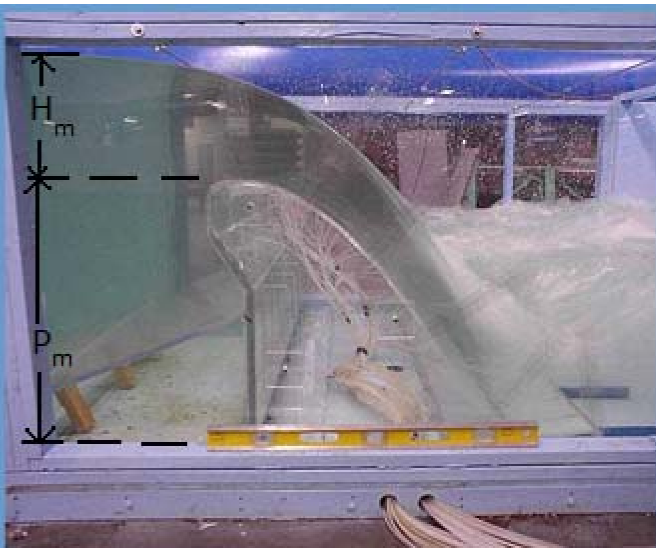
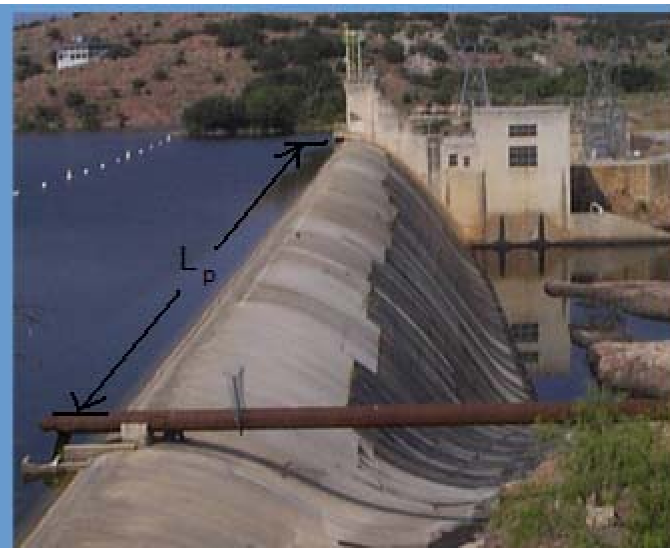
[1]. The photograph to the left shows the model of the Inks Dam spillway conducted at the Utah Water Research Laboratory, while the photograph to the right shows the actual Inks Dam spillway (the prototype). The model was build to measure the pressure distribution on the spillway's face. Manometric taps along the model spillway face can be seen attached to a myriad of plastic tubes in the photograph to the left.



Inks Dam Model.



Inks Dam Prototype.

The model spillway height is Pm = 1.33 ft and its length is Lm = 6 ft (not shown). For the test shown above, the model spillway head (water elevation above the spillway crest measured in the reservoir upstream) was Hm = 0.5 ft. The prototype length is Lp = 120 ft. (a) If the maximum pressure measured on the model spillway face was pm = 0.17 psi, what would be the corresponding maximum pressure on the prototype spillway face, pp = ?

Spillway discharge in the model Qm can be calculated using the equation,

$$Qm = Cwm \cdot Lm \cdot Hm^{\frac{3}{2}}$$

where (Cw)m is known as the model weir coefficient. The calibration performed at the laboratory indicates that the weir coefficient for the model spillway is Cwm = 2.3 cfs/ft^(5/2) , where cfs = cubic feet per second (ft^3/s). (b) If a flow equivalent to the one tested in th ewere to occur on prototype, what would be the equivalent discharge in the prototype, Qp = ?

Solution:

$Pm := 1.33\ ft$ $\quad Lm := 6\ ft$ $\quad Lp := 120\ ft$ $\quad Hm := 0.5\ ft$

$pm := 0.17\ psi$ $\quad Cwm := 2.3\ \frac{cfs}{ft^{\frac{5}{2}}}$

The length ratio is: $Lr := \frac{Lp}{Lm}$ i.e., $Lr = 20$

From page 242 - Finnemore and Franzini - using Froude similarity we find the pressure ratio to be:

$$pr = Lr \cdot \rho r \cdot gr$$

However, since we are using water in both model and prototype, both $\rho r = 1$ and $gr = 1$, therefore, $pr = Lr = 20$, and the maximum pressure in the prototype will be:

$pr := Lr$ or, $pr = 20$, and $pp := pr \cdot pm$, or, $pp = 3.4$ psi

The discharge in the model is:

$Qm := Cwm \cdot Lm \cdot Hm^{\frac{3}{2}}$, or, $Qm = 4.879$ cfs

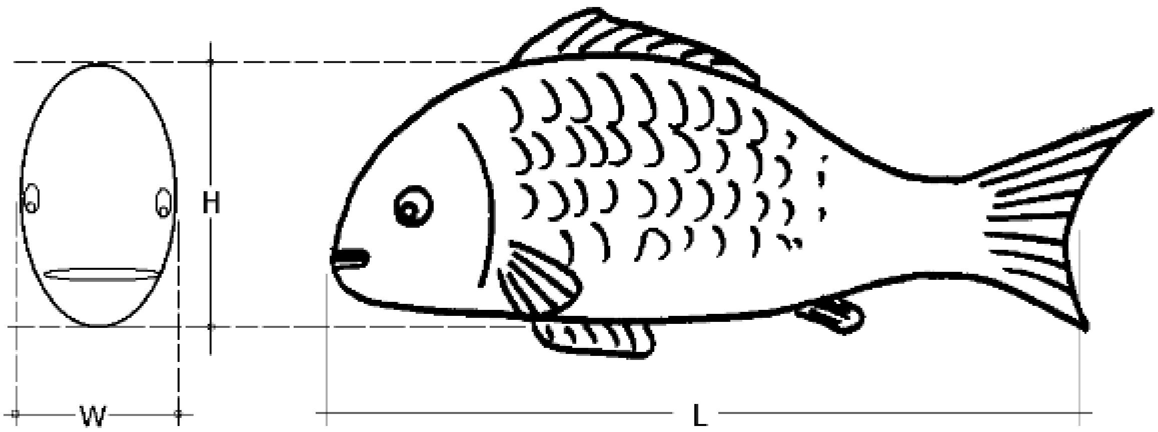
From page 242 - Finnemore and Franzini - using Froude similarity, we find the discharge ratio to be:

$Qr = Lr^{\frac{5}{2}} \cdot gr^{\frac{1}{2}} = Lr^{\frac{5}{2}}$, since $gr = 1$, thus $Qr := Lr^{\frac{5}{2}}$, or, $Qr = 1788.8544$

Consequently, the discharge in the prototype would be:

$Qp := Qr \cdot Qm$, i.e., $Qp = 8727.8863$ cfs

[2]. For the purpose of designing a fish passageway near the turbine intake in a dam you are asked to design an experimental study in which you are to measure the drag force FD (N) on a fish of length L (m) moving through water at a velocity V (m/s). The fish frontal cross-section is assumed to be an ellipse of width W (m) and heighth H (m) as shown below.



The study will be performed on a plastic replica of the fish submerged in a water tunnel, therefore, the density, ρ (kg/m3), and dynamic viscosity, μ (N.s/m^2), of water are important parameters to consider. To account for the fish own propulsion, the flapping frequency of the fish tail f (Hz) will also be taken into account. Thus, the relationship sought is of the form

$$\text{function}(FD, L, W, H, V, f, \rho, \mu) = 0.$$

Using H (geometric variable), V (kinematic variable), and ρ (dynamic variable) as the repeating variables in the application of Buckinham's P theorem, determine the corresponding dimensionless parameters for this experimental study.

Solution: Here is a listing of the dimensions that describe the variables involved:

$H = L \quad V = L \cdot T^{-1} \quad \rho = M \cdot L^{-3} \quad L = L$

$W = L \quad f = T^{-1} \quad \mu = M \cdot L^{-1} \cdot T^{-1} \quad FD = M \cdot L \cdot T^{-2}$

We have $n := 8$ variables, with $k := 3$ dimensions (L,M,T).

We need $n - k = 5$ dimensionless Π terms.

