Vertical Velocity Distributions
in a Hydraulically Smooth Open Channel Flow

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This worksheet develops the vertical velocity distributions in a smooth, trapezoidal open channel.

1. Properties of water

$\rho := 1.94 \frac{slug}{ft^3}$ — density of water

$\nu := 1.50 \cdot 10^{-5} \frac{ft^2}{sec}$ — kinematic viscosity of water

$g := 32.17 \frac{ft}{sec^2}$ — gravitational acceleration

$\mu := \rho \cdot \nu$ — $\mu = 2.91 \cdot 10^{-5} \frac{lbf\ sec}{ft^2}$ — dynamic viscosity of water

$\gamma := \rho \cdot g$ — $\gamma = 62.4098 \frac{lbf}{ft^3}$ — unit weight of water

2. Channel Properties

$Y := 4\ ft$ — depth of water

$B := 50\ ft$ — bottom width of channel

$n := 0.01$ — Manning's coefficient

$S_0 := 0.0004$ — channel longitudinal slope

$z := 2$ — channel side slope

$T := B + 2 \cdot Y \cdot z$ — $T = 66\ ft$ — top width of channel

$A := Y \cdot \left(\frac{B+T}{2}\right)$ — $A = 232\ ft^2$ — cross-sectional area

$P := B + 2 \cdot \sqrt{Y \cdot Y \cdot z}$ — $P = 61.3137\ ft$ — wetted perimeter

$R := \frac{A}{P}$ — $R = 3.7838\ ft$ — hydraulic radius

3. Open Channel Hydraulics

$C := 1.486 \frac{ft^{\frac{1}{3}}}{sec}$ — unit conversion coefficient

$Q := \frac{C}{n} \cdot A \cdot R^{\left(\frac{2}{3}\right)} \cdot \sqrt{S_0}$ — $Q = 1674.2632 \frac{ft^3}{sec}$ — flow in channel

$V := \frac{Q}{A}$ — $V = 7.2167 \frac{ft}{sec}$ — average velocity in channel

4. Shear Stress at Channel Bottom

$\tau_0 := \gamma \cdot R \cdot S_0$ | $\tau_0 = 0.0945 \frac{lbf}{ft^2}$ | (uniform) shear stress

$v_s := \sqrt{g \cdot Y \cdot S_0}$ | $v_s = 0.2269 \frac{ft}{sec}$ | shear velocity

$y_0 := \frac{\nu}{9 \cdot v_s}$ | $y_0 = 7.3462 \cdot 10^{-6}\ ft$ | smooth channel intercept; solution valid if k, the roughness element length is smaller than y0.

5. Velocity Distributions

$v(d) := 2.5 \cdot v_s \cdot \ln\left(\frac{Y - d}{y_0}\right)$ velocity distribution function

6. Plots

$k := 1, 1+1 .. 20$ create a dummy index

$n := \text{length}(k)$ length of index

for $i \in 1 .. n$
$d_i := (k_i - 1) \cdot \frac{Y}{n}$
for loop assigns depth for each element in k

for $i \in 1 .. n$
$y_i := (Y - d_i) \cdot \frac{1}{ft}$
for loop assigns height for each element in k

for $i \in 1 .. n$
$x_i := v(d_i) \frac{sec}{ft}$
for loop assigns velocity for each element in k

