Solution to ENGR 2030 - DYNAMICS Test 02 - Spring Semester 2010 -
Utah State University - Instructor: Gilberto Urroz

The values of the problem parameters were randomly generated by Blackboard from a range of values provided by the instructor. This is one of the tests generated by Blackboard. The parameters, therefore, would most likely be different than the ones in your test, but the procedure is common for all tests thus generated. These solutions were calculated using SMath Studio, a free WYSIWYG math worksheet:

http://www.neng.usu.edu/cee/faculty/gurro/SMathStudio.html

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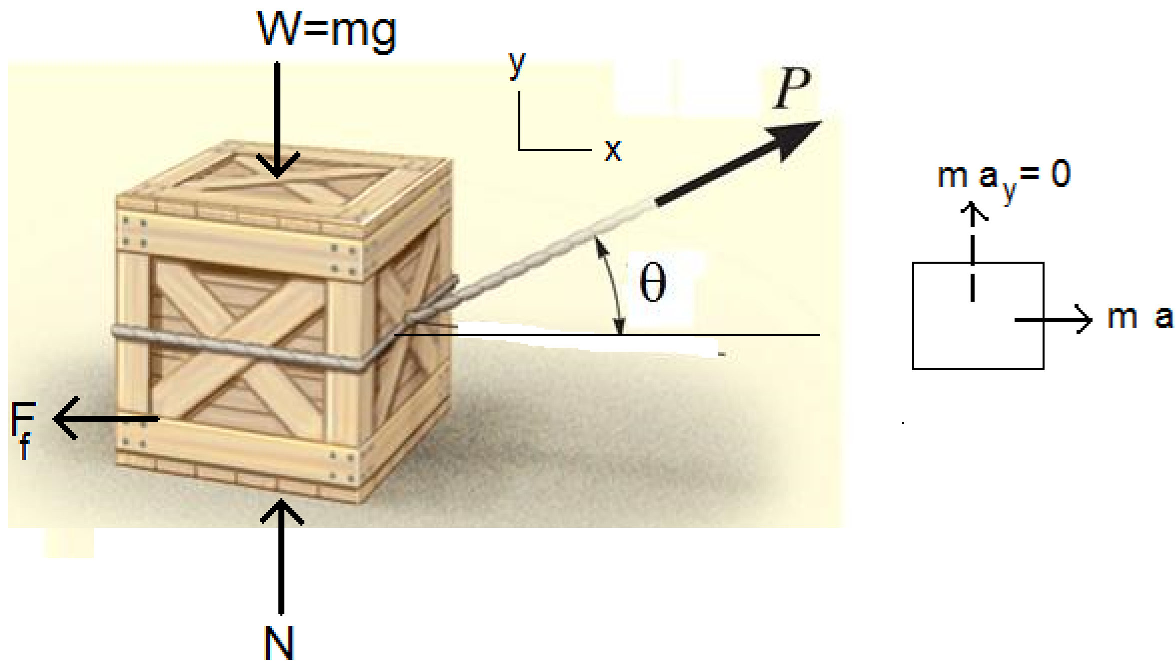
1. [1]. Kinetics of a particle - Problem 1 (Points: 10)

In order to pull the crate, a force P = 305 N is applied to the cord. The angle that the cord forms with the horizontal is q (theta) = 12°, the coefficient of kinetic friction between the crate and the floor is 0.12, and the mass of the crate is 198 kg. Determine the acceleration of the crate.



SOLUTION: The figure above shows the free-body diagram and the kinetic diagram for the crate. The equations of motion are:

$\Sigma F_x = m \cdot a_x$, i.e., $P \cdot \cos(\theta) - F_f = m \cdot a$ [1]

$\Sigma F_y = m \cdot a_y = 0$, i.e., $N + P \cdot \sin(\theta) - m \cdot g = 0$ [2]

From [2], one finds: $N = m \cdot g - P \cdot \sin(\theta)$

The friction force is: $F_f = \mu_k \cdot N = \mu_k \cdot (m \cdot g - P \cdot \sin(\theta))$

which replaced into [1] produces:

$P \cdot \cos(\theta) - \mu_k \cdot (m \cdot g - P \cdot \sin(\theta)) = m \cdot a$, and ,

$$a = \frac{P}{m} \cdot \cos(\theta) - \mu_k \cdot \left(g - \frac{P}{m} \cdot \sin(\theta)\right)$$