Select as repeating variables: H(geometric), V(kinematic), ρ(dynamic)

Form the following five Π terms:

$\Pi 1 = H^{x1} \cdot V^{y1} \cdot \rho^{z1} \cdot L \qquad \Pi 2 = H^{x2} \cdot V^{y2} \cdot \rho^{z2} \cdot W \qquad \Pi 3 = H^{x3} \cdot V^{y3} \cdot \rho^{z3} \cdot f$

$\Pi 4 = H^{x4} \cdot V^{y4} \cdot \rho^{z4} \cdot \mu \qquad \Pi 5 = H^{x5} \cdot V^{y5} \cdot \rho^{z5} \cdot FD$

Build the table of dimensions for the variables:

	REPEATING VARIABLES			NON-REPEATING VARIABLES (change sign)				
dimensions	H	V	ρ	L	W	f	μ	F_D
M or F	0	0	1	0	0	0	−1	−1
L	1	1	-3	−1	−1	0	1	−1
T	0	-1	0	0	0	1	1	2

Create the corresponding matrices:

$$A := \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & -3 \\ 0 & -1 & 0 \end{pmatrix} \qquad B := \begin{pmatrix} 0 & 0 & 0 & -1 & -1 \\ -1 & -1 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 & 2 \end{pmatrix} \qquad X = \begin{pmatrix} x1 & x2 & x3 & x4 & x5 \\ y1 & y2 & y3 & y4 & y5 \\ z1 & z2 & z3 & z4 & z5 \end{pmatrix}$$

and solve the matrix equation: $A \cdot X = B$ for X:

$$X := A^{-1} \cdot B \qquad \text{, i.e.,} \qquad X = \begin{pmatrix} -1 & -1 & 1 & -1 & -2 \\ 0 & 0 & -1 & -1 & -2 \\ 0 & 0 & 0 & -1 & -1 \end{pmatrix}$$

Thus,

$$\Pi 1 = H^{x1} \cdot V^{y1} \cdot \rho^{z1} \cdot L = H^{-1} \cdot V^{0} \cdot \rho^{0} \cdot L = \frac{L}{H}$$

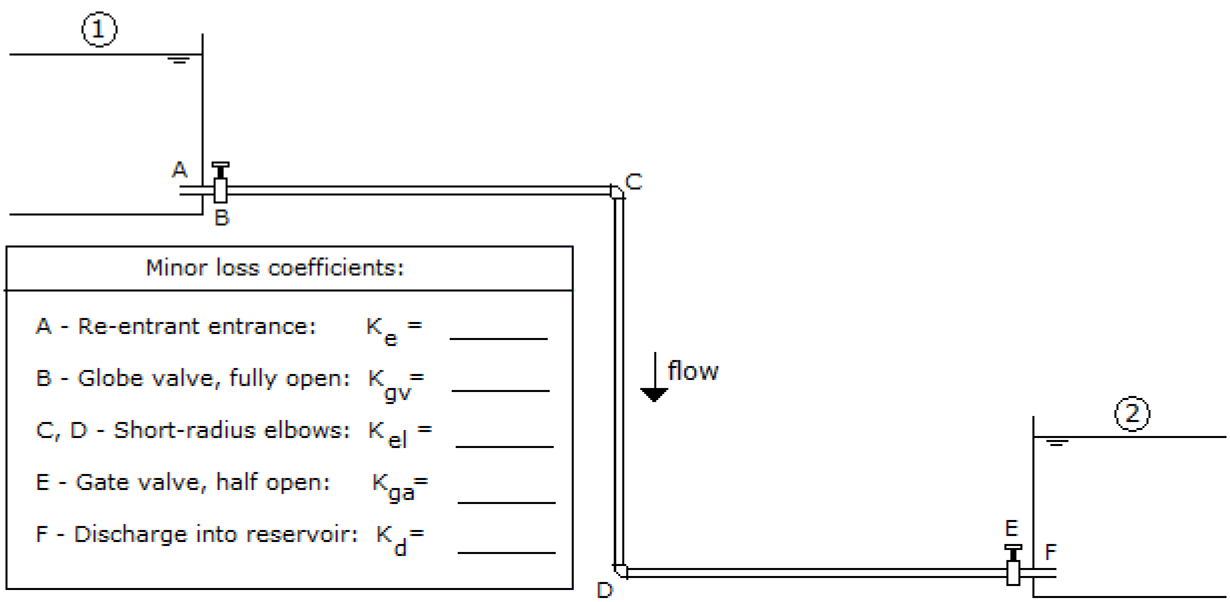
$$\Pi 2 = H^{x2} \cdot V^{y2} \cdot \rho^{z2} \cdot W = H^{-1} \cdot V^{0} \cdot \rho^{0} \cdot W = \frac{W}{H}$$

$$\Pi 3 = H^{x3} \cdot V^{y3} \cdot \rho^{z3} \cdot f = H^{1} \cdot V^{-1} \cdot \rho^{0} \cdot f = \frac{H \cdot f}{V}$$

