Solution to ENGR 2030 - DYNAMICS Test 01 - Spring Semester 2010 -
Utah State University - Instructor: Gilberto Urroz

The values of the problem parameters were randomly generated by Blackboard from a range of values provided by the instructor. This is one of the tests generated by Blackboard. The parameters, therefore, would most likely be different than the ones in your test, but the procedure is common for all tests thus generated. These solutions were calculated using SMath Studio, a free WYSIWYG math worksheet:

http://www.neng.usu.edu/cee/faculty/gurro/SMathStudio.html

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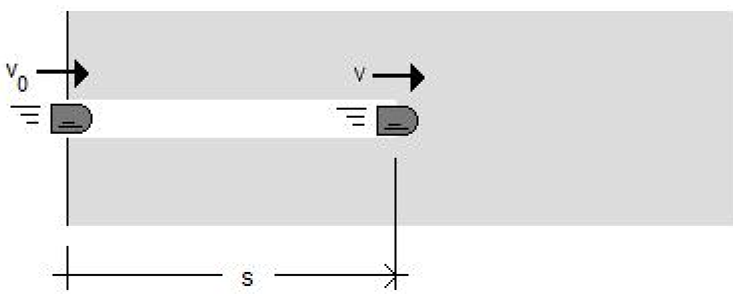
1. EX01-Q01-Rectilinear kinematics (Points: 10)

The acceleration, a, in ft/s^2, of a bullet fired into a porous material is given by the equation $a = -0.04\,s$, where s is the distance traveled by the bullet, in feet,. If the bullet enters the porous material with a speed v_0 = 0.44 ft/s, determine the bullet velocity, in ft/s, after it has traveled a distance of 0.34 ft into the porous material.



Solution: Start with the equation: $v \cdot dv = a \cdot ds$,since a is given as a function of s, and insert the value: $a = -0.04 \cdot s$,where s(ft) and a(ft/s^2). Combining these two equations you get the ordinary differential equation (ODE):

$$v \cdot dv = -0.04 \cdot s \cdot ds$$

The initial condition to use is: $v = v_0 = 0.44 \cdot \frac{ft}{s}$ at $s = 0$

Integrating the ODE, we get:

$$\int_{v_0}^{v} v\,dv = -0.04 \cdot \int_0^s s\,ds \quad \text{-->} \quad \left.\frac{v^2}{2}\right|_{v_0}^{v} = -0.04 \cdot \left.\frac{s^2}{2}\right|_0^s$$

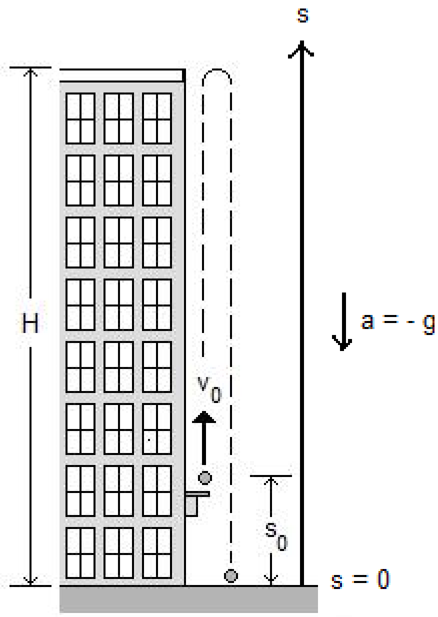
$$\frac{v^2}{2} - \frac{v_0^2}{2} = -0.04 \cdot \frac{s^2}{2} \quad \text{-->} \quad v^2 = v_0^2 - 0.04 \cdot s^2$$

$$v = \sqrt{v_0^2 - 0.04 \cdot s^2}$$ With $s := 0.34$ ft and $v0 := 0.44 \frac{ft}{s}$

$$v := \sqrt{v0^2 - 0.04 \cdot s^2} \quad \text{-->} \quad v = 0.4347 \frac{ft}{s}$$

2. EX01-Q02-Rectilinear kinematics (Points: 10)

A ball is launched vertically upward from a balcony located at a height of s_0 = 4.3 m above the ground in a building of height *H* = 19.0 m as illustrated in the figure. If the ball reaches the top of the building before it starts falling back to the ground, determine the total time of flight of the ball (in seconds) between the moment of launch and the time the ball hits the ground.