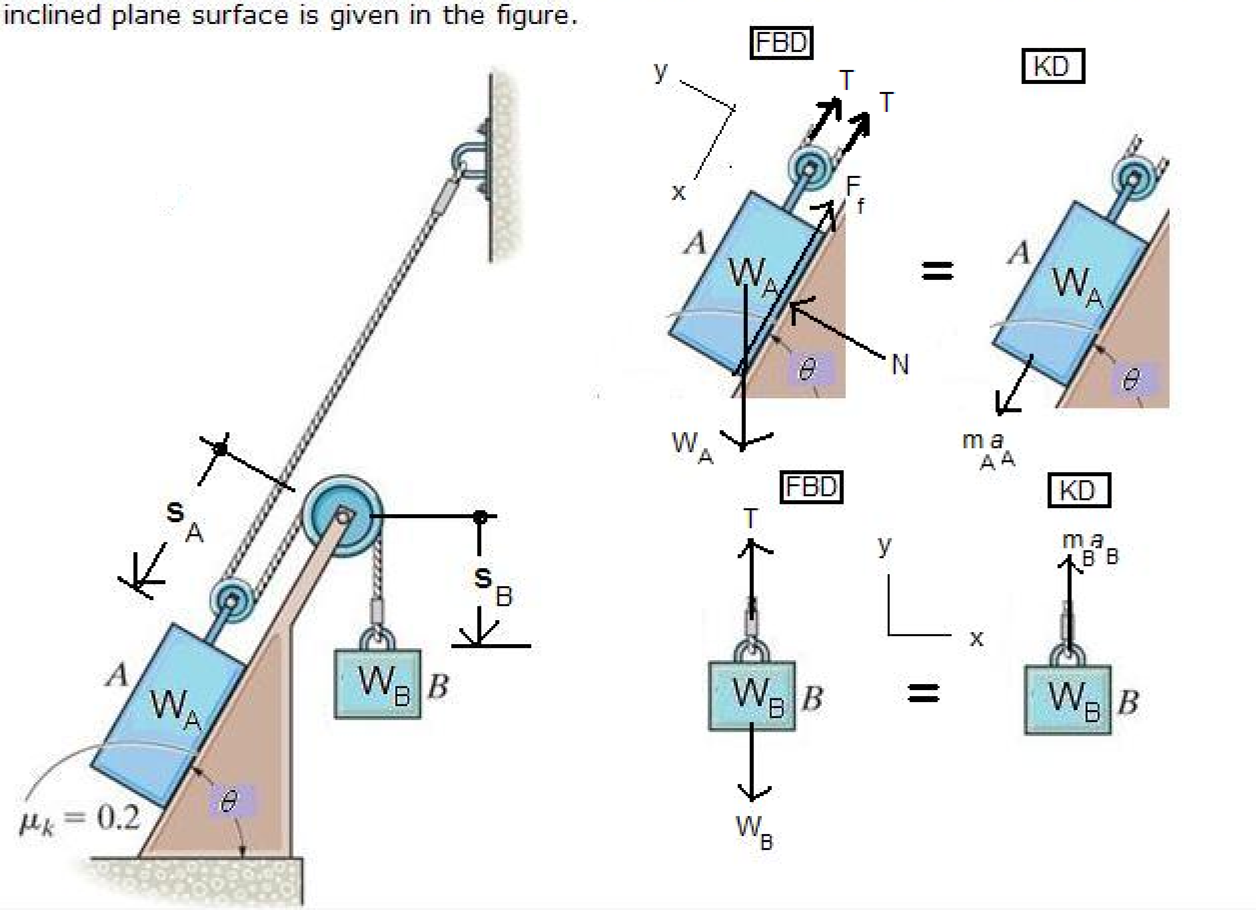
For this problem: $P := 305\ N$, $m := 198\ kg$, $\theta := 12 \cdot \frac{\pi}{180}$, or , $\theta = 0.2094\ rad$

$\mu k := 0.12$, and, $g := 9.81 \frac{m}{s^2}$, therefore,

$a := \frac{P}{m} \cdot \cos(\theta) - \mu k \cdot \left(g - \frac{P}{m} \cdot \sin(\theta)\right)$ --> $a = 0.368 \frac{m}{s^2}$

2. [2] - Kinetics of a particle - Problem 2 (Points: 10)

The system shown includes two blocks, A and B, joined by a cord-pulley system. Block B weights 29 pounds, while block A is 4 times as heavy. If the system is released from rest, determine the speed of block A after it has traveled 2.1 ft down the inclined plane. The angle of the inclined plane is q (theta) = 53°. The coefficient of friction between block A and the inclined plane surface is given in the figure.