$$\Pi 4 = H^{x4} \cdot V^{y4} \cdot \rho^{z4} \cdot \mu = H^{-1} \cdot V^{-1} \cdot \rho^{-1} \cdot \mu = \frac{\mu}{\rho \cdot V \cdot H} = \frac{1}{Re}$$

$$\Pi 5 = H^{x5} \cdot V^{y5} \cdot \rho^{z5} \cdot FD = H^{-2} \cdot V^{-2} \cdot \rho^{-1} \cdot FD = \frac{FD}{\rho \cdot V^{2} \cdot H^{2}}$$

3]. The sketch below shows a pipeline system connecting two 60oF-water reservoirs for a small farming operation. The pipe lengths AC, CD, DF are 25 ft, 6 ft, and 30 ft, respectively. The pipeline will be fitted with two short-radius elbows, and the flow will be controlled by a globe valve, fully open, near reservoir (1), and a gate valve, half open, near reservoir (2). The water surface elevation of reservoir (2), measured with respect to mean sea level is z2 = 4567.8 ft. A 6-inch-diameter commercial steel (absolute roughness, e = 150x10^(-6) ft) pipeline is selected. If the flow velocity is to be kept at 6.0 fps (feet per second = ft/s) determine the water surface elevation of reservoir (1).
(b) After selecting the pipe diameter, determine the actual discharge in the pipeline for that diameter.
(c) What is the actual flow velocity in the pipe?



For water at 60oF, $\nu := 12.17 \cdot 10^{-6} \frac{ft^2}{s}$

$L := 25 + 6 + 30$, i.e., $L = 61$ ft $\qquad z1 := 4567.8$ ft

$ee := 150 \cdot 10^{-6}$ ft $\quad V := 6$ fps $\quad g := 32.2 \frac{ft}{s^2}$ $\qquad D := \frac{6}{12}$ ft , i.e., $D = 0.5$

Energy terms: $z1 = ?$ $\qquad z2 := 4567.8$ ft $\qquad$ Pump head: hP = 0

$p1 = 0 \qquad p2 = 0 \qquad$ Turbine head: hT = 0

$V1 = 0 \qquad V2 = 0$

Energy equation (1)-(2): $z1 + \frac{p1}{\gamma} + \frac{V1^2}{2 \cdot g} - hf - \Sigma hm + hP - hT = z2 + \frac{p2}{\gamma} + \frac{V2^2}{2 \cdot g}$

Replacing energy terms: $z1 + 0 + 0 - hf - \Sigma hm + 0 - 0 = z2 + 0 + 0$

or, $z1 - z2 = \Delta z = hf + \Sigma hm$,i.e., $\Delta z = f \cdot \frac{L}{D} \cdot \frac{V^2}{2 \cdot g} + \Sigma K \cdot \frac{V^2}{2 \cdot g}$

which results in: $\Delta z = \frac{V^2}{2 \cdot g} \cdot \left(f \cdot \frac{L}{D} + \Sigma K \right)$, with $\Delta z = z1 - z2$

Minor losses coefficients: $Ke := 0.8$ $Kgv := 10$ $Kel := 0.9$ $Kga := 2.06$ $Kd := 1.0$

Total sum of minor loss coefficients: $\Sigma K := Ke + Kgv + 2 \cdot Kel + Kga + Kd$

i.e., $\Sigma K = 15.66$

To calculate the friction factor we'll use the Swamee-Jain equation, defined here as a function fSJ(eD,R), where eD = relative roughness, and R = Reynolds number:

$$fSJ(eD, R) := \frac{0.25}{\log_{10}\left(\frac{eD}{3.7} + \frac{5.74}{R^{0.9}} \right)^2}$$

The relative roughness is: $eD := \frac{ee}{D}$, i.e., $eD = 3 \cdot 10^{-4}$

The Reynolds number is: $R := \frac{V \cdot D}{\nu}$, i.e., $R = 2.4651 \cdot 10^{5}$

With these values of eD and R, the friction factor, using the Swamee-Jain equation is:

