

# Nonlinear system of equations solving with Broyden's method

$$f(x) := \begin{pmatrix} x_1^2 - 4 \\ 2 \cdot x_1 + 4 \cdot x_2 \end{pmatrix} \quad \text{Equations to be solved, } f(x) \equiv 0$$

$$x := \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \text{Initial estimate:}$$

## Broyden's method

$$\varepsilon := 10^{-6} \quad \text{Accuracy:}$$

$$\text{maxiter} := 100 \quad \text{Max. number of iterations}$$

$$\text{next} := 1 \quad \text{iter} := 1$$

$$n := \text{length}(x) \quad A := \text{identity}(n)$$

while next

$$s := \text{eval}(-A \cdot f(x))$$

$$\text{if } \left( \frac{|\min(s)|}{|\min(x) + \varepsilon|} < \varepsilon \right) \wedge \left( \frac{|\max(s)|}{|\min(x) + \varepsilon|} < \varepsilon \right)$$

$$\text{next} := 0$$

else

$$x := s + x$$

$$u := \text{eval}(A \cdot f(x))$$

$$v := \frac{s}{s^2}$$

$$\sigma := \text{eval}(1 + v \cdot u)$$

$$A := \text{eval} \left( \left( \text{identity}(n) - \frac{u \cdot v^T}{\sigma} \right) \cdot A \right)$$

$$\text{iter} := \text{iter} + 1$$

$$\text{if } \text{iter} > \text{maxiter}$$

$$\text{next} := 0$$

else

$$\text{next} := 1$$

$$\text{iter} = 11 \quad \text{maxiter} = 100$$

Check for convergence  
should be:  $\text{iter} < \text{maxiter}$

Result:

$$x = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

Control:

$$f(x) = \begin{pmatrix} 1.3715 \cdot 10^{-8} \\ -3.3619 \cdot 10^{-9} \end{pmatrix}$$