Kinematics of the two blocks (absolute dependent motion): $2 \cdot s_A + s_B = L$

Take derivatives with respect to time, twice: $2 \cdot a_A + a_B = 0$ i.e., $a_B = -2 \cdot a_A$

Working with magnitudes only: $a_B = 2 \cdot a_A$ [1]

Kinetics of B: $\Sigma F_y = m_B \cdot a_B$ $T - W_B = \frac{W_B}{g} \cdot a_B$ [2]

With [1] into [2]: $T - W_B = 2 \cdot \frac{W_B}{g} \cdot a_A$ $T = W_B \cdot \left(1 + 2 \cdot \frac{a_A}{g}\right)$ [2']

Kinetics of A: $\Sigma F_y = 0$ $N - W_A \cdot \cos(\theta) = 0$

Normal force:

$$N = W_A \cdot \cos(\theta)$$

Friction force: $F_f = \mu_k \cdot N = \mu_k \cdot W_A \cdot \cos(\theta)$

More kinetics of A: $\Sigma F_x = m_A \cdot a_A$ $\qquad W_A \cdot \sin(\theta) - 2 \cdot T - F_f = \frac{W_A}{g} \cdot a_A$

Replace the friction force: $W_A \cdot \sin(\theta) - 2 \cdot T - \mu_k \cdot W_A \cdot \cos(\theta) = \frac{W_A}{g} \cdot a_A$

With [2']: $W_A \cdot \sin(\theta) - 2 \cdot W_B \cdot \left(1 + 2 \cdot \frac{a_A}{g}\right) - \mu_k \cdot W_A \cdot \cos(\theta) = \frac{W_A}{g} \cdot a_A$

Manipulating the equation to solve for the acceleration of A:

$$W_A \cdot \sin(\theta) - 2 \cdot W_B - 4 \cdot W_B \cdot \frac{a_A}{g} - \mu_k \cdot W_A \cdot \cos(\theta) = \frac{W_A}{g} \cdot a_A$$

$$W_A \cdot \sin(\theta) - 2 \cdot W_B - \mu_k \cdot W_A \cdot \cos(\theta) = \frac{W_A}{g} \cdot a_A + 4 \cdot W_B \cdot \frac{a_A}{g}$$

$$W_A \cdot \left(\sin(\theta) - \mu_k \cdot \cos(\theta)\right) - 2 \cdot W_B = \frac{a_A}{g} \cdot \left(W_A + 4 \cdot W_B\right)$$

With $W_A = n \cdot W_B$ --> $n \cdot W_B \cdot \left(\sin(\theta) - \mu_k \cdot \cos(\theta)\right) - 2 \cdot W_B = \frac{a_A}{g} \cdot \left(n \cdot W_B + 4 \cdot W_B\right)$

$n \cdot \left(\sin(\theta) - \mu_k \cdot \cos(\theta)\right) - 2 = \frac{a_A}{g} \cdot (n + 4)$ --> $a_A = \frac{n \cdot \left(\sin(\theta) - \mu_k \cdot \cos(\theta)\right) - 2}{n + 4} \cdot g$

Kinematics of block A: For constant acceleration:

$v_A = vA_0^{\,2} + 2 \cdot a_A \cdot \left(s_A - SA_0\right)$, with $vA_0 = 0$ at $sA_0 := 0$, then

$$v_A = \sqrt{2 \cdot s_A \cdot a_A} = \sqrt{2 \cdot s_A \cdot \frac{n \cdot \left(\sin(\theta) - \mu_k \cdot \cos(\theta)\right) - 2}{n + 4} \cdot g}$$

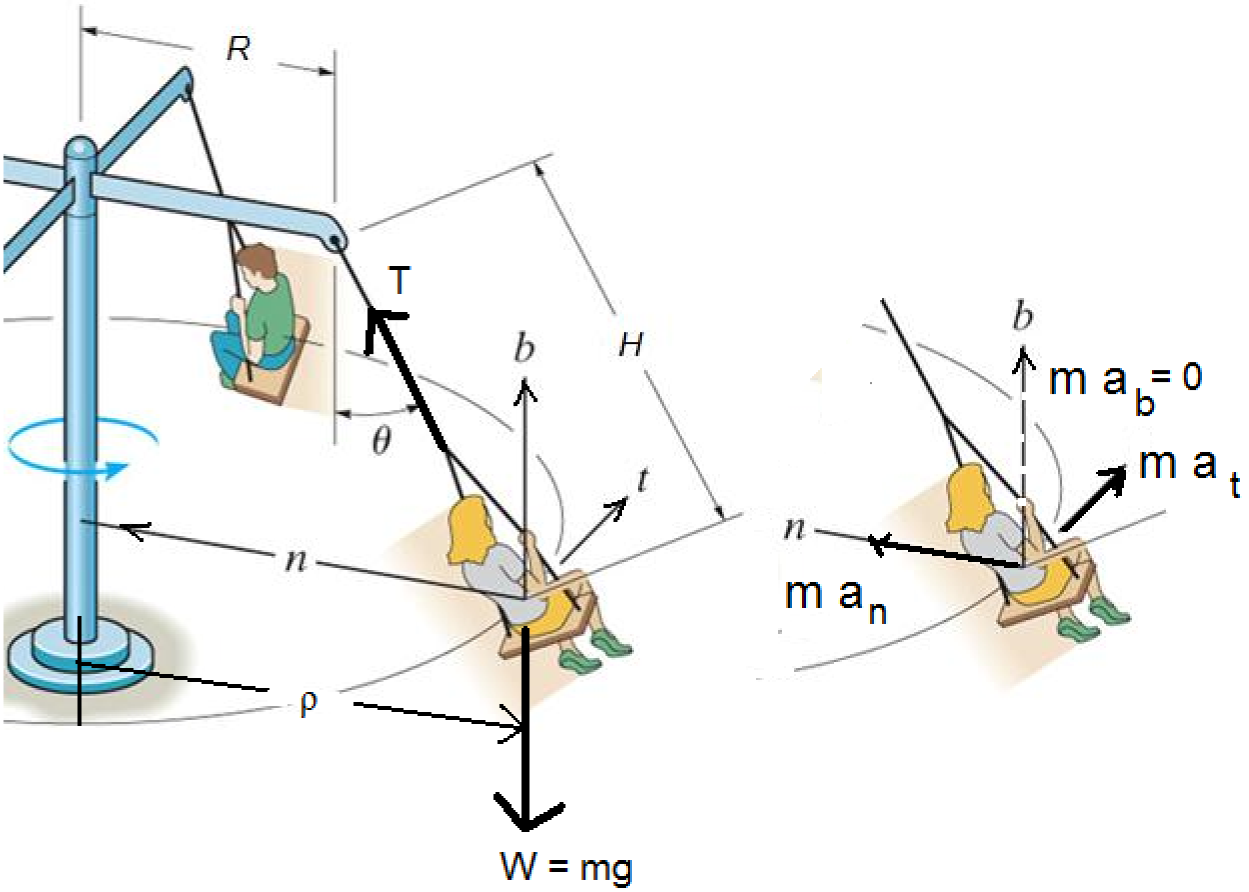
We can now calculate the required velocity, vA by using:

$\mu k := 0.2$ $\quad g := 32.2\ \frac{ft}{s^2}$ $\quad sA := 2.1\ ft$ $\quad n := 4$ $\quad \theta := 53 \cdot \frac{\pi}{180}\ rad$ $\quad WB := 29\ lb$

$vA := \left|\sqrt{2 \cdot sA \cdot \frac{n \cdot \left(\sin(\theta) - \mu k \cdot \cos(\theta)\right) - 2}{n + 4} \cdot g}\right|$ $\qquad vA = 3.472\ \frac{m}{s}$

3. [3] - Kinetics of a particle - Problem 3 (Points: 10)

The girl in the amusement park ride shown in the picture has a mass of 54 kg. As the ride turns about its axis at a constant angular rate the cable attached to the girl's seat forms an angle q (theta) = 27° with the vertical. If R = 3 m and H = 4.4 m, determine the constant speed of the girl in the circular path described in the figure. (H is the length from the point of attachment of the cable, to the girl's center of mass). For your analysis, use the normal (n) - tangential (t) - binormal (b) coordinate system shown in the figure.



The radius of curvature of the path described by the girl is: $\rho = R + H \cdot \sin(\theta)$

Her angular acceleration is: $an = \frac{v^2}{\rho} = \frac{v^2}{R + H \cdot \sin(\theta)}$

Equations of motion: $\Sigma F_b = 0$ $\quad T \cdot \cos(\theta) - m \cdot g = 0$ $\quad T = \frac{m \cdot g}{\cos(\theta)}$

$\Sigma F_n = m \cdot an$ $\quad T \cdot \sin(\theta) = m \cdot \frac{v^2}{\rho}$ $\quad \frac{m \cdot g}{\cos(\theta)} \cdot \sin(\theta) = m \cdot \frac{v^2}{R + H \cdot \sin(\theta)}$

Solving for v: $v = \sqrt{g \cdot \tan(\theta) \cdot (R + H \cdot \sin(\theta))}$

