```
PV = Present Value     (pv#)
FV = Future Value      (fv#)
i  = Interest Rate     (rate#)
n  = Number of periods (nper#)
CF = Variable Cash Flow
m  = Compounding Period (freq#)
n  = Period of interest  (per#)
```

Calculate the number of periods n a PV takes to achieve a FV at a fixed i

$$n = \log_{(1+i)}\left(\frac{FV}{PV}\right)$$

$$cnper(rate\#, pv\#, fv\#) := \log_{(1+rate\#)}\left(\frac{fv\#}{pv\#}\right)$$

$$nper := cnper(0.05, 1000, 2000) \qquad nper = 14.2067$$

$$1000 \cdot (1 + 0.05)^{nper} = 2000$$

Calculate the i for a PV to achieve a FV over a period n

$$i = \sqrt[n]{\frac{FV}{PV}} - 1$$

$$crate(nper\#, pv\#, fv\#) := \sqrt[nper\#]{\frac{fv\#}{pv\#}} - 1$$

$$crate(14, 1000, 2000) = 0.0508$$

$$rate := \sqrt[14]{\frac{2000}{1000}} - 1 \qquad rate = 0.0508$$

Calculate the payment to amortize a loan PV over a period of n at i

$$PMT = PV \frac{i \cdot (1+i)^n}{(1+i)^n - 1}$$

$$pmt(rate\#, nper\#, pv\#) := pv\# \cdot \frac{rate\# \cdot (1 + rate\#)^{nper\#}}{(1 + rate\#)^{nper\#} - 1}$$

$$pmt(0.05, 14, 2000) = 202.0479$$

$$pmt\# := 2000 \cdot \frac{0.05 \cdot (1 + 0.05)^{14}}{(1 + 0.05)^{14} - 1} \qquad pmt\# = 202.0479$$

Calculate FV for PV over a period n at i

$$FV = PV \cdot (1+i)^n$$

$fv(rate\#, nper\#, pv\#) := pv\# \cdot (1 + rate\#)^{nper\#}$

Calculate FV for PV over a period n at i where compounding is more frequent

$$FV = PV \cdot \left(1 + \frac{i}{m}\right)^{n \cdot m}$$

$fv(rate\#, nper\#, pv\#, freq\#) := pv\# \cdot \left(1 + \frac{rate\#}{freq\#}\right)^{(nper\# \cdot freq\#)}$

$fv(0.05, 14, 2000) = 3959.8632$

$fv1 := 2000 \cdot (1 + 0.05)^{14}$ $\qquad fv1 = 3959.8632$

$fv(0.05, 14, 2000, 12) = 4021.6525$

$fv2 := 2000 \cdot \left(1 + \frac{0.05}{12}\right)^{14 \cdot 12}$ $\qquad fv2 = 4021.6525$

Calculate the FV of a variable CF over a period n at i

$$FV = \sum_{j=1}^{n} CF_j \cdot (1 + i)^{j-1}$$

$$fvc(rate\#, pmt\#) := \sum_{j=1}^{rows(pmt\#)} \left[pmt\#_j \cdot (1 + rate\#)^{rows(pmt\#) - j} \right]$$

$$cf := \begin{pmatrix} 100 \\ 200 \\ 300 \\ 400 \\ 500 \\ 600 \\ 700 \\ 800 \\ 900 \\ 1000 \\ 1100 \\ 1200 \\ 1300 \\ 1400 \end{pmatrix}$$ Cash flow

$fvc(0.05, cf) = 13157.1272$

$fvc\# := \sum_{j=1}^{14} \left[cf_j \cdot (1 + 0.05)^{14 - j} \right]$ $\qquad fvc\# = 13157.1272$

