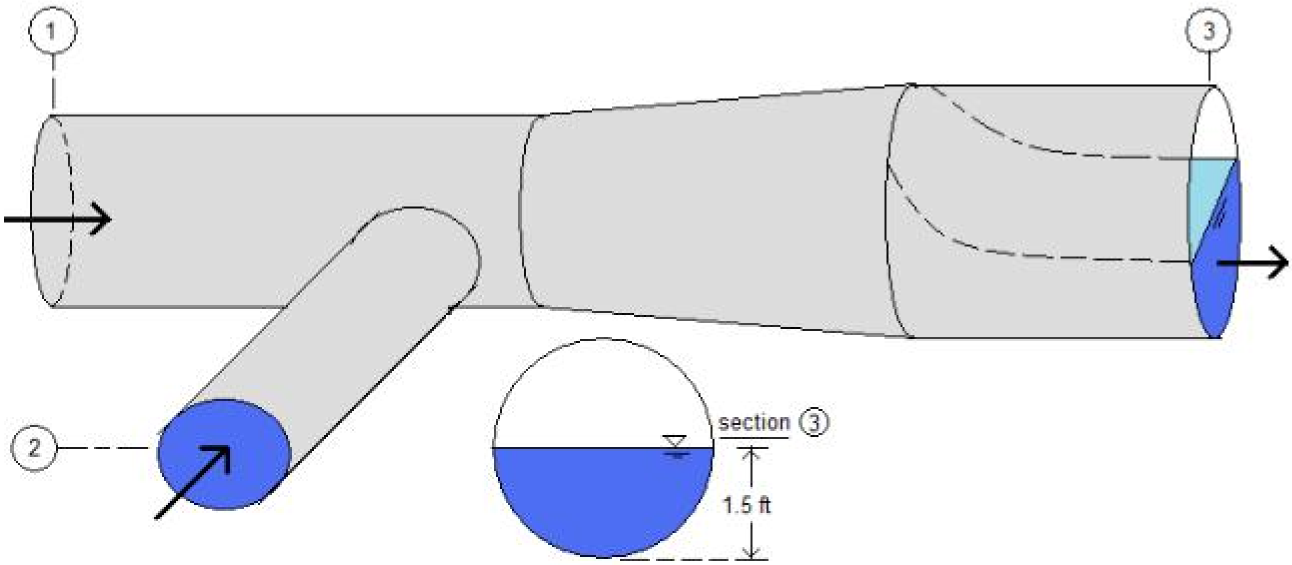
[1]. The diagram below shows the outlet from a water drainage system consisting of a main line (1) and a secondary line (2) draining through an outlet pipeline (3) located downstream from a pipeline expansion. The main and secondary lines are flowing full (i.e., under pressurized conditions). The diameters of the main line (1) and of the secondary line (2) are 2.0 ft and 1.0 ft, respectively. At the outlet pipeline (3) the flow is under open-channel conditions, and the water depth there is half the diameter of the pipeline. The flow velocities at sections (1) and (2) are 2.5 ft/s and 3.2 ft/s, respectively. (a) Determine the total discharge draining out of the main pipeline at section (3);
(b) Determine the flow velocity at section (3);



$D1 := 2.0 \ ft$ $\quad V1 := 2.5 \ \frac{ft}{s}$ $\quad Q1 := \frac{\pi \cdot D1^2}{4} \cdot V1$ $\quad Q1 = 7.854 \ cfs$

$D2 := 1.0 \ ft$ $\quad V2 := 3.2 \ \frac{ft}{s}$ $\quad Q2 := \frac{\pi \cdot D2^2}{4} \cdot V2$ $\quad Q2 = 2.5133 \ cfs$

Solution (a):