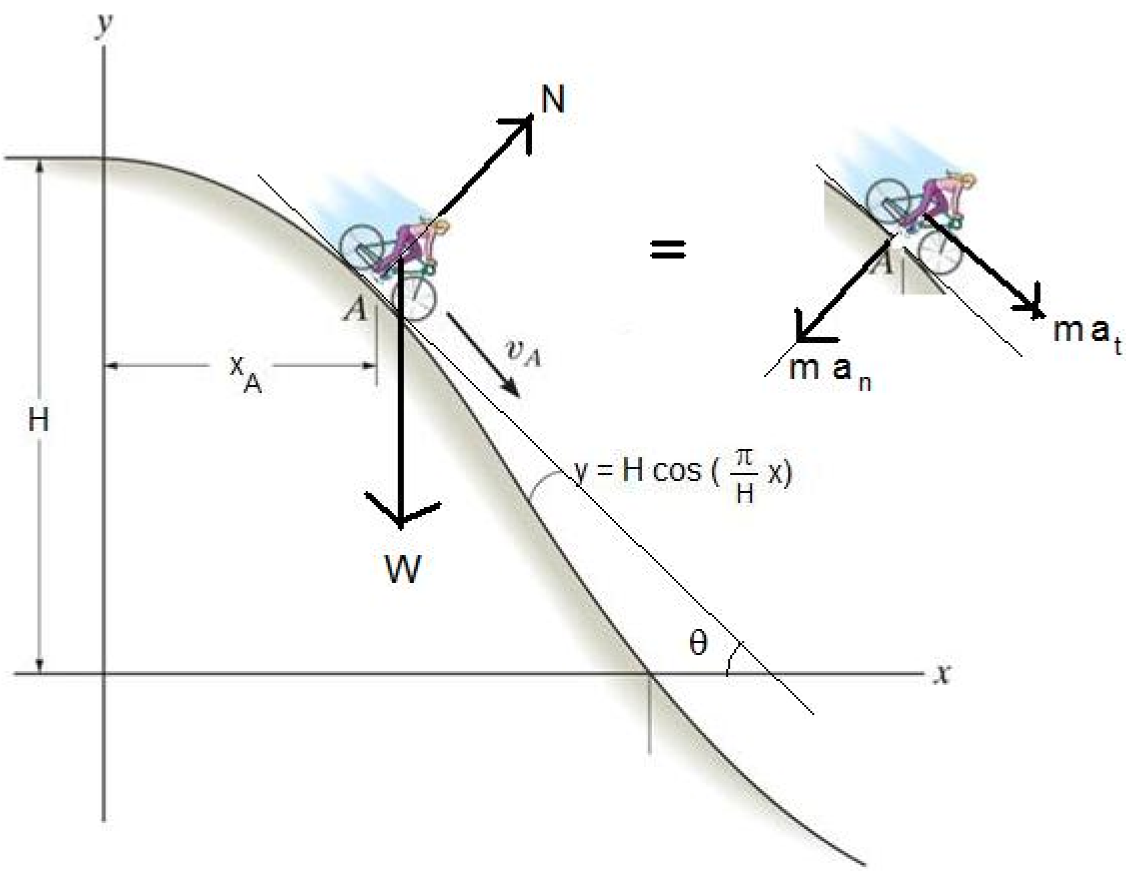
Replacing known values: $g := 9.81 \frac{m}{s^2}$ $\quad m := 54\ kg$

$R := 3\ m$ $\quad H := 4.4\ m$ and $\theta := 27 \cdot \frac{\pi}{180}$ rad

$v := \sqrt{g \cdot \tan(\theta) \cdot (R + H \cdot \sin(\theta))}$ --> $v = 4.998 \frac{m}{s}$

4. [4] - Kinematics of a particle - Problem 4 (Points: 20)

The cyclist is riding downhill on a hill described by the equation $y = H\cos\left(\frac{\pi}{H}x\right)$, with H = 37 ft, and both x and y in ft. If the cyclist has a speed v_A = 23 ft/s when she reaches point A, for which x_A = 16 ft, determine the normal reaction of the surface on the cyclist at that point. The combined weight of the bycicle and cyclist is 146 lb.



First, we'll find an expression for the radius of curvature, starting from:

$y = H\cdot\cos\left(\frac{\pi}{H}\cdot x\right)$. The first an dsecond derivatives are:

$$yp = \frac{d}{dx}y = \frac{d}{dx}\left(H\cdot\cos\left(\frac{\pi}{H}\cdot x\right)\right) = -H\cdot\sin\left(\frac{\pi}{H}\cdot x\right)\cdot\left(\frac{\pi}{H}\right) = -\pi\cdot\sin\left(\frac{\pi}{H}\cdot x\right)$$

$$ypp = \frac{d}{dx}yp = -\pi\cdot\cos\left(\frac{\pi}{H}\cdot x\right)\cdot\frac{\pi}{H} = -\frac{\pi^2}{H}\cdot\cos\left(\frac{\pi}{H}\cdot x\right)$$

Thus, $$\rho = \frac{\left(1+yp^2\right)^{\frac{2}{3}}}{|ypp|} = \frac{\left[1+\left(-\pi\cdot\sin\left(\frac{\pi}{H}\cdot x\right)\right)^2\right]^{\frac{3}{2}}}{\left|-\frac{\pi^2}{H}\cdot\cos\left(\frac{\pi}{H}\cdot x\right)\right|} = \frac{\left(1+\pi^2\cdot\sin\left(\frac{\pi}{H}\cdot x\right)^2\right)^{\frac{3}{2}}}{\frac{\pi^2}{H}\cdot\left|\cos\left(\frac{\pi}{H}\cdot x\right)\right|}$$

The normal acceleration is: $a_n = \frac{v^2}{\rho} = \frac{v^2 \cdot \frac{\pi^2}{H} \cdot \left|\cos\left(\frac{\pi}{H} \cdot x\right)\right|}{\left(1 + \pi^2 \cdot \sin\left(\frac{\pi}{H} \cdot x\right)^2\right)^{\frac{3}{2}}}$