$f := fSJ(eD, R)$, or, $f = 0.0174$

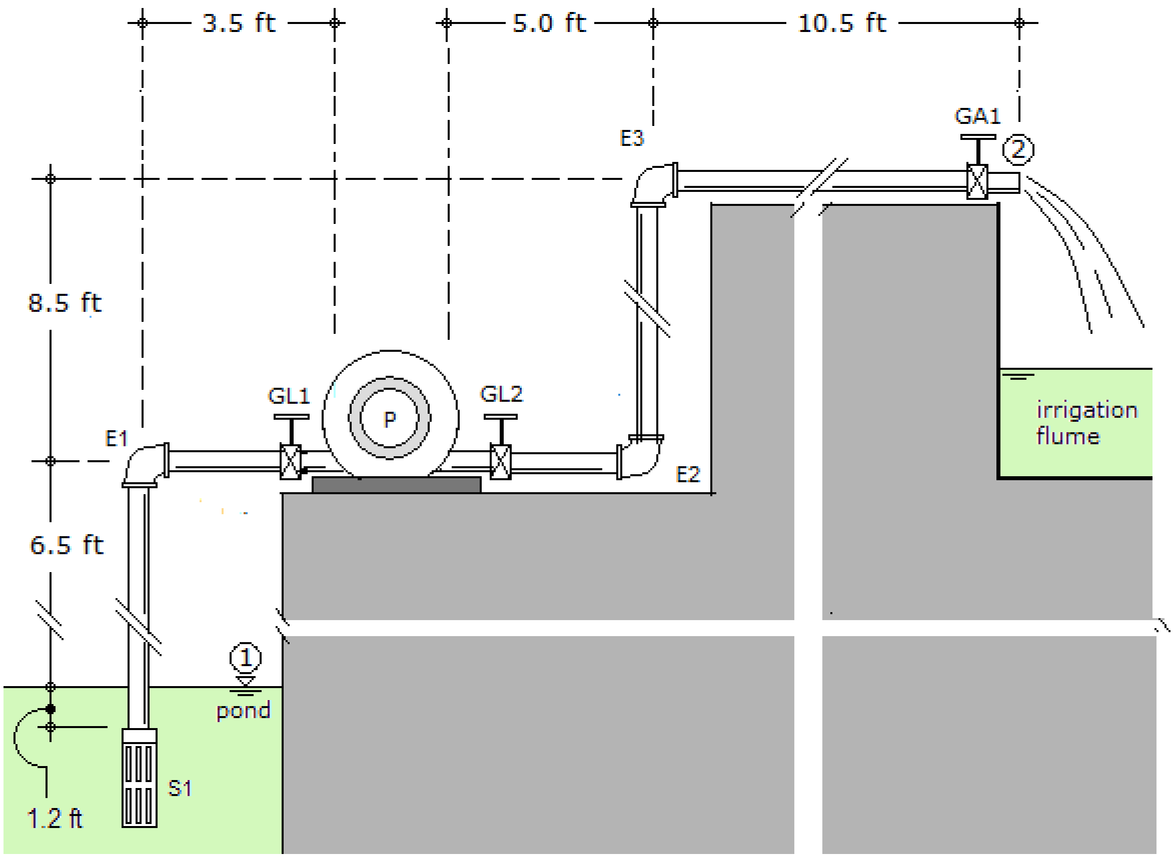
With this value of f, and the other variables defined above, the difference in reservoir elevations is:

$\Delta z := \frac{V^2}{2 \cdot g} \cdot \left(f \cdot \frac{L}{D} + \Sigma K \right)$, i.e., $\Delta z = 9.9401$ ft

From which, $z1 := z2 + \Delta z$, or, $z1 = 4577.7401$

The discharge in the pipe is: $Q := \frac{\pi \cdot D^2}{4} \cdot V$, i.e., $Q = 1.1781$ cfs

[4]. The figure below shows a pump P lifting water from a pond (1) and delivering it to an irrigation flume at point (2). The suction pipeline is provided by a trash screen, S1, with a minor loss coefficient KS1 = 0.6, and one short-radius elbow, E1. As shown in the figure, the discharge pipeline is fitted with two short-radius elbows, E2 and E3. The system is provided with three, fully-open, globe valves, GL1, GL2, and GL3. The pipeline is made of wrought iron (e = 0.00015 ft) and will carry water at 50oF. The pipeline diameter is 6.0 inches.



