Double Pendulum

⊞— d

⊞— equrep

⊞— isol

⊟— Lagrangians

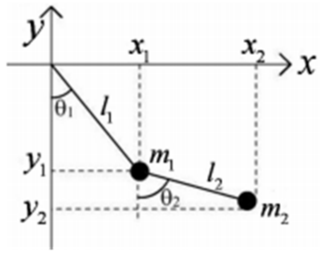
Lagrangian Dynamics

dSymbol := "∂"

Double Pendulum

Constants $\partial m_1 := 0 \quad \partial m_2 := 0 \quad \partial g := 0$

$\partial L_1 := 0 \quad \partial L_2 := 0$



Gen Coords $\partial \theta_1 := \theta'_1 \cdot \partial t \quad \partial \theta'_1 := \theta''_1 \cdot \partial t$

$\partial \theta_2 := \theta'_2 \cdot \partial t \quad \partial \theta'_2 := \theta''_2 \cdot \partial t$

Position $x_1 := L_1 \cdot \sin(\theta_1) \qquad y_1 := -L_1 \cdot \cos(\theta_1)$

$x_2 := x_1 + L_2 \cdot \sin(\theta_2) \qquad y_2 := y_1 - L_2 \cdot \cos(\theta_2)$

$$x'_1 := \frac{d(x_1)}{\partial t} \qquad y'_1 := \frac{d(y_1)}{\partial t} \qquad x'_2 := \frac{d(x_2)}{\partial t} \qquad y'_2 := \frac{d(y_2)}{\partial t}$$

Lagrangian

$$L := \frac{1}{2} \cdot m_1 \cdot \left(x'^{\,2}_1 + y'^{\,2}_1\right) + \frac{1}{2} \cdot m_2 \cdot \left(x'^{\,2}_2 + y'^{\,2}_2\right) - m_1 \cdot g \cdot y_1 - m_2 \cdot g \cdot y_2$$

Equations of motion

$$eq_1 := \mathrm{numden}\left(\mathrm{isol}\left(\frac{1}{\partial t} \cdot d\left(\frac{\partial(L, \theta'_1)}{\partial \theta'_1}\right) - \frac{\partial(L, \theta_1)}{\partial \theta_1}, \theta''_1\right) - \theta''_1\right)_1$$

$$eq_2 := \mathrm{numden}\left(\mathrm{isol}\left(\frac{1}{\partial t} \cdot d\left(\frac{\partial(L, \theta'_2)}{\partial \theta'_2}\right) - \frac{\partial(L, \theta_2)}{\partial \theta_2}, \theta''_2\right) - \theta''_2\right)_1$$

$$eq_1 = -\left(m_2 \cdot \left(L_2 \cdot \left(\theta'^{\,2}_2 \cdot \left(-\sin(\theta_2) \cdot \cos(\theta_1) + \cos(\theta_2) \cdot \sin(\theta_1)\right) + \left(\cos(\theta_2) \cdot \cos(\theta_1) + \sin(\theta_2) \cdot \sin(\theta_1)\right) \cdot \theta''\right.\right.\right.$$

$$eq_2 = -\left(L_1 \cdot \left(\theta'^{\,2}_1 \cdot \left(-\sin(\theta_1) \cdot \cos(\theta_2) + \cos(\theta_1) \cdot \sin(\theta_2)\right) + \left(\cos(\theta_1) \cdot \cos(\theta_2) + \sin(\theta_1) \cdot \sin(\theta_2)\right) \cdot \theta''_1\right) + \right.$$

Equations from https://scienceworld.wolfram.com/physics/DoublePendulum.html

$$eq_1 := (m_1 + m_2) \cdot L_1 \cdot \theta 1'' + m_2 \cdot L_2 \cdot \theta 2'' \cdot \cos(\theta 1 - \theta 2) + m_2 \cdot L_2 \cdot \theta 2'^{\,2} \cdot \sin(\theta 1 - \theta 2) + g \cdot (m_1 + m_2) \cdot \sin(\theta 1)$$

$$eq_2 := m_2 \cdot L_2 \cdot \theta 2'' + m_2 \cdot L_1 \cdot \theta 1'' \cdot \cos(\theta 1 - \theta 2) - m_2 \cdot L_1 \cdot \theta 1'^{\,2} \cdot \sin(\theta 1 - \theta 2) + m_2 \cdot g \cdot \sin(\theta 2)$$

