Bi-Linear Interpolation   Brett Towler, USFWS, 1/11/2015

Bi-linear interpolation is an extension of linear interpolation for interpolating functions
of two variables (e.g., x and y) on a regular 2D grid. The procedure, below, performs linear
interpolation first in one direction, and then again in the other direction.
Note that although each step is linear, the interpolation as a whole is not linear but
instead quadratic.

1. The matrix, T, below contains a table of values. The x values are contained in elements
2 through 5 of row 1; the y values are contained in rows 2 through 5 of column 1. Assume
element 1,1 is an empty dummy variable. T is a sample matrix; the code below would work for
any size matrix.

$$T := \begin{pmatrix} 0 & 2 & 4 & 8 & 10 \\ 1 & .3 & .45 & .6 & .6 \\ 2 & .4 & .5 & .62 & .65 \\ 3 & .4 & .48 & .6 & .61 \\ 4 & .38 & .45 & .55 & .59 \end{pmatrix}$$

2. The following custom user function performs bi-linear interpolation of the matrix, M, based
on the arguments x and y.

$$\text{minterp}(x, y, M) := \begin{vmatrix} A := \text{submatrix}(M, 1, 1, 2, \text{cols}(M)) \\ B := \text{submatrix}(M, 2, \text{rows}(M), 1, 1) \\ C := \text{submatrix}(M, 2, \text{rows}(M), 2, \text{cols}(M)) \\ \text{for } j \in 1, 2 .. \text{length}(A) \\ \quad CC_j := \text{linterp}(B, \text{submatrix}(C, 1, \text{rows}(C), j, j), y) \\ \text{linterp}(A^T, CC, x) \end{vmatrix}$$

3. Example application

$$ANS := \text{minterp}(3, 2, T)$$

$$ANS = 0.45$$