The figure in next page shows the pump curve for pump P. Out of that curve extract the required values of hP(ft) for the discharges Q(gpm) shown in the curve and fill the table to the right. The data in this table will be used to obtain the pump curve equation, hP =a+bQ+cQ^2, with Q in (cfs). (B) Use Excel to obtain the coefficients a, b, and c, for the pump curve. For details see the links pumps and pump data fitting in the class schedule entry corresponding to the date 11/04/09. The class schedule is available in the link shown below. Determine: (C) the operating point conditions, i.e., the pump head, hP , and the discharge, Q for this pump-pipeline system by solving simultaneously the system equation and the pump curve equation from (A) and (B).

Class schedule link:
http://www.neng.usu.edu/cee/faculty/gurro/Classes/Classes_Fall2009/CEE3500/CEE3500_Schedule.htm

SOLUTION:

The system equation for this pipeline is obtained by writing the energy equation between points (1) and (2). The energy terms are:

Point (1): Point (2):

$z1 = ?$ $z2 = z1 + \Delta z$, where Δz = difference in elevation (1)-(2)

$V1 = 0$ $V2 = V = $ pipe velocity

$p1 = 0$ $p2 = 0$ both points at atmospheric pressure

The energy equation (1)-(2), with a pump, is written as:

$$z1 + \frac{p1}{\gamma} + \frac{V1^2}{2 \cdot g} - hf - \Sigma hm + hP = z2 + \frac{p2}{\gamma} + \frac{V2^2}{2 \cdot g}$$

where hP = pump head. Replacing known values, we have:

$$z1 + \frac{0}{\gamma} + 0 - hf - \Sigma hm + hP = z1 + \Delta z + \frac{0}{\gamma} + \frac{V^2}{2 \cdot g}$$

from which, we isolate hP as (this is the system equation):

$$hP = \Delta z + hf + \Sigma hm + \frac{V^2}{2 \cdot g}$$

Friction losses are calculated using the Darcy-Weisbach equation:

$$hf = f \cdot \frac{L}{D} \cdot \frac{V^2}{2 \cdot g}$$

while minor losses are calculated using: $\Sigma hm = \Sigma K \cdot \frac{V^2}{2 \cdot g}$

With friction and minor losses in terms of the velocity head, the system equation becomes:

$$hP = \Delta z + \frac{V^2}{2 \cdot g} \cdot \left(f \cdot \frac{L}{D} + \Sigma K + 1 \right)$$

Since we typically seek a solution involving the pump head, hP and the flow discharge, Q, at the operating point, it is convenient to replace V in terms of Q, using the continuity equation, namely:

$V = \frac{4 \cdot Q}{\pi \cdot D^2}$, from which the velocity head is: $\frac{V^2}{2 \cdot g} = \frac{8 \cdot Q^2}{\pi^2 \cdot g \cdot D^4}$

The resulting system equation is now: $hP = \Delta z + \frac{8 \cdot Q^2}{\pi^2 \cdot g \cdot D^4} \cdot \left(f \cdot \frac{L}{D} + \Sigma K + 1 \right)$

From the figure: $\Delta z := 6.5 + 8.5$, i.e., $\Delta z = 15$ ft , also

$L := 1.2 + 6.5 + 3.5 + 5.0 + 8.5 + 10.5$,i.e., $L = 35.2$ ft

D is given, $D := \frac{6}{12}$ ft ,i.e., $D = 0.5$ ft , and $g := 32.2 \frac{ft}{s^2}$

Kinematic viscosity for water at 50oF: $\nu := 1.41 \cdot 10^{-5} \frac{ft^2}{s}$

Pipe roughness: $ee := 0.00015$ ft

The minor loss coefficients involved are:

* Trash screen: $KS1 := 0.6$

* Each short-radius elbow: $KEL := 0.9$

* Each, fully-open, globe valve: $KGA := 10.0$

Sum of minor loss coeff.: $\Sigma K := KS1 + 3 \cdot KEL + 3 \cdot KGA$, or, $\Sigma K = 33.3$

To calculate the friction factor we'll use the Swamee-Jain equation, defined here as a function fSJ(eD,R), where eD = relative roughness, and R = Reynolds number:

$$fSJ(eD, R) := \frac{0.25}{\left(\log_{10}\left(\frac{eD}{3.7} + \frac{5.74}{R^{0.9}}\right)\right)^2}$$

Using $eD = \frac{ee}{D}$ and $R = \frac{4 \cdot Q}{\pi \cdot \nu \cdot D}$, f becomes: $f = fSJ\left(\frac{ee}{D}, \frac{4 \cdot Q}{\pi \cdot \nu \cdot D}\right)$