$Q3 := Q1 + Q2$ $\quad Q3 = 10.3673 \ cfs$

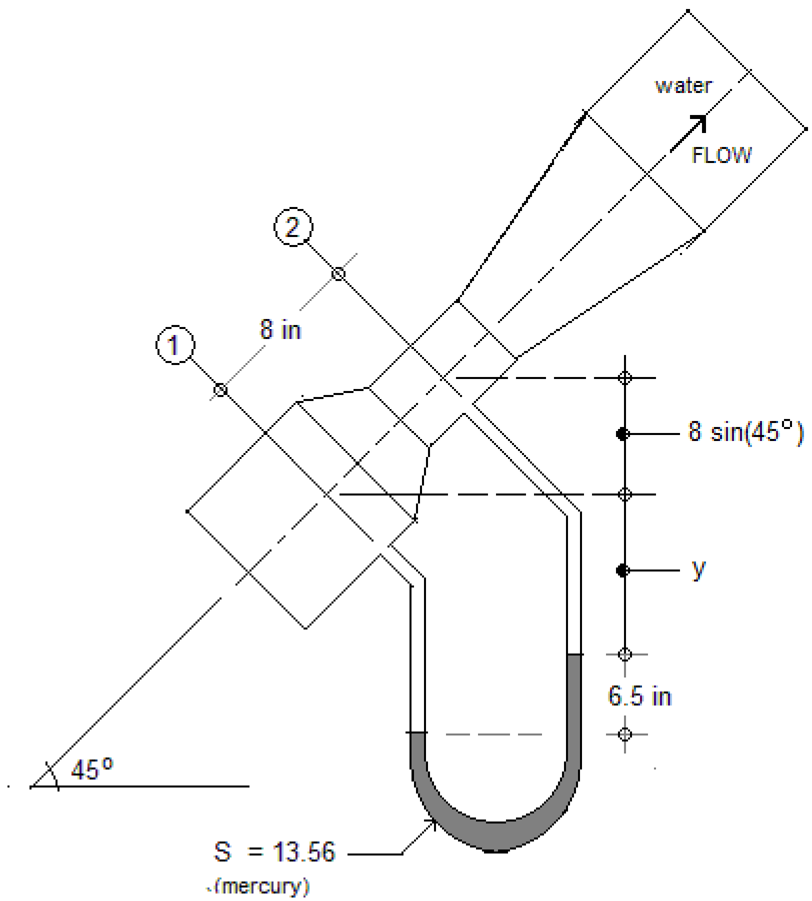
From the figure: $D3 := 2 \cdot 1.5 \ ft$

$A3 := \frac{1}{2} \cdot \left(\frac{\pi \cdot D3^2}{4}\right)$ $\quad A3 = 3.5343 \ ft^2$

Solution (b):

$V3 := \frac{Q3}{A3}$ $\quad V3 = 2.9333 \ \frac{ft}{s}$

[2] A Venturi meter is placed on a pipe at an angle of 45o, as shown in the figure below. (a) Using the manometer equation, determine the pressure difference Δp = p2 – p1, in psi. (b) If the diameters of the Venturi meter are D1 = 8 in and D2 = 4 in, determine the discharge through the Venturi meter. Neglect all energy losses.



Manometer equation:

$$p2+62.4\cdot\left(\frac{8}{12}\right)\cdot\sin\left(\frac{\pi}{4}\right)+62.4\cdot y+13.56\cdot 62.4\cdot\left(\frac{6.5}{12}\right)-62.4\cdot\left(\frac{6.5}{12}\right)-62.4\cdot y=p1$$

The term "62.4*y" is cancelled, thus the equation reduces to:

$$p2+62.4\cdot\left(\frac{8}{12}\right)\cdot\sin\left(\frac{\pi}{4}\right)+13.56\cdot 62.4\cdot\left(\frac{6.5}{12}\right)-62.4\cdot\left(\frac{6.5}{12}\right)=p1$$

From which, we can calculate:

$$\Delta p=p2-p1=62.4\cdot\left(\frac{6.5}{12}\right)-\left(62.4\cdot\left(\frac{8}{12}\right)\cdot\sin\left(\frac{\pi}{4}\right)+13.56\cdot 62.4\cdot\left(\frac{6.5}{12}\right)\right)$$

$$\Delta p:=\left(62.4\cdot\left(\frac{6.5}{12}\right)-\left(62.4\cdot\left(\frac{8}{12}\right)\cdot\sin\left(\frac{\pi}{4}\right)+13.56\cdot 62.4\cdot\left(\frac{6.5}{12}\right)\right)\right) \qquad \Delta p=-453.9436\ \text{psf}$$

To convert to "psi" divide by 144: $\Delta p\ psi:=\frac{\Delta p}{144}$

Solution (a): $\Delta p\ psi=-3.1524$ psi

Equation of continuity (1) - (2): $Q=\frac{\pi\cdot D1^2}{4}\cdot V1=\frac{\pi\cdot D2^2}{4}\cdot V2$

i.e., $D1^2\cdot V1=D2^2\cdot V2$, from which: $V2=\left(\frac{D1}{D2}\right)^2\cdot V1$

