Solution to ENGR 2030 - DYNAMICS Test 03 - Spring Semester 2010 -
Utah State University - Instructor: Gilberto Urroz

The values of the problem parameters were randomly generated by Blackboard from a range of values provided by the instructor. This is one of the tests generated by Blackboard. The parameters, therefore, would most likely be different than the ones in your test, but the procedure is common for all tests thus generated. These solutions were calculated using SMath Studio, a free WYSIWYG math worksheet:

http://www.neng.usu.edu/cee/faculty/gurro/SMathStudio.html

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[1]. Principle of work-energy.

Shock-absorbing barriers like the one shown in the picture are used to protect highway structures and ensure survivability in the event of a crash. Assume that on a head-on crash this horizontal shock-adsorbing barrier will act as a linear spring. Let a vehicle that weights W = 2,830 lbs hits the barrier of length L = 25 ft, head on, going at a speed v = 44 mph. In this crash, the driver was able to apply the breaks so that the friction force between the locked tires and the pavement helped in slowing down the car. Let the coefficient of kinetic friction between the tires and the pavement be 0.28. If the barrier deforms by an amount equal to 1/3 of its length before bringing the vehicle to a stop, what is the corresponding spring constant, k, in kips/ft (1 kip = 1000 lb)? Use the principle of work-energy to solve this problem.

SOLUTION:
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Principle of work-energy:

Position (1): Spring is unstretched, thus $s = 0$

and the elastic potential energy is:

$$Ve1 = \frac{1}{2} \cdot k \cdot s^2 = 0$$

The car is moving with velocity, v. Thus, the kinetic energy is:

$$T1 = \frac{1}{2} \cdot m \cdot v^2 = \frac{W \cdot v^2}{2 \cdot g}$$

Since the motion occurs in a horizontal path, the gravitational potential energy is constant, i.e.,

$$Vg1 = Vg2$$

Position (2): Spring is compressed to its maximum deformation:

$$smax = \frac{L}{n}$$

Thus, the elastic potential energy is:

$$Ve2 = \frac{1}{2} \cdot k \cdot smax^2 = \frac{1}{2} \cdot k \cdot \left(\frac{L}{n}\right)^2$$

Since the vehicle comes to a stop at that point, v = 0, and the kinetic energy is:

$$T2 = \frac{1}{2} \cdot m \cdot v^2 = 0$$

The friction force is calculated as: $Ff = \mu k \cdot N$

and, since motion occurs in a horizontal plane, $N = W$, and $Ff = \mu k \cdot W$

The work performed by the friction force as the barrier deforms by a lenght smax is:

$$UF12 = -Ff \cdot smax = -\mu k \cdot W \cdot \frac{L}{n}$$

The principle of work-energy will be written as:

$$T1 + Ve1 + Vg1 + UF12 = T2 + Ve2 + Vg2$$

Cancelling Vg1 and Vg2, since they're equal, and replacing the known values we get:

$$\frac{W \cdot v^2}{2 \cdot g} + 0 + \left(-\frac{\mu k \cdot W \cdot L}{n}\right) = 0 + \frac{1}{2} \cdot k \cdot \left(\frac{L}{n}\right)^2$$

Multiply by 2 and take W common factor on the left-hand side:

$$W \cdot \left(\frac{v^2}{g} - \frac{2 \cdot \mu k \cdot L}{n}\right) = k \cdot \left(\frac{L}{n}\right)^2$$

Solving for k:

$$k = W\cdot\left(\frac{n}{L}\right)^2\cdot\left(\frac{v^2}{g} - \frac{2\cdot \mu k\cdot L}{n}\right)$$

Since v is given in mph, we need to convert it to ft/s to make the units in this equation consistent. Note: L is given in ft, and g = 32.2 ft/s^2. Thus, v must be multiplied by:

$$1\cdot mph = 1\cdot\frac{mi}{hr} = \frac{5280\cdot ft}{3600\cdot s} = 1.4667$$

With, $1.4667^2 = 2.1512$, the solution for k becomes:

$$k = W\cdot\left(\frac{n}{L}\right)^2\cdot\left(2.1512\cdot\frac{v^2}{g} - \frac{2\cdot \mu k\cdot L}{n}\right)$$

