Triple Pendulum

Created by Valery Ochkov (**http://twt.mpei.ac.ru/ochkov/v_ochkov.htm**)

Area 1: Disclaimer

Area 2: User Notices

$g := 9.81\,m/s^2$ $t_e := 8s$ $t_{end} := t_e \frac{FRAME + 1}{500}$ $FRAME = 499$

1-st pendulum	$m_1 := 2kg$	$L_1 := 1m$	$\theta_{1begin} := \frac{\pi}{2}$
2-nd pendulum	$m_2 := 0.5kg$	$L_2 := 0.5m$	$\theta_{2begin} := \frac{\pi}{2}$
2-nd pendulum	$m_3 := 0.1kg$	$L_3 := 0.2m$	$\theta_{3begin} := \frac{\pi}{2}$

3. Solution

Given

1-st pendulum

Initial position (x1-y1), force (F1) and velocity (x1'-y1')

$x1(0s) = L_1 \sin(\theta_{1begin})$

$y1(0s) = -L_1 \cos(\theta_{1begin})$

$F1(0s) = 0kgf$

$x1'(0s) = 0\,m/s \qquad y1'(0s) = 0\,m/s$

System of equation

$$x1(t)^2 + y1(t)^2 = L_1^2$$

$$m_1 x1''(t) = F1(t)\frac{x1(t)}{L_1} - F2(t)\frac{x2(t) - x1(t)}{L_2}$$

$$m_1 y1''(t) + g m_1 = F1(t)\frac{y1(t)}{L_1} - F2(t)\frac{y2(t) - y1(t)}{L_2}$$

2-nd pendulum

Initial position (x2-y2), force (F2) and velocity (x2'-y2')

$$x2(0s) = L_1 \sin\left(\theta_{1begin}\right) + L_2 \sin\left(\theta_{2begin}\right)$$

$$y2(0s) = -\left(L_1 \cos\left(\theta_{1begin}\right) + L_2 \cos\left(\theta_{2begin}\right)\right)$$

$F2(0s) = 0\,kgf$

$x2'(0s) = 0\,m/s \qquad y2'(0s) = 0\,m/s$

System of equation

$$(x2(t) - x1(t))^2 + (y2(t) - y1(t))^2 = L_2^2$$

$$m_2 x2''(t) = F2(t)\frac{x2(t) - x1(t)}{L_2} - F3(t)\frac{x3(t) - x2(t)}{L_3}$$

$$m_2\, y2''(t) + m_2\, g = F2(t)\,\frac{y2(t) - y1(t)}{L_2} - F3(t)\,\frac{y3(t) - y2(t)}{L_3}$$

3-d pendulum

Initial position (x3-y3), force (F3) and velocity (x3'-y3')

$$x3(0s) = L_1 \sin\left(\theta_{1begin}\right) + L_2 \sin\left(\theta_{2begin}\right) + L_3 \sin\left(\theta_{3begin}\right)$$

$$y3(0s) = -\left(L_1 \cos\left(\theta_{1begin}\right) + L_2 \cos\left(\theta_{2begin}\right) + L_3 \cos\left(\theta_{3begin}\right)\right)$$

$$F3(0s) = 0kgf$$

$$x3'(0s) = 0m/s \qquad y3'(0s) = 0m/s$$

System of equation

$$(x3(t) - x2(t))^2 + (y3(t) - y2(t))^2 = L_3{}^2$$

$$m_3\, x3''(t) = F3(t)\,\frac{x3(t) - x2(t)}{L_3}$$

$$m_3\, y3''(t) + m_3\, g = F3(t)\,\frac{y3(t) - y2(t)}{L_3}$$

Solution

$$\begin{pmatrix} x1 \\ y1 \\ F1 \\ x2 \\ y2 \\ F2 \\ x3 \\ y3 \\ F3 \end{pmatrix} := \text{Odesolve}\left[\begin{pmatrix} x1 \\ y1 \\ F1 \\ x2 \\ y2 \\ F2 \\ x3 \\ y3 \\ F3 \end{pmatrix}, t, 7t_{end}\right] \qquad tt := 0, \frac{t_{end}}{1000} .. t_{end}$$