Also, $\frac{V2}{V1}=\left(\frac{D1}{D2}\right)^2$

Energy terms in points (1) and (2)

Point (1): Point (2):

$z1:=0$ $\qquad z2:=z1+\left(\frac{8}{12}\right)\cdot\sin\left(\frac{\pi}{4}\right)$, i.e., $z2=0.4714$ ft , and $\Delta z=z2-z1=0.4712$ ft

$p1=?$ $\qquad p2=?$ We do know, that $\Delta p=p2-p1=-453.9436$ psf

$V1 = ?$ $V2 = ?$ We do know, that $V2 = \left(\frac{D1}{D2}\right)^2 \cdot V1$

so, that $\frac{V2^2}{2 \cdot g} - \frac{V1^2}{2 \cdot g} = \frac{V1^2}{2 \cdot g} \cdot \left[\left(\frac{V2}{V1}\right)^2 - 1\right] = \frac{V1^2}{2 \cdot g} \cdot \left[\left(\frac{D1}{D2}\right)^4 - 1\right]$

Bernoulli Equation (1)-(2): $z1 + \frac{p1}{\gamma} + \frac{V1^2}{2 \cdot g} = z2 + \frac{p2}{\gamma} + \frac{V2^2}{2 \cdot g}$

$$(z2 - z1) + \left(\frac{p2 - p1}{\gamma}\right) + \left(\frac{V2^2}{2 \cdot g} - \frac{V1^2}{2 \cdot g}\right) = 0$$

i.e., $\Delta z + \frac{\Delta p}{\gamma} + \frac{V1^2}{2 \cdot g} \cdot \left[\left(\frac{D1}{D2}\right)^4 - 1\right] = 0$

with $\Delta z := z2 - z1$ i.e, $\Delta z = 0.4714$, $\Delta p = -453.9436\text{psf}$

$D1 := \frac{8}{12}$ ft, or, $D1 = 0.6667\text{ft}$, also, $D2 := \frac{4}{12}$ ft, or, $D2 = 0.3333\text{ft}$

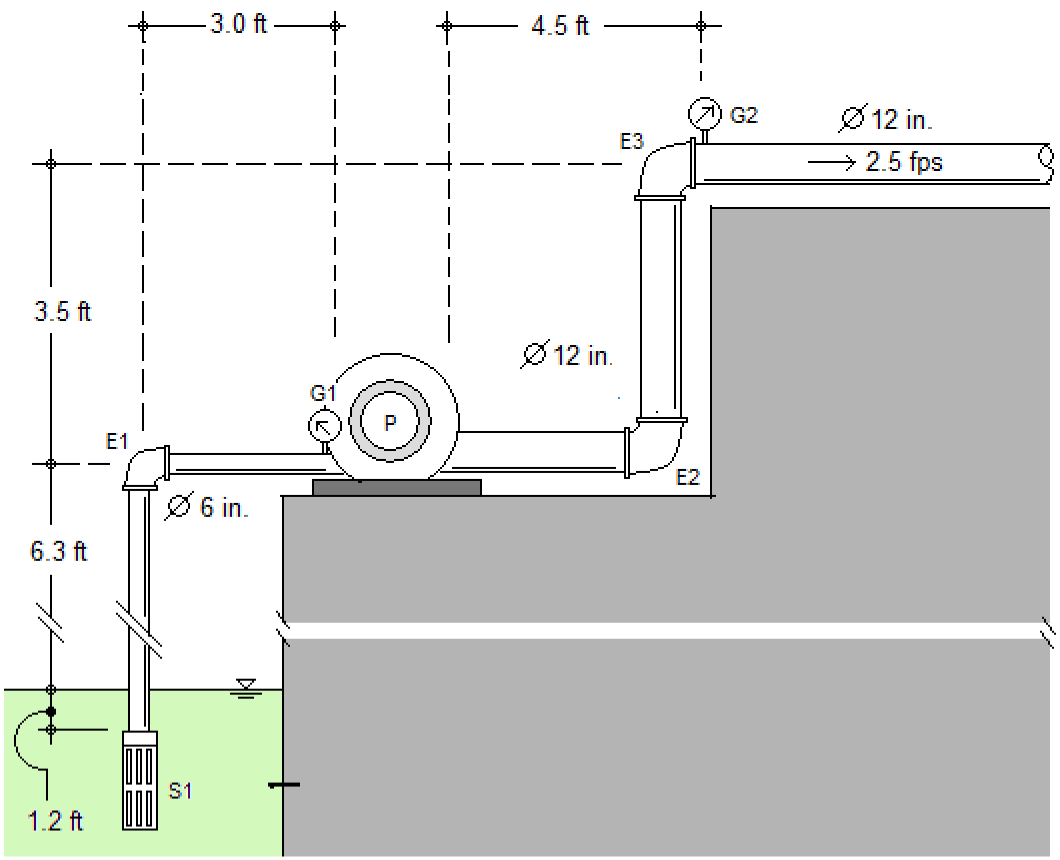
$\gamma := 62.4 \frac{\text{lb}}{\text{ft}^3}$ and, $g := 32.2 \frac{\text{ft}}{\text{s}^2}$, we solve for V1 by using:

$$\text{solve}\left[\Delta z + \frac{\Delta p}{\gamma} + \frac{V1^2}{2 \cdot g} \cdot \left[\left(\frac{D1}{D2}\right)^4 - 1\right] = 0, V1\right] = \begin{pmatrix} -5.4045 \\ 5.4045 \end{pmatrix}$$

Select: $V1 := 5.4045 \frac{\text{ft}}{\text{s}}$, then $Q := V1 \cdot \left(\frac{\pi \cdot D1^2}{4}\right)$, i.e.,

Solution (b): $Q = 1.8865$ cfs

[3] The figure below shows a pump P lifting water from a pond through a 6-in-diameter suction pipeline and delivering it at a velocity of 2.5 fps through a 12-in-diameter discharge pipeline. The suction pipeline is provided by a trash screen, S1, with a minor loss coefficient KS1 = 0.6, and one elbow, E1, with a minor loss coefficient KE1 = 1.2. As shown in the figure, the delivery pipeline is fitted with two elbows, E2 and E3, with discharge coefficients KE2 = KE3 = 0.8. The pump-pipeline system is provided with two pressure gages: G1, located in the suction end of the pump, and G2, located after the second elbow in the discharge pipeline and 3.5 ft above the pump. Gage G2 shows a reading of 2.0 psi. Determine (a) the power that the pump delivers to the flow in horsepower; and (b) the pressure in gage G1 in psi.



The minor losses in the screen and elbows are calculated using the equation:

$D1 := \frac{6}{12}\ ft$, i.e., $D1 = 0.5\ ft$

$D2 := \frac{12}{12}\ ft$, i.e., $D2 = 1\ ft$

$V2 := 2.5\ \frac{ft}{s}$ $\quad g := 32.2\ \frac{ft}{s^2}$ $\quad \gamma := 62.4\ \frac{lb}{ft^3}$

From continuity: $\frac{\pi \cdot D1^2}{4} \cdot V1 = \frac{\pi \cdot D2^2}{4} \cdot V2$

$V1 := \left(\frac{D2}{D1}\right)^2 \cdot V2$ $\qquad V1 = 10\ \frac{ft}{s}$

Energy losses:

Minor losses:

$KS1 := 0.6$ $\quad KE1 := 1.2$ $\quad KE2 := 0.8$ $\quad KE3 := 0.8$

$hS1 := KS1 \cdot \frac{V1^2}{2 \cdot g}$ $\qquad hS1 = 0.9317\ ft$

$hE1 := KE1 \cdot \frac{V1^2}{2 \cdot g}$ $\qquad hE1 = 1.8634\ ft$

$hE2 := KE2 \cdot \frac{V2^2}{2 \cdot g}$ $\qquad hE2 = 0.0776\ ft$

$hE3 := KE3 \cdot \frac{V2^2}{2 \cdot g}$ $\qquad hE3 = 0.0776\ ft$

Friction losses:

$L1 := 1.2 + 6.3 + 3.0$ i.e., $L1 = 10.5\ ft$ $\quad f1 := 0.021$

$L2 := 4.5 + 3.5$ i.e., $L2 = 8\ ft$ $\quad f2 := 0.012$

$hf1 := f1 \cdot \frac{L1}{D1} \cdot \frac{V1^2}{2 \cdot g}$ i.e., $hf1 = 0.6848\ ft$

$$hm = Km \cdot \frac{V^2}{2 \cdot g}$$

where Km is the corresponding minor loss coefficient and V is the mean velocity in the pipeline where the fitting (screen, or elbow) is located. The friction losses in a pipeline of length L and diameter D are calculated using the equation:

$$hf = f \cdot \frac{L}{D} \cdot \frac{V^2}{2 \cdot g}$$

where f is a friction factor, and V is the velocity in the pipeline. For the 6-in pipeline in the figure the friction factor is f6 = 0.021, while for the 12-in pipeline the friction factor is f12 = 0.012.

