CALCULATE AREA PROPERTIES OF A SERIES OF RECTANGULAR SHAPES

by Will Massie, SOMAR
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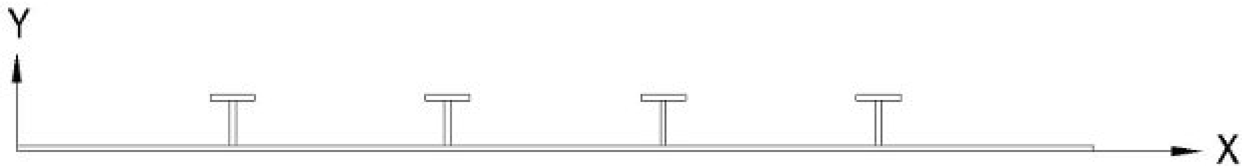
(DISCLAIMER: This worksheet is shared only as an example and should be used with caution. The calculations are not guaranteed to be error free.)

A. EXAMPLE GRAPHIC



B. INPUT VALUES

$$h = \begin{pmatrix} 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \\ 4 \\ 4 \\ 4 \\ 4 \end{pmatrix} in$$

(height dimension for each member)

$$b = \begin{pmatrix} 96 \\ 4 \\ 4 \\ 4 \\ 4 \\ 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \end{pmatrix} in$$

(base dimension for each member)

$$x_{bar} = \begin{pmatrix} 48 \\ 19.2 \\ 38.4 \\ 57.6 \\ 76.8 \\ 19.2 \\ 38.4 \\ 57.6 \\ 76.8 \end{pmatrix} in$$

(center of gravity x-value for each member)

$$y_{bar} = \begin{pmatrix} 0.25 \\ 4.75 \\ 4.75 \\ 4.75 \\ 4.75 \\ 2.5 \\ 2.5 \\ 2.5 \\ 2.5 \end{pmatrix} in$$

(center of gravity y-value for each member)

C. CALCULATE GEOMETRY BOUNDING VALUES

for $n \in 1..rows(h)$

$$x_{max_{n\,1}} = x_{bar_{n\,1}} + \frac{b_{n\,1}}{2}$$

$$x_{min_{n\,1}} = x_{bar_{n\,1}} - \frac{b_{n\,1}}{2}$$

$$y_{max_{n\,1}} = y_{bar_{n\,1}} + \frac{h_{n\,1}}{2}$$

$$y_{min_{n\,1}} = y_{bar_{n\,1}} - \frac{h_{n\,1}}{2}$$

$X_{max} = max(x_{max})$ $X_{max} = 96\ in$ (x-value of right-most edge)

$X_{min} = min(x_{min})$ $X_{min} = 0\ in$ (x-value of left-most edge)

$Y_{max} = max(y_{max})$ $Y_{max} = 5\ in$ (y-value of top-most edge)

$Y_{min} = min(y_{min})$ $Y_{min} = 0\ in$ (y-value of bottom-most edge)

D. CALCULATE TOTAL AREA

$A_{total} = b \cdot h$ $A_{total} = 64\ in^2$

E. CALCULATE FIRST MOMENTS OF AREAS FOR COMPOSITE AREA

$Q_x = \sum_{i=1}^{rows(h)} \left(x_{bar_{i\,1}} \cdot b_{i\,1} \cdot h_{i\,1} \right)$ $Q_x = 3072\ in^3$ (first moment of area with respect to the x-axis)

$Q_y = \sum_{i=1}^{rows(h)} \left(y_{bar_{i\,1}} \cdot b_{i\,1} \cdot h_{i\,1} \right)$ $Q_y = 70\ in^3$ (first moment of area with respect to the y-axis)

F. CALCULATE THE CENTROID OF COMPOSITE AREA

$X_{bar} = \frac{Q_x}{A_{total}}$ $X_{bar} = 48\ in$

$Y_{bar} = \frac{Q_y}{A_{total}}$ $Y_{bar} = 1.0938\ in$

G. CALCULATE THE MOMENT OF INERTIA OF THE COMPOSITE AREA

$$I_x = \sum_{i=1}^{rows(h)} \left(\left(\frac{1}{12} \cdot b_{i\,1} \cdot h_{i\,1}^{3} \right) + \left(b_{i\,1} \cdot h_{i\,1} \right) \cdot y_{bar_{i\,1}}^{2} \right)$$

$I_x = 245.3333\,in^4$ (moment of inertia with respect to the x-axis)

$$I_y = \sum_{i=1}^{rows(h)} \left(\left(\frac{1}{12} \cdot h_{i\,1} \cdot b_{i\,1}^{3} \right) + \left(b_{i\,1} \cdot h_{i\,1} \right) \cdot x_{bar_{i\,1}}^{2} \right)$$

$I_y = 1.917 \cdot 10^5\,in^4$ (moment of inertia with respect to the y-axis)

H. CALCULATE THE MOMENT OF INERTIA OF THE COMPOSITE AREA
WITH RESPECT TO THE CENTROIDAL AXES

$I'_x = I_x - A_{total} \cdot Y_{bar}^2$ $\quad I'_x = 168.7708\,in^4$ (moment of inertia with respect to the centroidal x-axis)

$I'_y = I_y - A_{total} \cdot X_{bar}^2$ $\quad I'_y = 44247.6333\,in^4$ (moment of inertia with respect to the centroidal y-axis)

I. CALCULATE THE SECTION MODULI OF THE COMPOSITE AREA

$S_{top} = \frac{I'_x}{Y_{max} - Y_{bar}}$ $\quad S_{top} = 43.2053\,in^3$ (section modulus to the top-most fiber)

$S_{bot} = \frac{I'_x}{Y_{bar} - Y_{min}}$ $\quad S_{bot} = 154.3048\,in^3$ (section modulus to the bottom-most fiber)

$S_{right} = \frac{I'_y}{X_{max} - X_{bar}}$ $\quad S_{right} = 921.8257\,in^3$ (section modulus to the right-most fiber)

$S_{left} = \frac{I'_y}{X_{bar} - X_{min}}$ $\quad S_{left} = 921.8257\,in^3$ (section modulus to the left-most fiber)

J. CALCULATE THE RADII OF GYRATION OF THE COMPOSITE AREA

$$k_x=\sqrt{\frac{I_x}{A_{total}}}$$ $k_x=1.9579\,in$ (radius of gyration with respect to the x-axis)

$$k_y=\sqrt{\frac{I_y}{A_{total}}}$$ $k_y=54.73\,in$ (radius of gyration with respect to the y-axis)

K. CALCULATE THE RADII OF GYRATION OF THE COMPOSITE AREA
WITH RESPECT TO THE CENTROIDAL AXES

$$k'_x=\sqrt{\frac{I'_x}{A_{total}}}$$ $k'_x=1.6239\,in$ (radius of gyration with respect to the centroidal x-axis)

$$k'_y=\sqrt{\frac{I'_y}{A_{total}}}$$ $k'_y=26.2939\,in$ (radius of gyration with respect to the centroidal y-axis)