begin{bmatrix} -172.4 \\ 172.4 \end{bmatrix} & \begin{bmatrix} -215.5 \\ 112.8 \end{bmatrix} & \begin{bmatrix} 56.4 \\ -21.6 \\ 21.6 \\ -56.4 \end{bmatrix} & \begin{bmatrix} 56.4 \\ -21.6 \\ 21.6 \\ -56.4 \end{bmatrix} \\ \begin{bmatrix} -36.9 \\ 193.8 \end{bmatrix} & \begin{bmatrix} -263.2 \\ 263.2 \end{bmatrix} & \begin{bmatrix} -193.8 \\ 36.9 \end{bmatrix} & \begin{bmatrix} 18.5 \\ 34.7 \\ -34.7 \\ -18.5 \end{bmatrix} & \begin{bmatrix} 18.5 \\ 34.7 \\ -34.7 \\ -18.5 \end{bmatrix} \end{bmatrix} kNm$$

← Case 1

← Case 2

← Case 3

$$\text{time}(0) - t_0 = 0.03\ s$$

Upper Column Moments: (from Column 4 of M_{all})

$$[[\ 48.9\ ]]$$

Lower Column Moments: (from Column 5 of M_{all})

$$[[\ 48.9\ ]]$$

$$M_{UC\_all} := M_{all\ [1..rows(M_{all})]\ 4} = \begin{bmatrix} \begin{bmatrix} 8.55 \\ -8.55 \\ -48.9 \end{bmatrix} \\ \begin{bmatrix} 56.38 \\ -21.58 \\ 21.58 \\ -56.38 \end{bmatrix} \\ \begin{bmatrix} 18.46 \\ 34.67 \\ -34.67 \\ -18.46 \end{bmatrix} \end{bmatrix} kNm \qquad M_{LC\_all} := M_{all\ [1..rows(M_{all})]\ 5} = \begin{bmatrix} \begin{bmatrix} 8.55 \\ -8.55 \\ -48.9 \end{bmatrix} \\ \begin{bmatrix} 56.38 \\ -21.58 \\ 21.58 \\ -56.38 \end{bmatrix} \\ \begin{bmatrix} 18.46 \\ 34.67 \\ -34.67 \\ -18.46 \end{bmatrix} \end{bmatrix} kNm$$

	Joint and member	End joint			Interior joint			
Row	Moments in members due to application of moments at joints	Upper column	Beam	Lower column	Upper column	End beam	Interior beam	Lower column
1	Unit moment applied at end joints	0.252	0.496	0.252		0.248		
2	Unit moment applied at interior joints		0.205		0.209	0.411	0.171	0.209
3	(Row 1)/0.248 – (Row 2)	1.016	1.795	1.016	–0.209	0.589	–0.171	–0.209
4	(Row 2)/0.205 – (Row 1)	–0.252	0.504	–0.252	1.017	1.752	0.832	1.017
5	(Row 3)/(1.016 + 1.795 + 1.016)	0.2655	0.469	0.2655	–0.055	0.154	–0.044	–0.055
6	(Row 4)/(1.017 + 1.752 + 0.832+ 1.017)	–0.055	0.110	–0.055	0.220	0.380	0.180	0.220
Case	Moments (kN m) in members for load case							
1	Maximum load (67.5 kN/m) on all spans							
	Fixed-end moments		–202.5			202.5	–291.6	
	(Row 5) × 202.5	53.8	94.9	53.8	–11.1	31.2	–9.0	–11.1
	(Row 6) × (291.6 – 202.5)	–4.9	9.8	–4.9	19.6	33.9	16.0	19.6
	Sum to obtain final moments	48.9	–97.8	48.9	8.5	267.6	–284.6	8.5
2	Maximum load (67.5kN/m) on end spans and minimum load (35.8 kN/m) on interior span							
	Fixed-end moments		–202.5			202.5	–154.7	
	(Row 5) × 202.5	53.8	94.9	53.8	–11.1	31.2	–9.0	–11.1
	(Row 6) × (154.7 – 202.5)	2.6	–5.2	2.6	–10.5	–18.2	–8.6	–10.5
	Sum to obtain final moments	56.4	–112.8	56.4	–21.6	215.5	–172.3	–21.6
3	Minimum load (35.8 kN/m) on end spans and maximum load (67.5 kN/m) on interior span							
	Fixed-end moments		–107.4			107.4	–291.6	
	(Row 5) × 107.4	28.5	50.4	28.5	–5.9	16.5	–4.7	–5.9
	(Row 6) × (291.6 – 107.4)	–10.1	20.2	–10.1	40.5	70.0	33.2	40.5
	Sum to obtain final moments	18.4	–36.8	18.4	34.6	193.9	–263.1	34.6