Also, the angle of the tangent at the point of interest is θ, as shown:

$$\theta = \operatorname{atan}\left(\frac{dy}{dx}\right) = \operatorname{atan}(yp) = \operatorname{atan}\left(-\pi \cdot \sin\left(\frac{\pi}{H} \cdot x\right)\right)$$

The angle is a negative angle, but we take the absolute value for the angle as shown in the figure above, i.e.,

$$\theta = \left|\operatorname{atan}\left(-\pi \cdot \sin\left(\frac{\pi}{H} \cdot x\right)\right)\right|$$

Equations of motion: $\Sigma F_n = m \cdot a_n$ $\qquad W \cdot \cos(\theta) - N = \frac{W}{g} \cdot a_n$

$$N = W \cdot \cos(\theta) - \frac{W}{g} \cdot a_n = W\left(\cos(\theta) - \frac{a_n}{g}\right) = W \cdot \left(\cos(\theta) - \frac{v^2 \cdot \frac{\pi^2}{H} \cdot \left|\cos\left(\frac{\pi}{H} \cdot x\right)\right|}{g \cdot \left(1 + \pi^2 \cdot \sin\left(\frac{\pi}{H} \cdot x\right)^2\right)^{\frac{3}{2}}}\right)$$

$$N = W \cdot \left(\cos\left(\left|\operatorname{atan}\left(-\pi \cdot \sin\left(\frac{\pi}{H} \cdot x\right)\right)\right|\right) - \frac{v^2 \cdot \frac{\pi^2}{H} \cdot \left|\cos\left(\frac{\pi}{H} \cdot x\right)\right|}{g \cdot \left(1 + \pi^2 \cdot \sin\left(\frac{\pi}{H} \cdot x\right)^2\right)^{\frac{3}{2}}}\right)$$

$H := 37\ ft$ $\qquad W := 146\ lb$ $\qquad v := 23\ \frac{ft}{s}$ $\qquad x := 16\ ft$ $\qquad g := 32.2\ \frac{ft}{s^2}$

$$N := W \cdot \left(\cos\left(\left|\operatorname{atan}\left(-\pi \cdot \sin\left(\frac{\pi}{H} \cdot x\right)\right)\right|\right) - \frac{v^2 \cdot \frac{\pi^2}{H} \cdot \left|\cos\left(\frac{\pi}{H} \cdot x\right)\right|}{g \cdot \left(1 + \pi^2 \cdot \sin\left(\frac{\pi}{H} \cdot x\right)^2\right)^{\frac{3}{2}}}\right)$$

$N = 41.2034\ lb$

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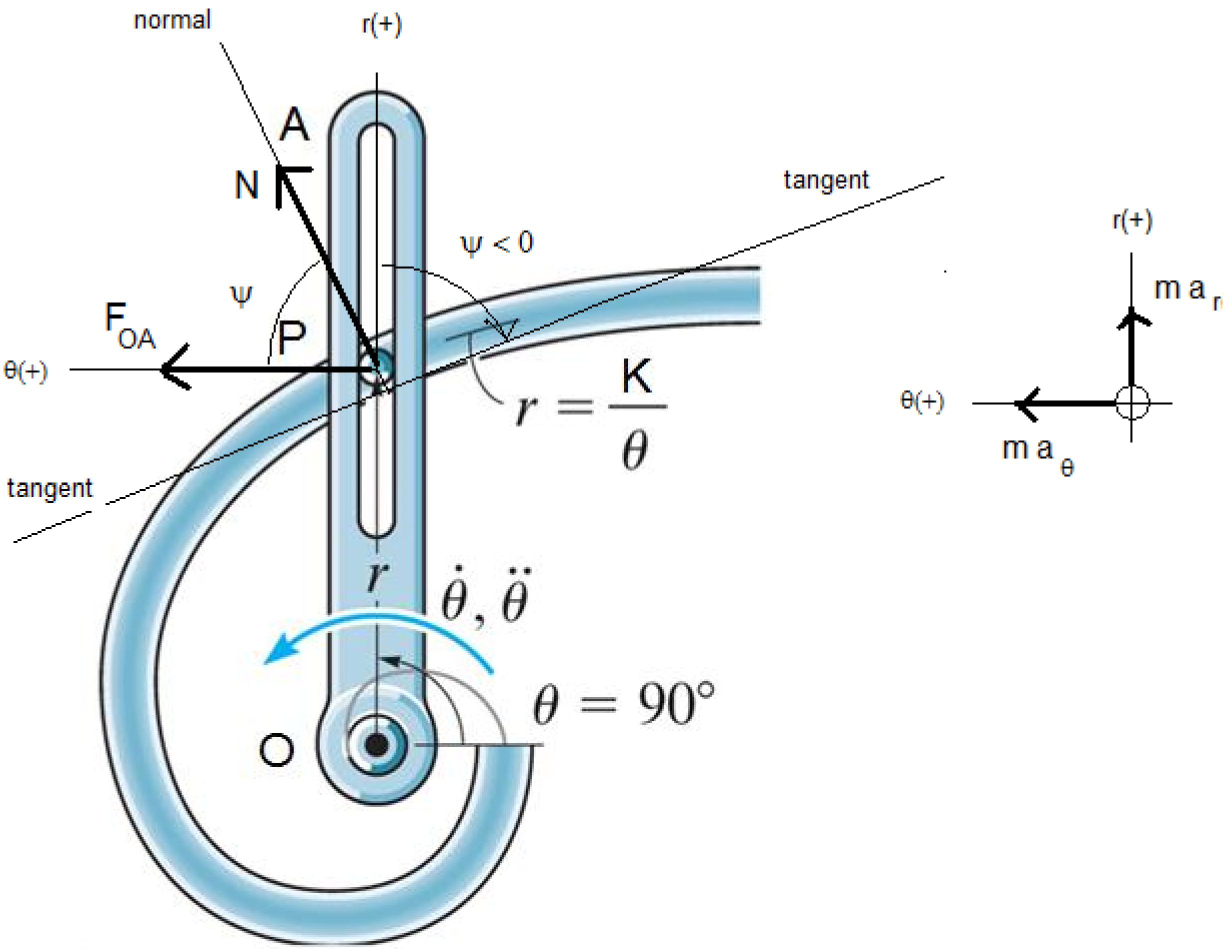
5. [5] - Kinetics of a particle - Problem 5 (Points: 20)

