## Fitting function to data
## -many independent variables,
## -linear on parameters

//Independent //Dependent

$$Re := \begin{pmatrix} 49000 \\ 68600 \\ 84800 \\ 34200 \\ 22900 \\ 1321 \\ 931 \\ 518 \\ 346 \\ 122.9 \\ 54 \\ 84.6 \\ 1249 \\ 1021 \\ 465 \\ 54.8 \end{pmatrix} \quad Pr := \begin{pmatrix} 2.3 \\ 2.28 \\ 2.27 \\ 2.32 \\ 2.36 \\ 246 \\ 247 \\ 251 \\ 273 \\ 1518 \\ 1590 \\ 1521 \\ 107.4 \\ 186 \\ 414 \\ 1302 \end{pmatrix} \quad Nu := \begin{pmatrix} 277 \\ 348 \\ 421 \\ 223 \\ 177 \\ 114.8 \\ 95.9 \\ 68.3 \\ 49.1 \\ 56 \\ 39.9 \\ 47 \\ 94.2 \\ 99.9 \\ 83.1 \\ 35.9 \end{pmatrix}$$

//The expression we are interested in
//Pr,Re-independent variables, b-vector of unknown parameters

$$\ln(Nu) = b_1 + b_2 \cdot \ln(Re) + b_3 \cdot \ln(Pr)$$

//Transforming original dependent variable

for $i \in 1 .. \text{length}(Nu)$

$y_i := \ln(Nu_i)$

//Matrix of independent variables

$xx := \text{augment}(Re, Pr)$

//from the function above we construct a vector function
//with vector argument

$$\varphi(x) := \begin{pmatrix} 1 \\ \ln(x_1) \\ \ln(x_2) \end{pmatrix}$$

$n := \text{length}(Nu)$ $k := \text{length}(\varphi(x))$

$n = 16$ $k = 3$

//X-matrix, matrix of experiment

for $j \in 1 .. k$

for $i \in 1 .. n$

$$X_{i,j} := \varphi\left(\begin{pmatrix} xx_{i,1} \\ xx_{i,2} \end{pmatrix}\right)_j$$

$\Phi := X^{T} \cdot X \qquad d := X^{T} \cdot y$

$b := \mathrm{eval}\left(\Phi^{-1}\right) \cdot d$

$$b = \begin{pmatrix} -0.412 \\ 0.5395 \\ 0.2454 \end{pmatrix}$$

//Unknown parameters
//found in the least-square sense

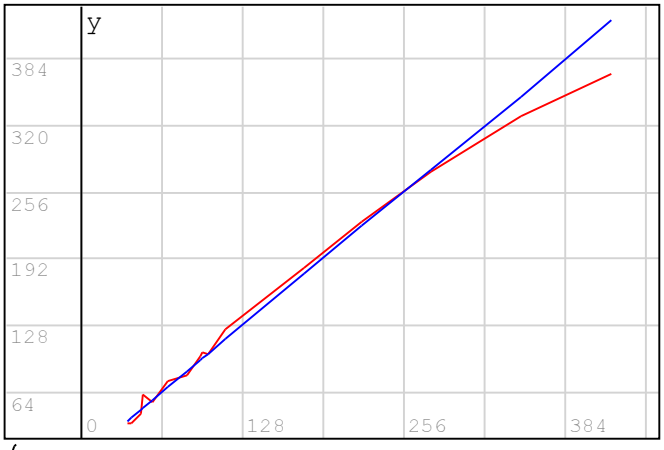
//Calculated Nu-values

$\mathrm{Nucalc}(x) := \mathrm{eval}\left(e^{b \cdot \varphi(x)}\right)$

for $i \in 1 .. \mathrm{length}(Nu)$

$\mathrm{Nuc}_i := \mathrm{Nucalc}\begin{pmatrix} xx_{i\,1} \\ xx_{i\,2} \end{pmatrix}$

| Calculated | Given | Plotting |
|---|---|---|
| $\mathrm{Nuc} =$ | $\mathrm{Nu} =$ | $\mathrm{NuNu} := \mathrm{augment}(Nu, Nu)$ |
| 275.6408 | 277 | $\mathrm{NuNu} := \mathrm{eval}(\mathrm{csort}(\mathrm{NuNu}, 1))$ |
| 329.8029 | 348 | |
| 369.3706 | 421 | $\mathrm{NuNuc} := \mathrm{augment}(Nu, Nuc)$ |
| 227.513 | 223 | $\mathrm{NuNuc} := \mathrm{eval}(\mathrm{csort}(\mathrm{NuNuc}, 1))$ |
| 184.0116 | 177 | |
| 123.4589 | 114.8 | |
| 102.3223 | 95.9 | |
| 74.8695 | 68.3 | |
| 61.4754 | 49.1 | |
| 53.5773 | 56 | |
| 34.7711 | 39.9 | |
| 43.8216 | 47 | |
| 97.7416 | 94.2 | |
| 100.3158 | 99.9 | |
| 79.861 | 83.1 | |
| 33.3711 | 35.9 | |




Data

Fitted

NuNu
NuNuc