Calculate the FV of an annuity with PMT over a period n at i Note this is similar to cash flow but for constant PMT

$$FV_a = PMT \cdot \frac{(1+i)^n - 1}{i}$$

$$\text{fva}(\text{rate\#}, \text{nper\#}, \text{pmt\#}) := \text{pmt\#} \cdot \frac{(1+\text{rate\#})^{\text{nper\#}} - 1}{\text{rate\#}}$$

$$\text{fva}(0.05, 14, 100) = 1959.8632$$

$$\text{fva1} := \sum_{j=1}^{14} \left(100 \cdot (1+0.05)^{j-1}\right) \qquad \text{fva1} = 1959.8632$$

$$\text{fva2} := 100 \cdot \frac{(1+0.05)^{14} - 1}{0.05} \qquad \text{fva2} = 1959.8632$$

Calculate PV of FV over a period n at i

$$PV = \frac{FV}{(1+i)^n}$$

$$\text{pv}(\text{rate\#}, \text{nper\#}, \text{fv\#}) := \frac{\text{fv\#}}{(1+\text{rate\#})^{\text{nper\#}}}$$

Calculate PV for FV over a period n at i where compounding is more frequent

$$PV = \frac{FV}{\left(1+\frac{i}{m}\right)^{n \cdot m}}$$

$$\text{pv}(\text{rate\#}, \text{nper\#}, \text{fv\#}, \text{freq\#}) := \frac{\text{fv\#}}{\left(1+\frac{\text{rate\#}}{\text{freq\#}}\right)^{\text{nper\#} \cdot \text{freq\#}}}$$

$$\text{pv}(0.05, 14, 2000) = 1010.1359$$

$$\text{pv}(0.05, 14, 2000, 12) = 994.616$$

Calculate the remaining balance on a loan

$$Balance = PV \cdot (1+i)^n - PMT \frac{(1+i)^n - 1}{i}$$

$$\text{bal}(\text{rate\#}, \text{nper\#}, \text{pv\#}, \text{per\#}) := \text{fv}(\text{rate\#}, \text{per\#}, \text{pv\#}) - \text{fva}(\text{rate\#}, \text{per\#}, \text{pmt}(\text{rate\#}, \text{nper\#}, \text{pv\#}))$$

$$\text{bal}(0.05, 14, 2000, 5) = 1436.1207$$

Calculate cummulative interest paid on a loan up to period n

$$CumInterest = PV\left[(1+i)^n - 1\right] + PMT \frac{(1+i)^n + ni + 1}{i}$$

$cumint(rate\#, nper\#, pv\#, per\#) := per\# \cdot pmt(rate\#, nper\#, pv\#) - pv\# + bal(rate\#, nper\#, pv\#, per\#)$

$cumint(0.05, 14, 2000, 5) = 446.3604$

Calculate the principle paid on a load at period n

$$CumPrinciple = PV\left[1 - (1+i)^n\right] + PMT\frac{(1+i)^n - 1}{i}$$

$cumprn(rate\#, nper\#, pv\#, per\#) := pv\# - bal(rate\#, nper\#, pv\#, per\#)$

$cumprn(0.05, 14, 2000, 5) = 563.8793$

$$2000\cdot\left(1-(1+0.05)^5\right) + pmt\#\cdot\frac{(1+0.05)^5 - 1}{0.05} = 563.8793$$

Amount of principle paid on payment n

$$Principle_n = (PMT - i \cdot PV)\cdot(1+i)^{n-1}$$

$prnpmt(rate\#, nper\#, pv\#, per\#) := (pmt(rate\#, nper\#, pv\#) - rate\# \cdot pv\#)\cdot(1 + rate\#)^{per\# - 1}$

$prnpmt(0.05, 14, 2000, 5) = 124.0399$

$bal(0.05, 14, 2000, 4) - bal(0.05, 14, 2000, 5) = 124.0399$

Amount of interest paid on payment n

$intpmt(rate\#, nper\#, pv\#, per\#) := bal(rate\#, nper\#, pv\#, per\# - 1)\cdot rate\#$

$intpmt(0.05, 14, 2000, 5) = 78.008$