The pin P is driven by the motion of the slotted arm OA and is forced to move along the path given by $r = \frac{K}{\theta}$, where K = 0.4 *ft*, and the angle θ is in *rad*. At the instant shown, when θ = 90°, the arm is rotating counterclockwise with angular velocity and acceleration of 0.1 rad/s and 17.5 rad/s^2, respectively (both counterclockwise). Determine the force that the slotted arm OA exerts on the pin at that instant. The pin weights 0.4 *lbs*. Motion takes place in a horizontal plane. Enter your result with 3 decimals.



Angle between r(+) and the tangent line: $\tan(\psi) = \dfrac{r}{\dfrac{dr}{d\theta}}$, with

$r = \frac{K}{\theta} = K \cdot \theta^{-1}$ --> $\frac{dr}{d\theta} = -K \cdot \theta^{-2} = -\frac{K}{\theta^2}$ and, $\tan(\psi) = \left(\frac{K}{\theta}\right) \cdot \left(-\frac{\theta^2}{K}\right) = -\theta$

i.e., $\psi = \operatorname{atan}(-\theta)$. Since, for the instant shown, $\theta = 90^{\circ} = \left(\frac{\pi}{2}\right)$ rad

then, $\psi := \operatorname{atan}\left(-\frac{\pi}{2}\right)$, i.e., $\psi = -1.0039$ rad

or, $\psi := \psi \cdot \frac{180}{\pi}$, i.e., $\psi = -57.5184$ deg

The angle ψ being negative indicates that the angle of 57.52 degrees is to be measure in a clockwise direction from r(+), as shown. In the rest of the calculations, we keep the magnitude of ψ, only, i.e.,

$$\psi := -\psi \quad , \text{ or,} \quad \psi = 57.5184$$

For the purpose of calculating trigonometric functions we re-write ψ as:

$$\psi := \psi \cdot \frac{\pi}{180} \quad , \text{ i.e.,} \quad \psi = 1.0039 \text{ rad}$$

$$\sin(\psi) = 0.8436 \quad \cos(\psi) = 0.537 \quad \tan(\psi) = 1.5708$$

Kinematics: given θd, and θdd, with $\theta d = \frac{d\theta}{dt}$ and $\theta dd = \frac{d(\theta d)}{dt}$, from

$$r = \frac{K}{\theta} = K \cdot \theta^{-1} \quad \text{-->} \quad rd = \frac{dr}{dt} = -K \cdot \theta^{-2} \cdot \frac{d\theta}{dt} = -K \cdot \theta^{-2} \cdot \theta d = -\frac{K \cdot \theta d}{\theta^2}$$

$$rdd = \frac{d(rd)}{dt} = \frac{d}{dt}\left(-K \cdot \theta^{-2} \cdot \theta d\right) = -K \cdot \frac{d}{dt}\left(\theta^{-2} \cdot \theta d\right) = -K \cdot \left(-2 \cdot \theta^{-3} \cdot \theta d^2 + \theta^{-2} \cdot \theta dd\right)$$

$$\text{i.e.,} \quad rdd = -K \cdot \left(-\frac{2 \cdot \theta d^2}{\theta^3} + \frac{\theta dd}{\theta^2}\right) = -\frac{K}{\theta^3} \cdot \left(-2 \cdot \theta d^2 + \theta \cdot \theta dd\right)$$