Point (C) : location of gage G1

$zC := 6.3$ ft $VC := V1$ $VC = 10$ fps $pC = ?$

Minor losses (A)-(C): $hm := hS1 + hE1$ $hm = 2.795$ ft

Friction losses (A)-(C): $hf := hf1$ $hf = 0.6848$ ft

Energy equation (A) - (C):

$$zA + \frac{pA}{\gamma} + \frac{VA^2}{2 \cdot g} - hm - hf = zC + \frac{pC}{\gamma} + \frac{VC^2}{2 \cdot g}$$

$$pC := \gamma \cdot \left[zA + \frac{pA}{\gamma} + \frac{VA^2}{2 \cdot g} - hm - hf - \left(zC + \frac{VC^2}{2 \cdot g} \right) \right]$$

$\frac{pC}{\gamma} = -11.3326$ ft $pC = -707.1548$ psf

$pC\ psi := \frac{pC}{144}$ $pC\ psi = -4.9108$ psi

$$hf2 := f2 \cdot \frac{L2}{D2} \cdot \frac{V2^2}{2 \cdot g}$$ i.e., $hf2 = 0.0093$ ft

Point (A): surface of pond, Point (B): location of gage G2

$VA := 0$ $pA := 0$ $zA := 0$ $VB := V2$, i.e., $VB = 2.5\ \frac{ft}{s}$ $pB := 2 \cdot 144$

$zB := 6.3 + 3.5$, i.e., $zB = 9.8$ ft $pB = 288$ psf

Total minor losses (A)-(B): $hm := hS1 + hE1 + hE2 + hE3$ $hm = 2.9503$ ft

Total friction losses (A)-(B): $hf := hf1 + hf2$ $hf = 0.6941$ ft

Energy equation (A)-(B):

$$zA + \frac{pA}{\gamma} + \frac{VA^2}{2 \cdot g} - hm - hf + hP = zB + \frac{pB}{\gamma} + \frac{VB^2}{2 \cdot g}$$

$$hP := solve\left(zA + \frac{pA}{\gamma} + \frac{VA^2}{2 \cdot g} - hm - hf + hP = zB + \frac{pB}{\gamma} + \frac{VB^2}{2 \cdot g}, hP \right)$$

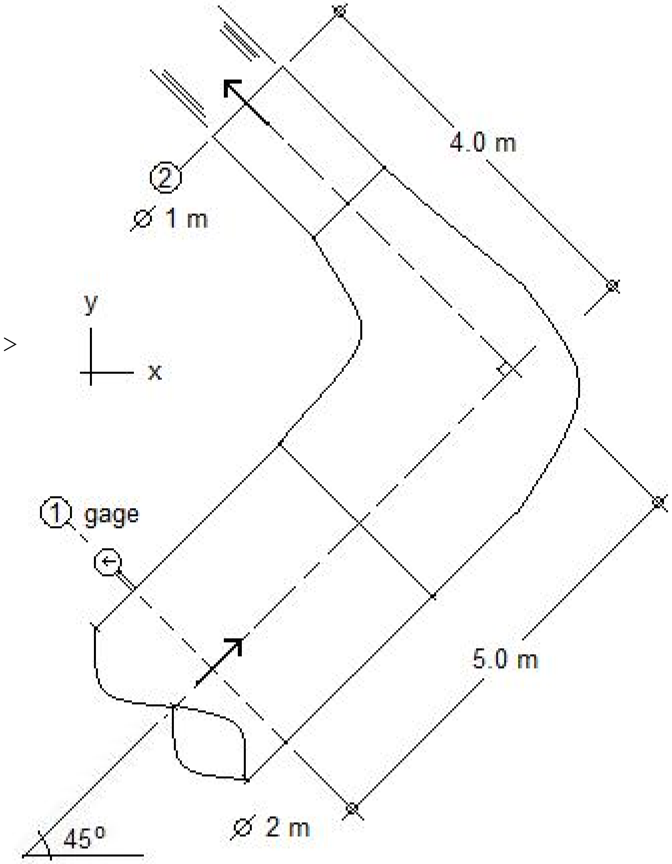
Then, $hP = 18.1568$ ft

Power delivered by pump: $Q := V1 \cdot \left(\frac{\pi \cdot D1^2}{4} \right)$ $Q = 1.9635$ cfs