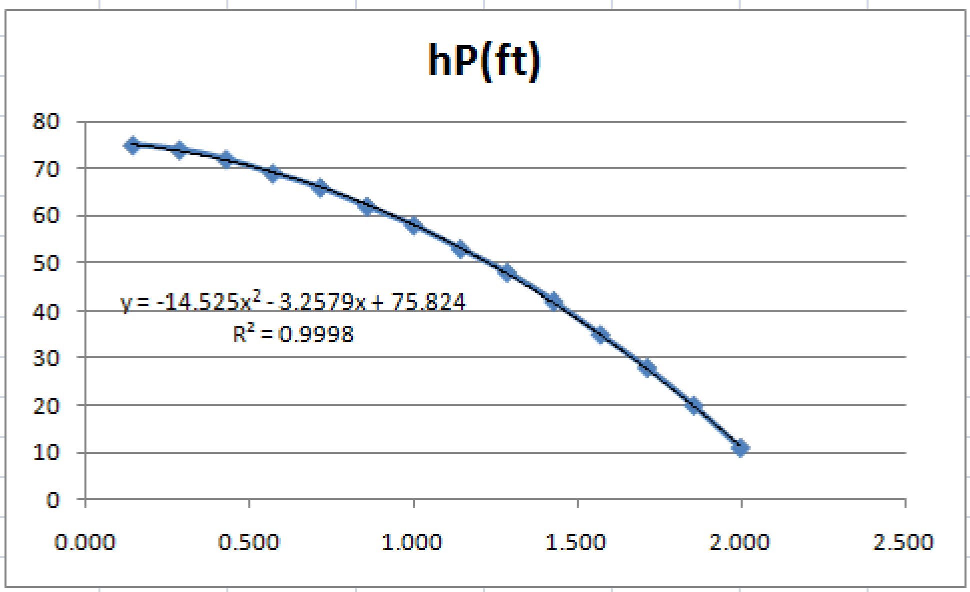
the system eq. becomes: $hP = \Delta z + \frac{8 \cdot Q^2}{\pi^2 \cdot g \cdot D^4} \cdot \left(fSJ\left(\frac{ee}{D}, \frac{4 \cdot Q}{\pi \cdot \nu \cdot D}\right) \cdot \frac{L}{D} + \Sigma K + 1\right)$

Function hPS(x), x=Q(cfs), is the system equation:

$$hPS(x) := \Delta z + \frac{8 \cdot x^2}{\pi^2 \cdot g \cdot D^4} \cdot \left(fSJ\left(\frac{ee}{D}, \frac{4 \cdot x}{\pi \cdot \nu \cdot D}\right) \cdot \frac{L}{D} + \Sigma K + 1\right)$$

Using EXCEL we find the following quadratic fitting for the pump data:

Q(gpm)	Q(cfs)	hP(ft)
64	0.143	75
128	0.285	74
192	0.428	72
256	0.570	69
320	0.713	66
384	0.856	62
448	0.998	58
512	1.141	53
576	1.283	48
640	1.426	42
704	1.569	35
768	1.711	28
832	1.854	20
896	1.996	11



For the pump equation we use: $a := 75.824$ $b := -3.2579$ $c := -14.525$

Function hPP(x), is the pump equation: $hPP(x) := a + b \cdot x + c \cdot x^2$

$Mxy := augment(xx, yy)$ $Mxz := augment(xx, zz)$

Generating data for a graphical solution:

$hPP(x) := a + b \cdot x + c \cdot x^2$ <-- This is the pump curve I came up with when setting up the problem.

$xx := 0.1, 0.3 .. 2.1$ $yy := matrix(length(xx), 1)$

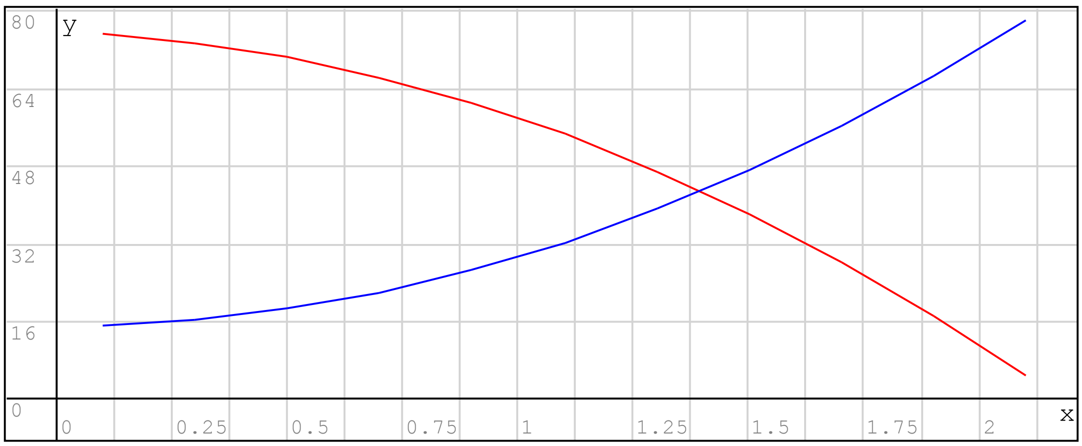
```
for k ∈ 1 .. length(yy)
  | yy_k := hPS(xx_k)
  | zz_k := hPP(xx_k)
```