Solution:

The ball is a particle subject to a constant acceleration, g = 9.81 m/s^2. For the coordinate system shown, that constant acceleration is a = -g = -9.81 m/s^2. The equations for velocity and position are, therefore:

$$v = v0 - g \cdot t \quad \text{and} \quad s = s0 + v0 \cdot t - \frac{1}{2} \cdot g \cdot t^2$$

Since the ball reaches the top of the building and then starts falling downwards, its velocity at that point must be zero: v = 0 at s = H. The time required to reach s = H, can be calculated from:

$$v = 0 = v0 - g \cdot t \quad \text{or} \quad t = \frac{v0}{g}$$

Using that value of t for s = H in the equation for position, we can obtain the initial velocity, i.e., insert s = H and t = v0/g into:

$s = s0 + v0 \cdot t - \frac{1}{2} \cdot g \cdot t^2$ to get: $H = s0 + v0 \cdot \left(\frac{v0}{g}\right) - \frac{1}{2} \cdot g \cdot \left(\frac{v0}{g}\right)^2$,which results in:

$H = s0 + \frac{v0^2}{2 \cdot g}$. Thus, $v0 = \sqrt{2 \cdot g \cdot (H - s0)}$ With $s0 := 4.3$ m, $H := 19.0$ m, and

$g := 9.81 \frac{m}{s^2}$ the initial velocity is $v0 := \sqrt{2 \cdot g \cdot (H - s0)}$ or $v0 = 16.9828$ m/s

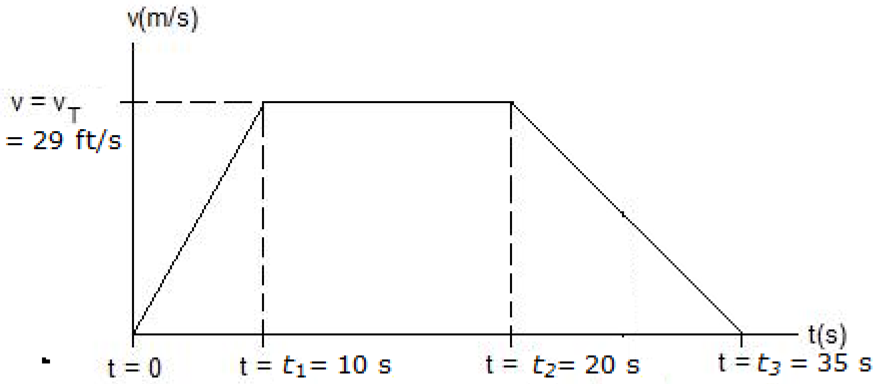
To find the total time of flight put s = 0 into $s = s0 + v0 \cdot t - \frac{1}{2} \cdot g \cdot t^2$ and solve for t, i.e.,

$$\text{solve}\left(s = s0 + v0 \cdot t - \frac{1}{2} \cdot g \cdot t^2, t\right) = \begin{pmatrix} -0.2193 \\ 3.6816 \end{pmatrix}$$

Taking the positive value (since flight started at t = 0), and rounding to 2 decimals, the solution is: t = 3.68 s

3. EX01-Q03-Erratic rectilinear motion (Points: 10)

The figure below shows the velocity-versus-time (v-vs-t) graph of a car moving in a straight line. In this graph v_T = 29 ft/s, t_1= 10 s, t_2= 20 s, and t_3 = 35 s. If the car starts at position s_0 = 4.13 ft at t = 0, what would be its position s at t = 35 s?



Solution:

From $v = \frac{ds}{dt}$ it follows that $ds = v \cdot dt$ --> $\int_{s0}^{s} 1\, ds = \int_{0}^{t} v\, dt$

--> $s - s0 = \int_{0}^{t} v\, dt$ --> $s = s0 + \int_{0}^{t} v\, dt = s0 + A$, where A = the area

under the v-t curve between 0 and time t. In this problem the area of interest is the area of the trapezoid formed by the v-t curve from t = 0 to t = t3 = 35 s. The trapezoid has bases b1 = 35 s (bottom) and b2 = 20 s - 10 s = 10 s (top). The height of the trapezoid is h=vT = 29 ft/s. The area of a trapezoid with bases b1 and b2 and height h is calculated as:

$A = \left(\frac{b1 + b2}{2}\right) \cdot h$. With $b1 := 35$ s , $b2 := 10$ s, and $h := 29$ ft/s, then

$A := \left(\frac{b1 + b2}{2}\right) \cdot h$, i.e., $A = 652.5$ ft . Also, $s0 := 4.13$ ft, thus