1. Support Moments for Loading Cases

BEAM

AB	BC	CD
BA	CB	DC

$$M\_Case1 := M_{all\ 1\ [1..3]} = \begin{bmatrix} \begin{bmatrix} -97.8 \\ 267.48 \end{bmatrix} & \begin{bmatrix} -284.58 \\ 284.58 \end{bmatrix} & \begin{bmatrix} -267.48 \\ 97.8 \end{bmatrix} \end{bmatrix} kNm$$

CASE 1

$$M\_Case2 := M_{all\ 2\ [1..3]} = \begin{bmatrix} \begin{bmatrix} -112.75 \\ 215.51 \end{bmatrix} & \begin{bmatrix} -172.36 \\ 172.36 \end{bmatrix} & \begin{bmatrix} -215.51 \\ 112.75 \end{bmatrix} \end{bmatrix} kNm$$

CASE 2

$$M\_Case3 := M_{all\ 3\ [1..3]} = \begin{bmatrix} \begin{bmatrix} -36.93 \\ 193.83 \end{bmatrix} & \begin{bmatrix} -263.16 \\ 263.16 \end{bmatrix} & \begin{bmatrix} -193.83 \\ 36.93 \end{bmatrix} \end{bmatrix} kNm$$

CASE 3

2. All SF, Max Span BM and X_max Cases using PROGRAM 3

$$Res\_All := Find\_All\left(M_{all}, FF, L\right) = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} 1.74\cdot10^{5}\ \frac{kg\ m}{s^{2}} \\ 2.43\cdot10^{5}\ \frac{kg\ m}{s^{2}} \\ 2.31\cdot10^{5}\ \frac{kg\ m}{s^{2}} \end{bmatrix} & \begin{bmatrix} 2.31\cdot10^{5}\ \frac{kg\ m}{s^{2}} \\ 2.43\cdot10^{5}\ \frac{kg\ m}{s^{2}} \\ 1.74\cdot10^{5}\ \frac{kg\ m}{s^{2}} \end{bmatrix} & \begin{bmatrix} 1.27\cdot10^{5}\ \frac{kg\ m^{2}}{s^{2}} \\ 1.53\cdot10^{5}\ \frac{kg\ m^{2}}{s^{2}} \\ 1.27\cdot10^{5}\ \frac{kg\ m^{2}}{s^{2}} \end{bmatrix} & \begin{bmatrix} 2.58\ m \\ 3.6\ m \\ 3.42\ m \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} 1.85\cdot10^{5}\ \frac{kg\ m}{s^{2}} \\ 1.29\cdot10^{5}\ \frac{kg\ m}{s^{2}} \\ 2.2\cdot10^{5}\ \frac{kg\ m}{s^{2}} \end{bmatrix} & \begin{bmatrix} 2.2\cdot10^{5}\ \frac{kg\ m}{s^{2}} \\ 1.29\cdot10^{5}\ \frac{kg\ m}{s^{2}} \\ 1.85\cdot10^{5}\ \frac{kg\ m}{s^{2}} \end{bmatrix} & \begin{bmatrix} 1.42\cdot10^{5}\ \frac{kg\ m^{2}}{s^{2}} \\ 59627.23\ \frac{kg\ m^{2}}{s^{2}} \\ 1.42\cdot10^{5}\ \frac{kg\ m^{2}}{s^{2}} \end{bmatrix} & \begin{bmatrix} 2.75\ m \\ 3.6\ m \\ 3.25\ m \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} 81249.87\ \frac{kg\ m}{s^{2}} \\ 2.43\cdot10^{5}\ \frac{kg\ m}{s^{2}} \\ 1.34\cdot10^{5}\ \frac{kg\ m}{s^{2}} \end{bmatrix} & \begin{bmatrix} 1.34\cdot10^{5}\ \frac{kg\ m}{s^{2}} \\ 2.43\cdot10^{5}\ \frac{kg\ m}{s^{2}} \\ 81249.87\ \frac{kg\ m}{s^{2}} \end{bmatrix} & \begin{bmatrix} 55274.82\ \frac{kg\ m^{2}}{s^{2}} \\ 1.74\cdot10^{5}\ \frac{kg\ m^{2}}{s^{2}} \\ 55274.82\ \frac{kg\ m^{2}}{s^{2}} \end{bmatrix} & \begin{bmatrix} 2.27\ m \\ 3.6\ m \\ 3.73\ m \end{bmatrix} \end{bmatrix} \end{bmatrix}$$