$Pp := \frac{\gamma \cdot Q \cdot hP}{550}$ $Pp = 4.0448$ horsepower

Problem [4]. (Take home). The figure below shows a 90o reducing elbow located in a vertical plane that delivers water to an outlet at section (2) open to the atmosphere. The diameters of sections (1) and (2) are 2.0 m and 1.0 m, respectively. A pressure gage at section (1) reads a value of 200.0 kPa. Determine (a) the water discharge through the elbow, Q; (b) the x-component of the force that the flowing water applies on the elbow; and, (c) the y-component of the force that the flowing water applies on the elbow. Assume negligible energy losses.

--->



NOTE: The volume of water contained within sections (1) and (2) is not known, therefore, you need to provide a reasonable guesstimate for this volume from the information in the figure (e.g., using two cylinders). The volume of water between sections (1) and (2) is required to estimate the weight of the water for the momentum equation.

$D1 := 2.0\ m$ $\quad D2 := 1.0\ m$ $\quad g := 9.81\ \frac{m}{s^2}$ $\quad \gamma := 9810\ \frac{N}{m^3}$

Continuity: $\frac{\pi \cdot D1^2}{4} \cdot V1 = \frac{\pi \cdot D2^2}{4} \cdot V2$ => $V1 = \left(\frac{D2}{D1}\right)^2 \cdot V2$

with $\left(\frac{D2}{D1}\right)^2 = 0.25$, then, $V1 = 0.25 \cdot V2$

Energy (1)-(2) with no losses, i.e., Bernoulli's eqn:

Point (1): $z1 := 0 \quad p1 := 200\cdot 10^{3}\,Pa \quad V1 = ?$

Point (2): $z2 := 5\cdot \sin\left(\frac{\pi}{4}\right) + 4\cdot \sin\left(\frac{\pi}{4}\right) \quad z2 = 6.364\ m$

$p2 := 0 \quad v2 = ?$

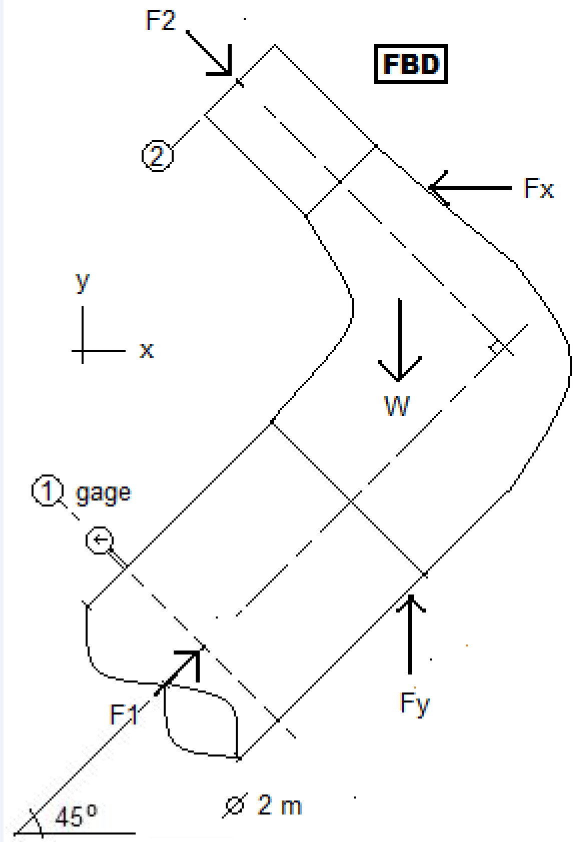
Bernoulli's equation: $z1 + \frac{p1}{\gamma} + \frac{V1^2}{2\cdot g} = z2 + \frac{p2}{\gamma} + \frac{V2^2}{2\cdot g}$