<-- yy are values of hP for the system curve, while zz are values of hP for the pump curve.

```
Mxy := augment(xx, yy)
```
<-- Data for the system curve (blue)

```
Mxz := augment(xx, zz)
```
<-- Data for the pump curve (red)

In the graph below, the x-axis is Q(cfs), and the y-axis is hP(ft). The operating point is the intersection point of the two curves. From the graph, I estimate Q = 1.40 cfs, hP = 40 ft.



Mxy
Mxz

Solution of the combined system and curve equations:

$$\text{solve}\left(a + b\cdot Q + c\cdot Q^2 = \Delta z + \frac{8\cdot Q^2}{\pi^2\cdot g\cdot D^4}\cdot\left(\text{fSJ}\left(\frac{ee}{D}, \frac{4\cdot Q}{\pi\cdot\nu\cdot D}\right)\cdot\frac{L}{D} + \Sigma K + 1\right), Q\right) = 1.397$$

Thus, $Q := 1.397$ cfs and $hP := a + b\cdot Q + c\cdot Q^2$, i.e., $hP = 42.9256$ ft

This part corresponds to the generation of the pump curve for the problem:

$hPP(x) := 75 - 2\cdot x - 15\cdot x^2$ <-- This is the pump curve I came up with when setting up the problem.

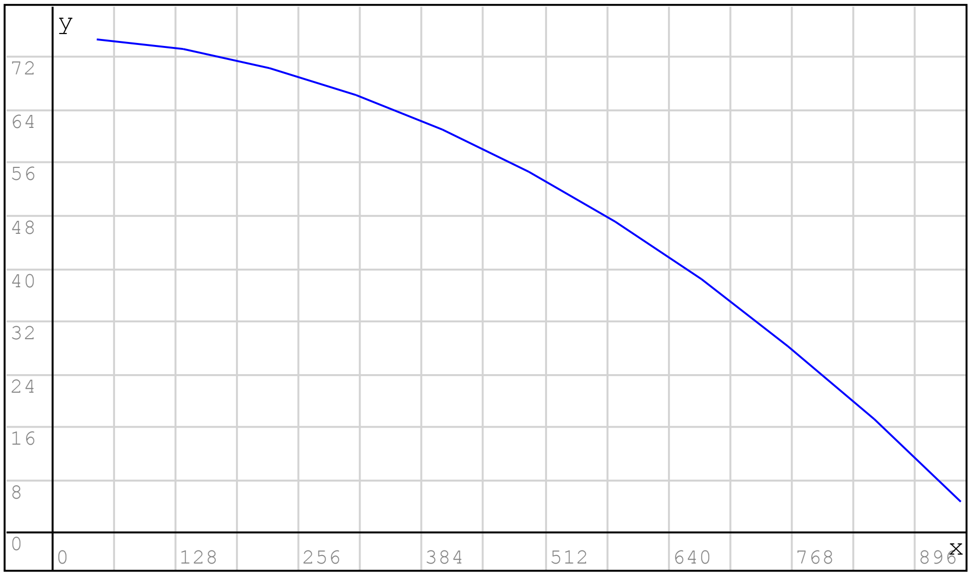
```
xx := 0.1 , 0.3 .. 2.1     yy := matrix(length(xx), 1)     zz := yy     qq := yy
```

```
for k ∈ 1 .. length(yy)
  | qq_k := 448.83·xx_k
  | zz_k := hPP(xx_k)
```

Note: 1 cfs = 448.83 cfs

This is the pump curve with x = qq(gpm) and y = hP(ft):

```
Mqz := augment(qq, zz)
```



Mqz