Extract: Shear Forces, Max Span Moments and X_max - Using PROGRAMS 4, 5, 6

PROGRAM 4

$SFF := SF\_All\left(Res\_All\right)$

$$SFF = \begin{bmatrix} \begin{bmatrix} 174.2 \\ 243 \\ 230.8 \end{bmatrix} & \begin{bmatrix} 230.8 \\ 243 \\ 174.2 \end{bmatrix} \\ \begin{bmatrix} 185.4 \\ 128.9 \\ 219.6 \end{bmatrix} & \begin{bmatrix} 219.6 \\ 128.9 \\ 185.4 \end{bmatrix} \\ \begin{bmatrix} 81.2 \\ 243 \\ 133.6 \end{bmatrix} & \begin{bmatrix} 133.6 \\ 243 \\ 81.2 \end{bmatrix} \end{bmatrix} kN$$

PROGRAM 5

$MM\_Span := Span\_M\_All\left(Res\_All\right)$

$$MM\_Span = \begin{bmatrix} \begin{bmatrix} 127 \\ 152.8 \\ 127 \end{bmatrix} \\ \begin{bmatrix} 141.8 \\ 59.6 \\ 141.8 \end{bmatrix} \\ \begin{bmatrix} 55.3 \\ 174.2 \\ 55.3 \end{bmatrix} \end{bmatrix} kNm$$

PROGRAM 6

$XX\_Max := X\_Max\_All\left(Res\_All\right)$

$$XX\_Max = \begin{bmatrix} \begin{bmatrix} 2.58 \\ 3.6 \\ 3.42 \end{bmatrix} \\ \begin{bmatrix} 2.75 \\ 3.6 \\ 3.25 \end{bmatrix} \\ \begin{bmatrix} 2.27 \\ 3.6 \\ 3.73 \end{bmatrix} \end{bmatrix} m$$

$time\left(0\right) - t_0 = 0.08\ s$

The shear forces at the ends of the span and the maximum sagging moment can now be calculated from the following expressions, where the support moments M_L and M_R both take positive values. For the end span,

$V_L = nl/2 - (M_R - M_L)/l$ and $V_R = nl - V_L$

Distance from end of span to point of zero shear, $a = V_L/n$

Maximum sagging moment, $M = V_L \times a/2 - M_L$

For the interior span,

$V_L = V_R = nl/2$ and $M = nl^2/8 - M_L$ (or M_R)

Load Case	Location and Member	End Support	End Span	Interior Support	Interior Support	Interior Span
No.	Bending Moment (kN m) in Members for Load Case					
1	Beam Upper column Lower column	–97.8 48.9 48.9	127.0	267.6 8.5 8.5	–284.6	152.8
2	Beam Upper column Lower column	–112.8 56.4 56.4	141.8	215.5 –21.6 –21.6	–172.3	59.7
3	Beam Upper column Lower column	–36.8 18.4 18.4	55.3	193.9 34.6 34.6	–263.1	174.3
No.	Shear Force (kN) in Members for Load Case					
1	Beam	174.2		230.8	243.0	
2	Beam	185.4		219.6	128.9	
3	Beam	81.2		133.6	243.0	

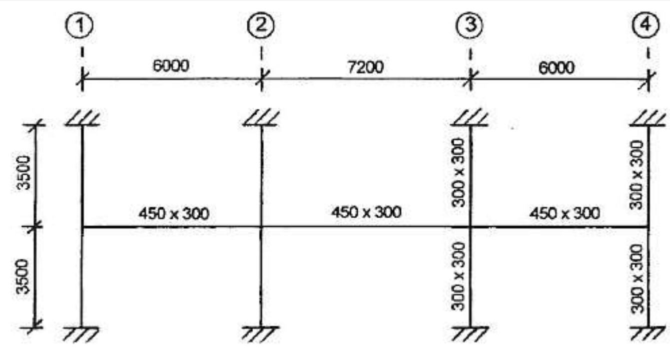
Allowing for some redistribution of moment, the maximum hogging moments in the beam will be taken as 112.8 kN m at the end supports for load cases 1 and 2, and 215.5 kN m at the interior supports for load cases 1 and 3. As a result, the maximum sagging moment in the end span for load case 1 will be the same as that for load case 2. In the interior span, the maximum sagging moment for load cases 1 and 3 will increase in order to maintain equilibrium, as follows:

$M = 67.5 \times 7.2^2/8 - 215.5 = 221.9$ kN m

All Beam Support Moments

$M\_Sup\_All := M_{all}{}_{[1..3][1..3]}$

$$M\_Sup\_All = \begin{bmatrix} \begin{bmatrix} -97.8 \\ 267.48 \end{bmatrix} & \begin{bmatrix} -284.58 \\ 284.58 \end{bmatrix} & \begin{bmatrix} -267.48 \\ 97.8 \end{bmatrix} \\ \begin{bmatrix} -112.75 \\ 215.51 \end{bmatrix} & \begin{bmatrix} -172.36 \\ 172.36 \end{bmatrix} & \begin{bmatrix} -215.51 \\ 112.75 \end{bmatrix} \\ \begin{bmatrix} -36.93 \\ 193.83 \end{bmatrix} & \begin{bmatrix} -263.16 \\ 263.16 \end{bmatrix} & \begin{bmatrix} -193.83 \\ 36.93 \end{bmatrix} \end{bmatrix} kNm$$



Dimensions of sub-frame

For All Cases : Arrange Beam Moments, Span Moments, Shear Forces, Max Span Moments and X_Max

b1 **b2** **b3**

$$M\_Sup := \text{stack}\left(\overrightarrow{M\_Sup\_All^{T}}\right) = \begin{bmatrix} [-97.8\ 267.5] & [-284.6\ 284.6] & [-267.5\ 97.8] \\ [-112.8\ 215.5] & [-172.4\ 172.4] & [-215.5\ 112.8] \\ [-36.9\ 193.8] & [-263.2\ 263.2] & [-193.8\ 36.9] \end{bmatrix} kNm$$

Support Moments

c1 **c2** **c3** **c4**

$$M_{UC} := \text{stack}\left(\overrightarrow{M_{UC\_all}^{T}}\right) = \begin{bmatrix} [48.9\ 8.6\ -8.6\ -48.9] \\ [56.4\ -21.6\ 21.6\ -56.4] \\ [18.5\ 34.7\ -34.7\ -18.5] \end{bmatrix} kNm$$

----------- Upper Col Moments

c1 c2 c3 c4

$$M_{LC} := \text{stack}\left(\overrightarrow{M_{LC\_all}^{\mathrm{T}}}\right) = \begin{bmatrix} [48.9\ 8.6\ -8.6\ -48.9] \\ [56.4\ -21.6\ 21.6\ -56.4] \\ [18.5\ 34.7\ -34.7\ -18.5] \end{bmatrix} kNm$$

------------ Lower Col Moments

$$SF1 := \text{stack}\left(\overrightarrow{SFF^{\mathrm{T}}}\right)$$

b1 b2 b3

$$SF := \overrightarrow{SF1} = \begin{bmatrix} \begin{bmatrix} 174.2 & 230.8 \\ 185.4 & 219.6 \\ 81.2 & 133.6 \end{bmatrix} & \begin{bmatrix} 243 & 243 \\ 128.9 & 128.9 \\ 243 & 243 \end{bmatrix} & \begin{bmatrix} 230.8 & 174.2 \\ 219.6 & 185.4 \\ 133.6 & 81.2 \end{bmatrix} \end{bmatrix} \mathrm{kN}$$

----------- Shear Forces

b1 b2 b3

$$M\_Span := \text{stack}\left(\overrightarrow{MM\_Span^{\mathrm{T}}}\right) = \begin{bmatrix} [127\ 152.8\ 127] \\ [141.8\ 59.6\ 141.8] \\ [55.3\ 174.2\ 55.3] \end{bmatrix} kNm$$

----------- Max Span Moments

b1 b2 b3

$$X\_Max := \text{stack}\left(\overrightarrow{XX\_Max^{\mathrm{T}}}\right) = \begin{bmatrix} [2.58\ 3.6\ 3.42] \\ [2.75\ 3.6\ 3.25] \\ [2.27\ 3.6\ 3.73] \end{bmatrix} \mathrm{m}$$

----------- Dist to Max BM

$$\text{time}(0) - t_0 = 0.09\ \mathrm{s}$$