Data for the plot below

$$v_{black}(t) := \sqrt{\left(\frac{d}{dt}x1(t)\right)^2 + \left(\frac{d}{dt}y1(t)\right)^2} \qquad v_{red}(t) := \sqrt{\left(\frac{d}{dt}x2(t)\right)^2 + \left(\frac{d}{dt}y2(t)\right)^2} \qquad v_{blue}(t) := \sqrt{\left(\frac{d}{dt}x3(t)\right)^2 + \left(\frac{d}{dt}y3(t)\right)^2}$$

$$m_{black} := m_1 \qquad m_{red} := m_2 \qquad m_{blue} := m_3 \qquad L_{black} := L_1 \qquad L_{red} := L_2 \qquad L_{blue} := L_3$$

$$F_{black}(t) := \frac{F1(t)}{g} \qquad F_{red}(t) := \frac{F2(t)}{g} \qquad F_{blue}(t) := \frac{F3(t)}{g} \qquad t := t_{end} - 0.0000001$$

$$\theta_{black} := -\text{atan}\left(\frac{x1(t)}{y1(t)}\right) \qquad \theta_{red} := -\text{atan}\left(\frac{x2(t) - x1(t)}{y2(t) - y1(t)}\right) \qquad \theta_{blue} := -\text{atan}\left(\frac{x3(t) - x2(t)}{y3(t) - y2(t)}\right)$$

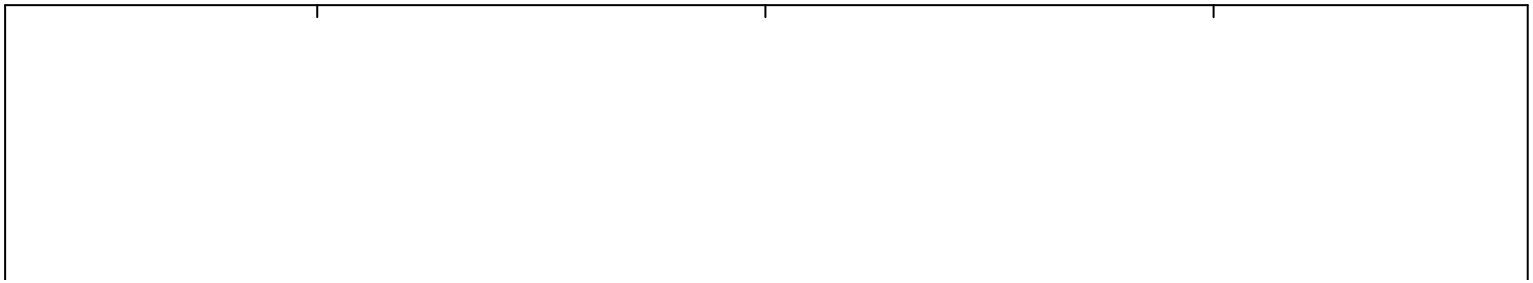
$$pe(t) := g\left[m_1(y1(t) - y1(0)) + m_2(y2(t) - y2(0)) + m_3(y3(t) - y3(0))\right]$$