$V_1 := \text{augment}(x, y)$ augment x and y vectors into matrix for plotting

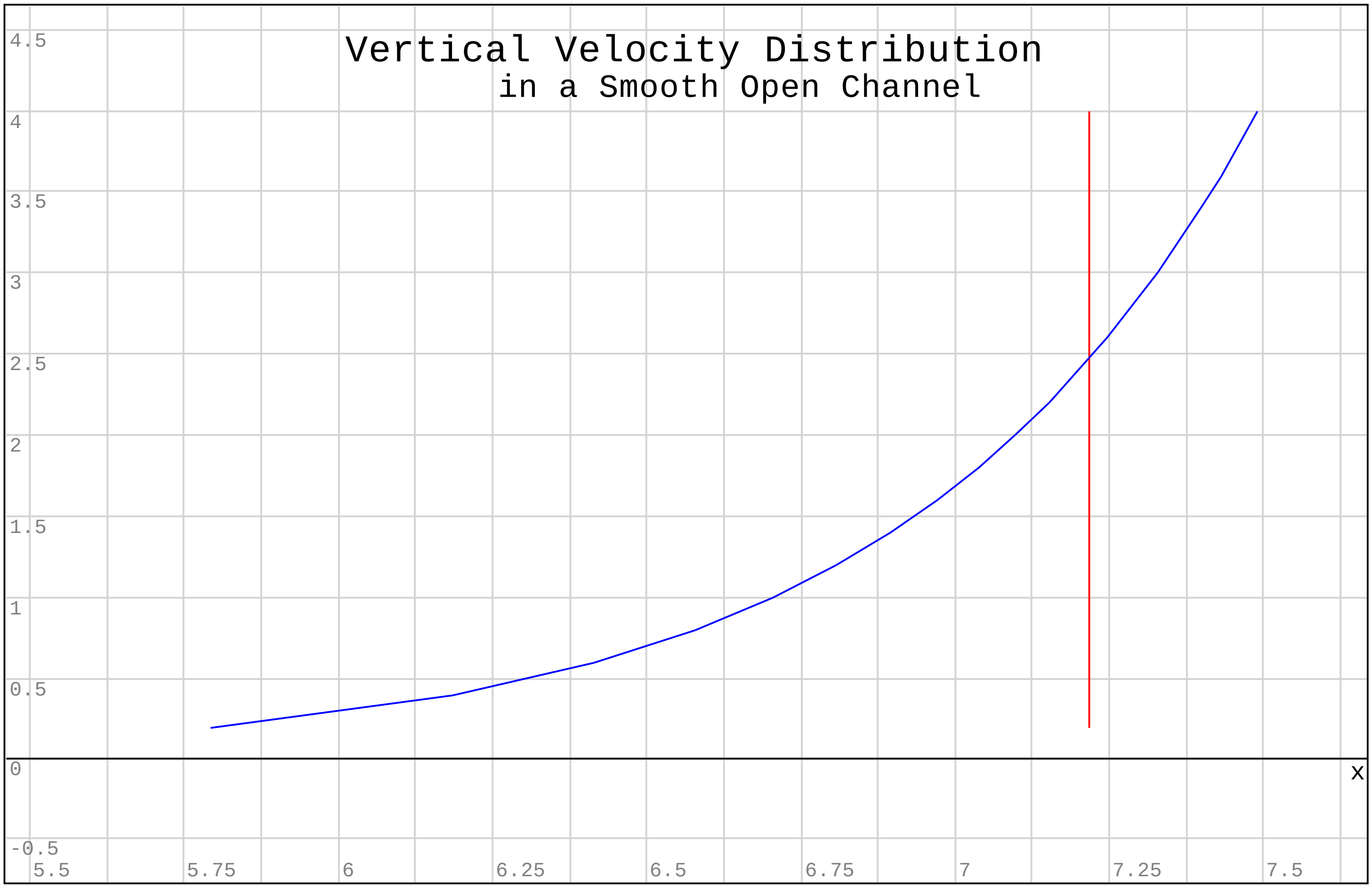
for $i \in 1 .. n$
$x_i := V \frac{sec}{ft}$
for loop assigns AVG velocity for each element in k

$V_2 := \text{augment}(x, y)$ augment x and y vectors into matrix for plotting

LABEL :=

6	4.5	"Vertical Velocity Distribution"	15	"black"
6.25	4.25	"in a Smooth Open Channel"	13	"black"

labels for plot



$$\begin{cases} V_1 \\ V_2 \\ \text{LABEL} \end{cases}$$