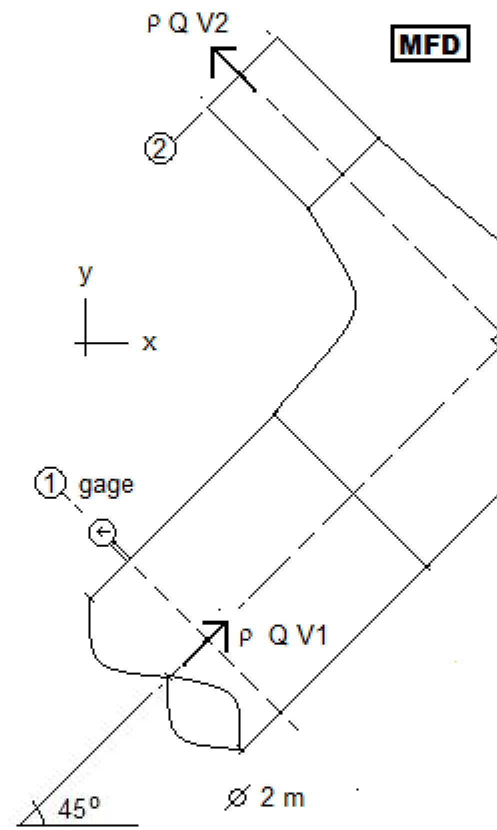
replace: $V1 = 0.25\cdot V2$ into Bernoulli's equation:

$$z1 + \frac{p1}{\gamma} + \frac{(0.25\cdot V2)^2}{2\cdot g} = z2 + \frac{p2}{\gamma} + \frac{V2^2}{2\cdot g}$$

Solving for V2:

$$V2 := \mathrm{solve}\left(z1 + \frac{p1}{\gamma} + \frac{(0.25\cdot V2)^2}{2\cdot g} - \left(z2 + \frac{p2}{\gamma} + \frac{V2^2}{2\cdot g}\right), V2, 0, 100\right)$$





$V2 = 17.1313\ \frac{m}{s} \quad V1 := 0.25\cdot V2 \quad V1 = 4.2828\ \frac{m}{s}$

Discharge: $Q := V2\cdot\left(\frac{\pi\cdot D2^2}{4}\right) \quad Q = 13.4549\ \frac{m^3}{2}$

Density: $\rho := \frac{\gamma}{g} \quad \rho = 1000\ \frac{kg}{m^3}$

Momentum equation:

Estimate volume of water $Vol := \frac{\pi\cdot D1^2}{4}\cdot 5 + \frac{\pi\cdot D2^2}{4}\cdot 4$ ==> $Vol = 18.8496\ m^3$ Weight: $W := \gamma\cdot Vol \quad W = 1.8491\cdot 10^{5}\ N$

-Other forces: $F1 := p1\cdot\left(\frac{\pi\cdot D1^2}{4}\right) \quad F1 = 6.2832\cdot 10^{5}\ N \quad F2 := p2\cdot\left(\frac{\pi\cdot D2^2}{4}\right) \quad F2 = 0$

Momentum eqn.x: $F1\cdot\cos\left(\frac{\pi}{4}\right) - Fx + F2\cdot\cos\left(\frac{\pi}{4}\right) = -\rho\cdot Q\cdot V2\cdot\cos\left(\frac{\pi}{4}\right) - \rho\cdot Q\cdot V1\cdot\cos\left(\frac{\pi}{4}\right)$

$Fx := F1\cdot\cos\left(\frac{\pi}{4}\right) + F2\cdot\cos\left(\frac{\pi}{4}\right) - \left(-\rho\cdot Q\cdot V2\cdot\cos\left(\frac{\pi}{4}\right) - \rho\cdot Q\cdot V1\cdot\cos\left(\frac{\pi}{4}\right)\right) \quad Fx = 6.4802\cdot 10^{5}\ N$

Momentum eqn.y: $F1 \cdot \sin\left(\frac{\pi}{4}\right) + Fy - W - F2 \cdot \sin\left(\frac{\pi}{4}\right) = \rho \cdot Q \cdot V2 \cdot \sin\left(\frac{\pi}{4}\right) - \rho \cdot Q \cdot V1 \cdot \sin\left(\frac{\pi}{4}\right)$

$Fy := \rho \cdot Q \cdot V2 \cdot \sin\left(\frac{\pi}{4}\right) - \rho \cdot Q \cdot V1 \cdot \sin\left(\frac{\pi}{4}\right) - \left(F1 \cdot \sin\left(\frac{\pi}{4}\right) - W - F2 \cdot \sin\left(\frac{\pi}{4}\right)\right)$ $Fy = -1.3713 \cdot 10^{5}\ N$