The data for this problem is as follows:

$W := 2830\ lb$ $L := 25\ ft$ $v := 44\ mph$ $\mu k := 0.28$ $n := 3$ $g := 32.2\ \frac{ft}{s^2}$

$$k := W\cdot\left(\frac{n}{L}\right)^2\cdot\left(2.1512\cdot\frac{v^2}{g} - \frac{2\cdot \mu k\cdot L}{n}\right) \quad \text{-->} \quad k = 5080.6563\ \frac{lb}{ft}$$

Using 1 kip = 1000 lb, the stiffness is: $k := 5.08\ \frac{kip}{lb}$

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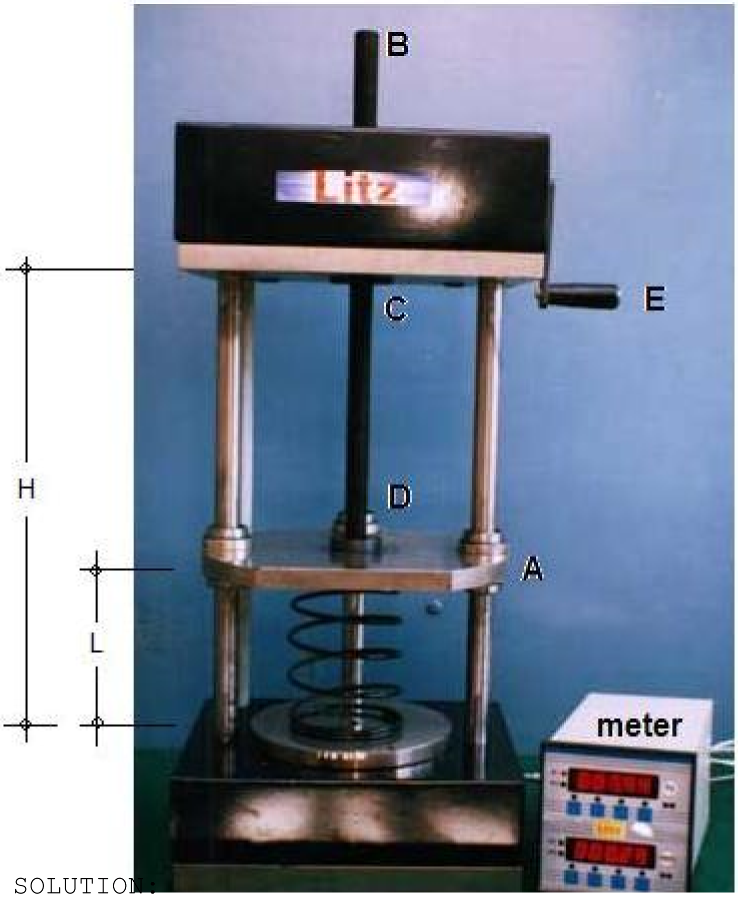
[2]. Conservation of Energy.

The device shown in the picture is used to determine the stiffness of springs. Cranking handle E forces rod BCD, of negligible mass, against plate A (mass = 32 g), which in turns presses against the spring, while the meter shown records the spring's force and deformation. Plate A can slide up and down guided by three smooth vertical rods as shown. Suppose that you are testing a spring whose unstretched length is L0 = 0.8 H, and H = 37 cm. At the instant shown, when L = 10.9 cm, rod BCD breaks and that the spring pushes plate A upwards forcing it to hit the top of the device at point C. If the spring stiffness is k = 7,505 dyn/cm, determine the velocity, in cm/s, with which plate A hits point C. [Note: since the units used belong to the cgs system, use g = 981 cm/s^2.