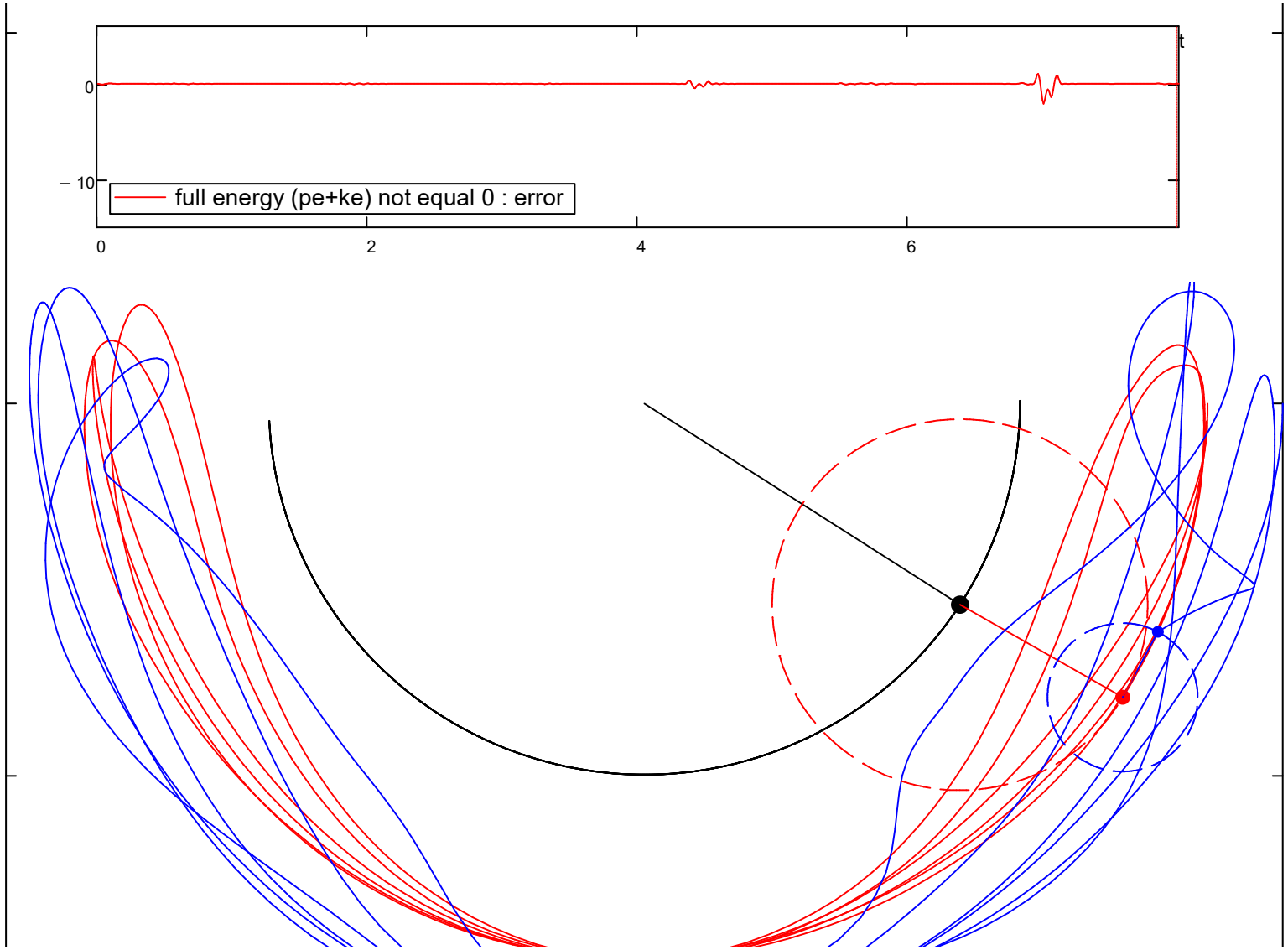
$$ke(t) := \frac{m_1 v_{black}(t)^2 + m_2 v_{red}(t)^2 + m_3 v_{blue}(t)^2}{2}$$

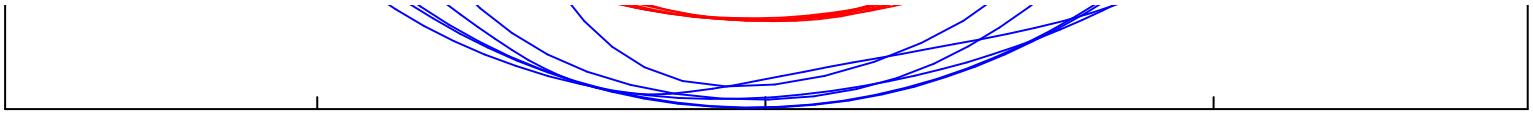
$$\alpha := 0, \frac{2\pi}{360} .. 2\pi$$

3. Solution

FRAME ≡ 499 You can change this value (1-700: manuel or by help Tools/Animation/Record) an see the plot below.




t
0
− 10
full energy (pe+ke) not equal 0 : error
0
2
4
6



$v_{black}(t) = 3.042\,m/s$

$v_{red}(t) = 4.307\,m/s$

$v_{blue}(t) = 4.997\,m/s$

$t = 8.00\,s$

$F_{black}(t) = -3.877\,kgf$

$F_{red}(t) = -0.906\,kgf$

$F_{blue}(t) = -0.192\,kgf$

$m_{black} = 2\,kg$

$L_{black} = 1\,m$

$m_{blue} = 0.1\,kg$

$L_{blue} = 0.2\,m$

$m_{red} = 0.5\,kg$

$L_{red} = 0.5\,m$

FRAME = 499

Created by Valery Ochkov

See an animation on PlanetPTC - **http://communities.ptc.com/videos/1956**

$\theta_{black} = 57.18°$

$\theta_{red} = 60.06°$

$\theta_{blue} = -27.93°$