Acceleration components in polar coordinates:

$$ar = rdd - r \cdot \theta d^2 = -\frac{K}{\theta^3} \cdot \left(-2 \cdot \theta d^2 + \theta \cdot \theta dd\right) - \frac{K}{\theta} \cdot \theta d^2$$

$$ar = \frac{2 \cdot K \cdot \theta d^2}{\theta^3} - \frac{K \cdot \theta dd}{\theta^2} - \frac{K \cdot \theta d^2}{\theta} = \frac{2 \cdot K \cdot \theta d^2 - K \cdot \theta \cdot \theta dd - K \cdot \theta^2 \cdot \theta d^2}{\theta^3}$$

$$ar = K \cdot \left(\frac{2 \cdot \theta d^2 - \theta \cdot \theta dd - \theta^2 \cdot \theta d^2}{\theta^3}\right) = \frac{K}{\theta^3} \cdot \left(\theta d^2 \cdot \left(2 - \theta^2\right) - \theta \cdot \theta dd\right)$$

$$a\theta = r \cdot \theta dd + 2 \cdot rd \cdot \theta d = \frac{K \cdot \theta dd}{\theta} + 2 \cdot \left(-\frac{K \cdot \theta d}{\theta^2}\right) \cdot \theta d = \frac{K}{\theta^2} \cdot \left(\theta \cdot \theta dd - 2 \cdot \theta d^2\right)$$

Equations of motion: $\Sigma F_r = m \cdot ar$ --> $N \cdot \sin(\psi) = \frac{W}{g} \cdot ar$ --> $N = \frac{W \cdot ar}{g \cdot \sin(\psi)}$

$\Sigma F_\theta = m \cdot a_\theta$ --> $F_{OA} + N \cdot \cos(\psi) = \frac{W}{g} \cdot a\theta$

$$F_{OA} = \frac{W}{g} \cdot a\theta - N \cdot \cos(\psi) = \frac{W \cdot a\theta}{g} - \frac{W \cdot ar}{g \cdot \sin(\psi)} \cdot \cos(\psi) \quad \text{-->} \quad F_{OA} = \frac{W}{g} \cdot \left(a\theta - \frac{ar}{\tan(\psi)}\right)$$

$$F_{OA} = \frac{W}{g} \cdot \left(\frac{K}{\theta^2} \cdot \left(\theta \cdot \theta dd - 2 \cdot \theta d^2\right) - \frac{K}{\theta^3 \cdot \tan(\psi)} \cdot \left(\theta d^2 \cdot \left(2 - \theta^2\right) - \theta \cdot \theta dd\right)\right)$$

Note: $\frac{K}{\theta} \cdot \theta dd - \frac{2 \cdot K}{\theta^2} \cdot \theta d^2 - \frac{K \cdot \left(2 - \theta^2\right)}{\theta^3 \cdot \tan(\psi)} \cdot \theta d^2 + \frac{K}{\theta^2 \cdot \tan(\psi)} \cdot \theta dd$

is expanded and factored as:

$$\frac{K}{\theta^2}\cdot\left(\theta+\frac{1}{\tan(\psi)}\right)\cdot\theta dd-\frac{K}{\theta^3}\cdot\left(2\cdot\theta+\left(\frac{2-\theta^2}{\tan(\psi)}\right)\right)\cdot\theta d^2$$

Thus, $$F_{OA}=\frac{W}{g}\cdot\left(\frac{K}{\theta^2}\cdot\left(\theta+\frac{1}{\tan(\psi)}\right)\cdot\theta dd-\frac{K}{\theta^3}\cdot\left(2\cdot\theta+\left(\frac{2-\theta^2}{\tan(\psi)}\right)\right)\cdot\theta d^2\right)$$

Using the data given:

$W := 0.4\ \text{lb}$ $\qquad K := 0.4\ \text{ft}$ $\qquad g := 32.2\ \frac{\text{ft}}{\text{s}^2}$

$\theta := \frac{\pi}{2}\ \text{rad}$ $\qquad \theta d := 0.1\ \frac{\text{rad}}{\text{s}}$ $\qquad \theta dd := 17.5\ \frac{\text{rad}}{\text{s}^2}$

$$FOA := \frac{W}{g}\cdot\left(\frac{K}{\theta^2}\cdot\left(\theta+\frac{1}{\tan(\psi)}\right)\cdot\theta dd-\frac{K}{\theta^3}\cdot\left(2\cdot\theta+\left(\frac{2-\theta^2}{\tan(\psi)}\right)\right)\cdot\theta d^2\right)$$

$FOA = 0.0778\ \text{lb}$