SOLUTION:
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Position (1): v = 0, thus T1 = 0, and spring deformation is:

$$s = L - L0 = L - 0.8 \cdot H$$

The elastic potential energy is:

$$Ve1 = \frac{1}{2} \cdot k \cdot s^2 = \frac{1}{2} \cdot k \cdot (L - 0.8 \cdot H)^2$$

while, we'll take the gravitational potential energy to be zero at this level, i.e., Vg1=0

Position (2): the kinetic energy is: $T2 = \frac{1}{2} \cdot m \cdot v^2$

Elastic potential energy is zero since the spring is unstretched, thus Ve2 = 0, whereas the gravitational potential energy is now:

$$Vg2 = m \cdot g \cdot y = m \cdot g \cdot (H - L)$$

Conservationof energy:

$$T1 + Ve1 + Vg1 = T2 + Ve2 + Vg2$$

$$0 + \frac{1}{2} \cdot k \cdot (L - 0.8 \cdot H)^2 + 0 = \frac{1}{2} \cdot m \cdot v^2 + 0 + m \cdot g \cdot (H - L)$$

$$k \cdot (L - 0.8 \cdot H)^2 - 2 \cdot m \cdot g \cdot (H - L) = m \cdot v^2$$

$$v = \sqrt{\frac{k}{m} \cdot (L - 0.8 \cdot H)^2 - 2 \cdot g \cdot (H - L)}$$

The data for this problem:

$k := 7505 \, \frac{dyn}{cm}$ $\quad m := 32 \, gm$ $\quad g := 981 \, \frac{cm}{s^2}$ $\quad H := 37 \, cm$ $\quad L := 10.9 \, cm$

$v := \sqrt{\frac{k}{m} \cdot (L - 0.8 \cdot H)^2 - 2 \cdot g \cdot (H - L)}$ --> $v = 175.5136 \, \frac{cm}{s}$

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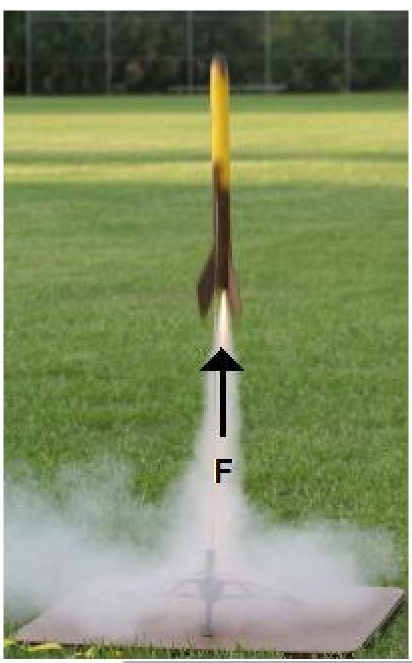
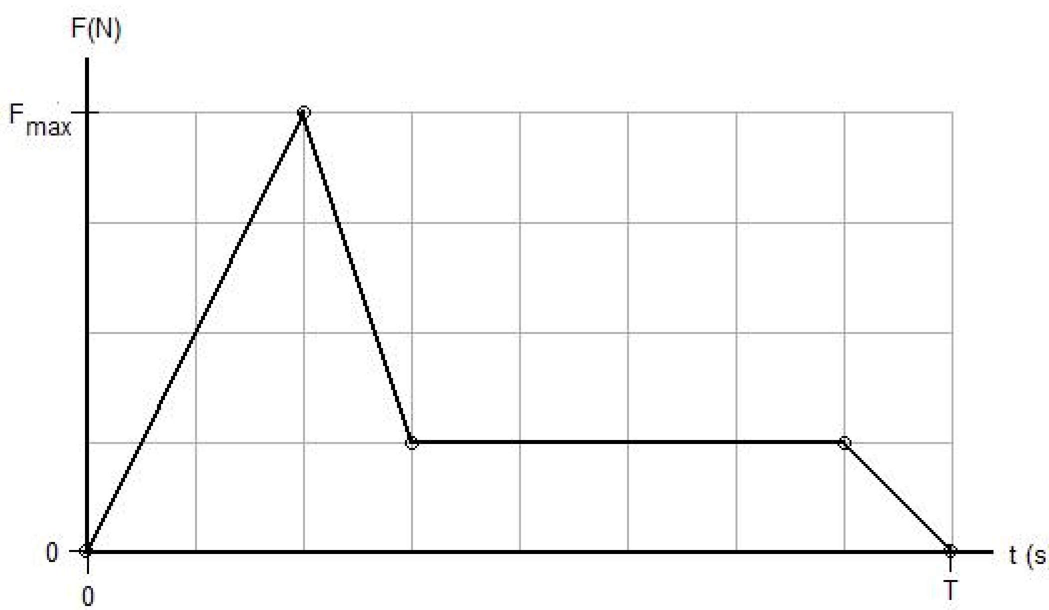
[3]. Principle Impulse-Momentum

A model rocket with a mass of 0.3 kg is launched vertically upwards. The thrust provided by the model rocket engine is approximated by the graph shown where Fmax = 28.1 N and T = 0.9 s. Using the principle of impulse-momentum determine the speed of the rocket (m/s) when the engine quits. Assume that the air resistance is negligible and that the rocket flies in a vertical path all the time.