$\theta_{01} := 180\,°$ $\quad L_1 := 1 \quad m_1 := 1 \quad \theta_{02} := 90\,° \quad L_2 := 0.2 \quad m_2 := 0.5 \quad g := 9.81 \quad t_{end} := 8$

$D(t, u, \theta'') := \Big| \begin{bmatrix} \theta_1 & \theta'_1 & \theta_2 & \theta'_2 \end{bmatrix} := \begin{bmatrix} u_1 & u_2 & u_3 & u_4 \end{bmatrix} \qquad \theta'' := \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

This way is for update the guess for θ''

$\left[\;{}_{1}\;\;{}_{2}\right] := \left[\;{}_{1}\;\;{}_{2}\right]$

$eq_3(v) := \text{stack}\left(eq_1, eq_2\right)$

$\theta'' := \text{al\_nleqsolve}\left(\theta'', eq_3\right)$

$\text{stack}\left(\theta'_1, \theta''_1, \theta'_2, \theta''_2\right)$

Although in this case the second derivatives can be isolated, it is much more practical to solve them numerically.

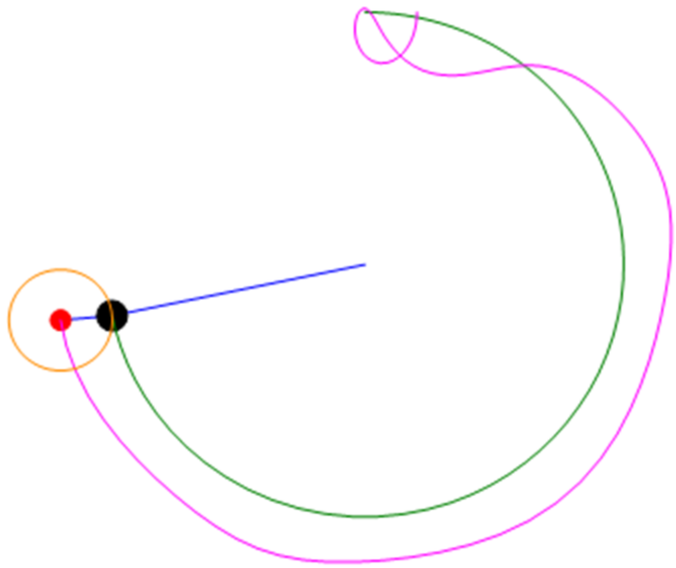
$U := \text{rkfixed}\left(\text{stack}\left(\theta_{01}, 0, \theta_{02}, 0\right), 0, t_{end}, 500, D\right)$

$\left[T \;\; \theta_1 \;\; \theta_2\right] := \left[\text{col}(U, 1) \;\; \text{col}(U, 2) \;\; \text{col}(U, 4)\right]$ $\qquad n := [1..\text{rows}(U)]$