SOLUTION:
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At the start of the motion, v = 0. The model rocket will be subject to the effects of its own weight, W = mg, and of the variable thrust force, F, as shown in the graph above. While the thrust force produces a positive impact, the weight produces a negative impact. At the moment the engine quits, the rocket speed is v = ?. The principle of impulse-momentum is written as:

$$\frac{1}{2}\cdot 2\cdot\frac{1}{8}\cdot 1+\frac{1}{2}\cdot\frac{1}{8}\cdot$$

[1] $$m\cdot 0+\int_0^T F(t)\,dt-\int_0^T m\cdot g\,dt = m\cdot v$$

with $\int_0^T F(t)\,dt = \text{Area in graph above}$, then

$$\int_0^T F(t)\,dt = \frac{1}{2}\cdot\left(2\cdot\frac{T}{8}\right)\cdot Fmax+\frac{1}{2}\cdot\left(\frac{T}{8}\right)\cdot\left(Fmax-\frac{Fmax}{4}\right)+\left(5\cdot\frac{T}{8}\right)\cdot\left(\frac{Fmax}{4}\right)+\frac{1}{2}\cdot\left(\frac{T}{8}\right)\cdot\left(\frac{Fmax}{4}\right)$$

$$\int_0^T F(t)\,dt = Fmax\cdot T\cdot\left(\frac{1}{2}\cdot 2\cdot\frac{1}{8}+\frac{1}{2}\cdot\frac{1}{8}\cdot\left(1-\frac{1}{4}\right)+5\cdot\frac{1}{8}\cdot\frac{1}{4}+\frac{1}{2}\cdot\frac{1}{8}\cdot\frac{1}{4}\right) = 0.3438\cdot Fmax\cdot T$$

Using:

$$\frac{1}{2}\cdot 2\cdot\frac{1}{8}+\frac{1}{2}\cdot\frac{1}{8}\cdot\left(1-\frac{1}{4}\right)+5\cdot\frac{1}{8}\cdot\frac{1}{4}+\frac{1}{2}\cdot\frac{1}{8}\cdot\frac{1}{4}\rightarrow\frac{11}{32}$$

$$\frac{1}{2}\cdot 2\cdot\frac{1}{8}+\frac{1}{2}\cdot\frac{1}{8}\cdot\left(1-\frac{1}{4}\right)+5\cdot\frac{1}{8}\cdot\frac{1}{4}+\frac{1}{2}\cdot\frac{1}{8}\cdot\frac{1}{4} = 0.3438$$

$$\int_0^T m\cdot g\,dt = m\cdot g\cdot T$$

Thus, [1] becomes: $0.3438\cdot Fmax\cdot T-m\cdot g\cdot T = m\cdot v$

and, $$v = \frac{0.3438\cdot Fmax\cdot T}{m}-g\cdot T$$

With: $m := 0.3\ kg$ $Fmax := 28.1\ N$ $T := 0.9\ s$ $g := 9.81\ \frac{m}{s^2}$

$$v := \frac{0.3438\cdot Fmax\cdot T}{m}-g\cdot T$$ --> $v = 20.1533\ \frac{m}{s}$

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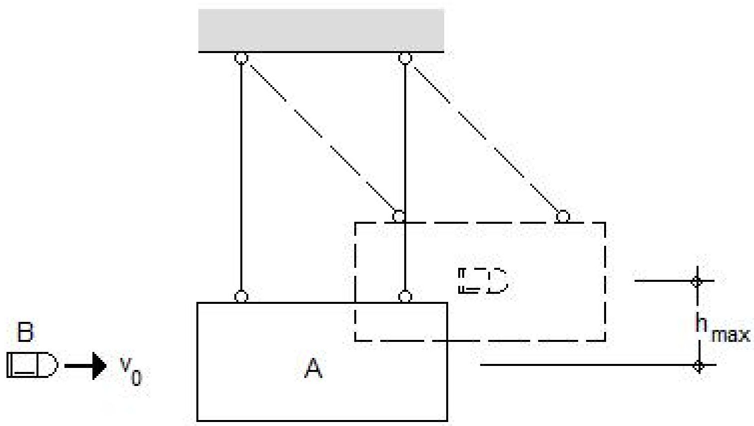
[4]. Conservation of momentum-conservation of energy

A bullet B with a weight of 0.082 lbs is fired horizontally at a velocity 63 ft/s against a wooden block A with a weight of 1.10 lbs so that the bullet becomes embedded in the block. Block A hangs from cords as shown. Use conservation of momentum to determine the initial velocity of the combined block+bullet mass and then use conservation of energy to determine how high will the block rise after hit by the bullet. Enter the value of hmax in ft as your answer.



SOLUTION:
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Conservation of momentum is used for position (1), right before the bullet hits the block, and position (2), immediately after the bullet is embeded within the block:

$$vA1 = 0 \qquad vB1 = v0 \qquad vA2 = vB2 = v$$

Conservation of momentum:

$$mA \cdot vA1 + mB \cdot vB1 = mA \cdot vA2 + mB \cdot vB2$$

$$mA \cdot 0 + mB \cdot v0 = mA \cdot v + mB \cdot v$$

$$v = \frac{mB}{mA + mB} \cdot v0 = \frac{WB}{WA + WB} \cdot v0$$

Conservation of energy is used for position (2), immediately after the bullet is embeded within the block, and position (3), the point where the combined block+bullet mass reaches the highest point at y = hmax, with v = 0.

The mass of the combined body is: $m = mA + mB$

Thus, for position (2): $T2 = \frac{1}{2} \cdot m \cdot v2^2 = \frac{1}{2} \cdot m \cdot v^2$ and $V2 = 0$

While for position (3): $T3 = \frac{1}{2} \cdot m \cdot v3^2 = 0$ and $V3 = m \cdot g \cdot y = m \cdot g \cdot hmax$

Conservation of energy: $T2 + V2 = T3 + V3$

$$\frac{1}{2} \cdot m \cdot v^2 + 0 = 0 + m \cdot g \cdot hmax \quad \text{-->} \quad hmax = \frac{v^2}{2 \cdot g}, \text{ with } v = \frac{WB}{WA + WB} \cdot v0$$

i.e., $hmax = \left(\frac{WB}{WA + WB}\right)^2 \cdot \frac{v0^2}{2 \cdot g}$

With: $WA := 1.10\ lbs$ $WB := 0.082\ lbs$ $v0 := 63\ \frac{ft}{s}$ $g := 32.2\ \frac{ft}{s^2}$

$hmax := \left(\frac{WB}{WA + WB}\right)^2 \cdot \frac{v0^2}{2 \cdot g}$ --> $hmax = 0.2966\ ft$

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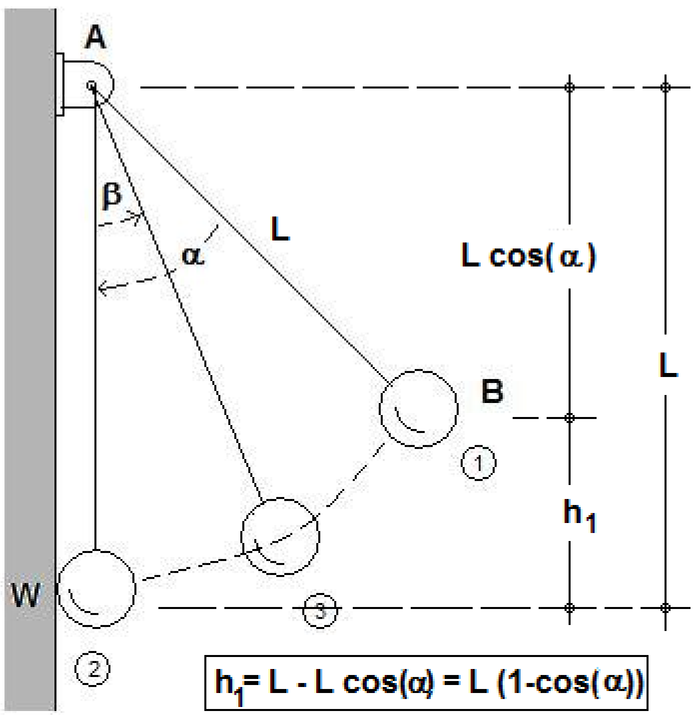
[5]. Conservation of energy - coefficient of restitution