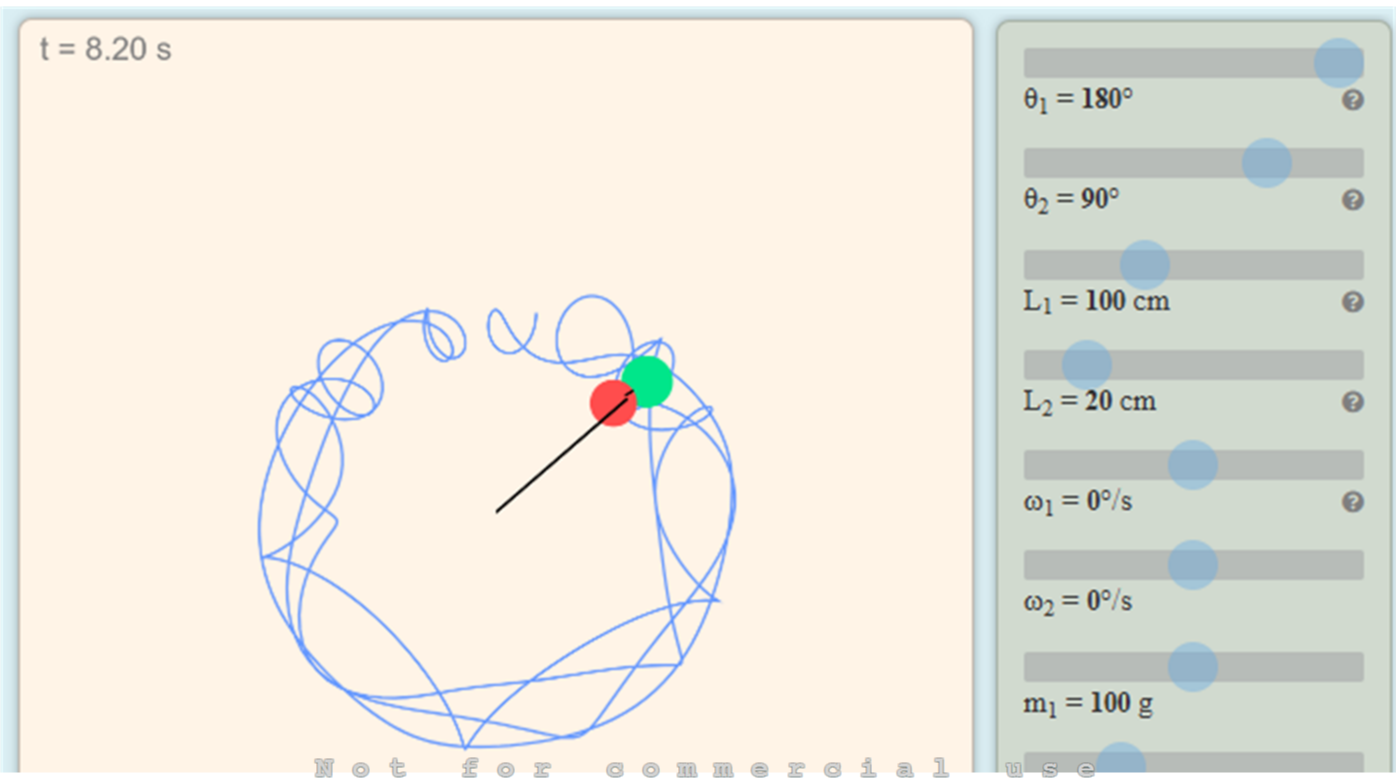
$X1 := \overrightarrow{x_1} \qquad Y1 := \overrightarrow{y_1} \qquad X2 := \overrightarrow{x_2} \qquad Y2 := \overrightarrow{y_2}$

$\varphi := [0, 4..360]\,°$ $\qquad S := \overrightarrow{\sin(\varphi)} \qquad C := \overrightarrow{\cos(\varphi)}$

$$\Pi(n) := \begin{cases} \begin{bmatrix} 0 & 0 \\ X1_n & Y1_n \\ X2_n & Y2_n \end{bmatrix} \\ \left[X1_n \;\; Y1_n \;\; \text{"."} \;\; 15 \;\; \text{"black"}\right] \\ \left[X2_n \;\; Y2_n \;\; \text{"."} \;\; 10 \;\; \text{"red"}\right] \\ \text{augment}\left(X1_{[1..n]}, Y1_{[1..n]}\right) \\ \text{augment}\left(X2_{[1..n]}, Y2_{[1..n]}\right) \\ \text{augment}\left(L_2 \cdot C + X2_n, L_2 \cdot S + Y2_n\right) \end{cases}$$



Notice that it looks very similar to https://simufisica.com/en/double-pendulum/



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