A ball B with a mass of 0.13 kg is tethered to point A by a cord of length L = 2.74 m. If the ball is released from rest at position (1), with an angle a = 46 o, determine the angle b, in degrees, at which the ball will raise (position (3)) after hitting the wall at point W (position (2)). The coefficient of restitution at position (2) is e = 0.36. Hint: Use conservation of energy from position (1) to position (2). Use the definition of coefficient of restitution, with wall velocities = 0, at impact for position (2), then use conservation of energy again from position (2), after bouncing, to position (3).



$h_1 = L - L\cos(\alpha) = L\,(1-\cos(\alpha))$

SOLUTION:
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Conservation of energy from position (1) to position (2), just before the ball hits the wall. At position (1):

$v1 = 0$ $T1 = \frac{1}{2} \cdot m \cdot v1^2 = 0$

$y1 = L - L \cdot \cos(\alpha) = L \cdot (1 - \cos(\alpha))$ $V1 = m \cdot g \cdot y1 = m \cdot g \cdot L \cdot (1 - \cos(\alpha))$

At position (2): $v2 = ?$ $T2 = \frac{1}{2} \cdot m \cdot v2^2$ $V2 = 0$

Conservation of energy: $T1 + V1 = T2 + V2$

$$0 + m \cdot g \cdot L \cdot (1 - \cos(\alpha)) = \frac{1}{2} \cdot m \cdot v2^2 + 0$$

$$v2 = \sqrt{2 \cdot g \cdot L \cdot (1 - \cos(\alpha))}$$

Use the definition of the coefficient of restitution to include position (2), immediately before impact, and position (2'), right after impact between ball and wall. Notice that the the wall velocity is zero at all times, thus:

Coefficient of restitution: $e = \frac{v2p - 0}{0 - v2}$, and $v2p = e \cdot v2$

Apply conservation of energy from the point (2'), when the ball bounces off the wall, to point (3), when the ball reaches its maximum height and v3 = 0:

Thus, at position (2'):

$$T2p = \frac{1}{2} \cdot m \cdot v2p^2 = \frac{1}{2} \cdot m \cdot (e \cdot v2)^2 = \frac{1}{2} \cdot m \cdot e^2 \cdot (2 \cdot g \cdot L \cdot (1 - \cos(\alpha)))$$

$T2p = m \cdot e^2 \cdot g \cdot L \cdot (1 - \cos(\alpha))$,while $V2p = 0$

While, at position (3): $T3 = 0$ and

$$V3 = m \cdot g \cdot y3 = m \cdot g \cdot L \cdot (1 - \cos(\beta))$$

Conservation of energy: $T2p + V2p = T3 + V3$

$$m \cdot e^2 \cdot g \cdot L \cdot (1 - \cos(\alpha)) + 0 = 0 + m \cdot g \cdot L \cdot (1 - \cos(\beta))$$

$e^2 \cdot (1 - \cos(\alpha)) = 1 - \cos(\beta)$ --> $\cos(\beta) = 1 - e^2 \cdot (1 - \cos(\alpha))$

and, $\beta = \mathrm{acos}\left(1 - e^2 \cdot (1 - \cos(\alpha))\right) \cdot \frac{180}{\pi}$

With: $m := 0.13$ kg $L := 2.74$ m $\alpha := 46 \cdot \frac{\pi}{180}$ rad $e := 0.36$

$\beta := \mathrm{acos}\left(1 - e^2 \cdot (1 - \cos(\alpha))\right) \cdot \frac{180}{\pi}$ --> $\beta = 16.1724$ degrees

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