$s := s0 + A$ i,e., $s = 656.63$ ft

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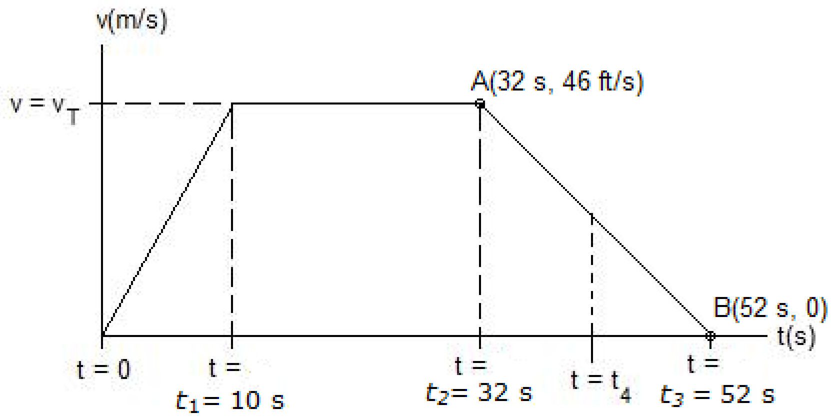
4. EX01-Q04-Erratic rectilinear motion (Points: 10)

The figure below shows the velocity-versus-time (v-vs-t) graph of a car moving in a straight line. In this graph v_T = 46 ft/s, t_1= 10 s, t_2= 32 s, and t_3 = 52 s. What is the acceleration *a*, in ft/s^2, of the car for any time $t = t_4$ in the interval 32 s < t_4 < 52 s?



Solution: From $a = \frac{dv}{dt}$ it follows that the acceleration, a, is the slope
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of the v-t curve at any point. Since at the point of interest, namely, t4, in the interval 32 s < t4 < 52 s, the v-t curve is a straight line, all we need to determine are the coordinates of the end points of segment AB, i.e., A(32 s, 46 ft/s) and B(52 s, 0), and calculate the slope of that straight line as:

$a = \frac{\Delta v}{\Delta t} = \frac{vB - vA}{tB - tA}$ or $a := \frac{0 - 46}{52 - 32}$ This results in: $a = -2.3 \frac{ft}{s^2}$

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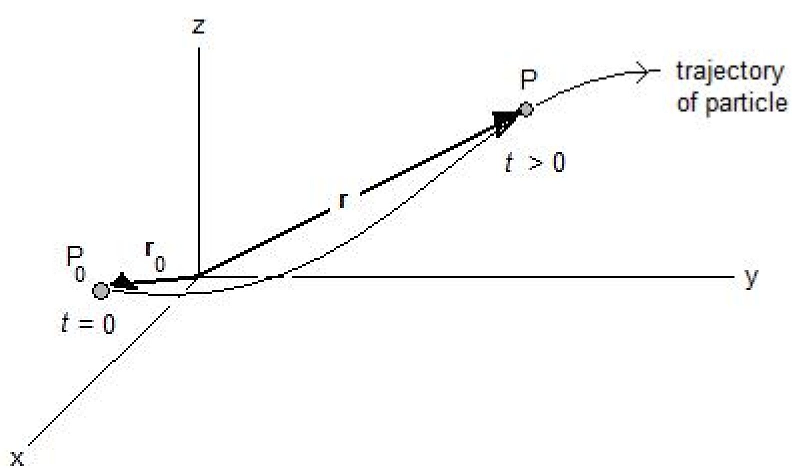
5. EX01-Q05-Curvilinear motion (Points: 10)

The velocity of a particle in curvilinear motion in Cartesian coordinates is given by the vector $\mathbf{v} = [(-5t^2)\mathbf{i} + (2t-3t^{1/2})\mathbf{j}+(1/(1+t^2))\mathbf{k}]$ m/s. If the particle starts its motion at point P_0(2 m,4 m,5 m) at t = 0, determine the magnitude of the position vector $r = |\mathbf{r}|$ at t = 4 s.



Solution: Vectors will be represented in here using column vectors. Thus, the
--------- velocity vector will be written as:

$$v = \begin{pmatrix} -5 \cdot t^2 \\ 2 \cdot t - 3 \cdot \sqrt{t} \\ \dfrac{1}{1+t^2} \end{pmatrix}$$,and the initial position as: $r0 = \begin{pmatrix} 2 \\ 4 \\ 5 \end{pmatrix}$ m

From the definition of the velocity vector (here the variables v, r, and r0, are vectors although they are not shown with the traditional bold face characters):

$v = \dfrac{dr}{dt}$, therefore we can write the ordinary (vector) differential equation:

$dr = v \cdot dt$, and integrate as: $\displaystyle\int_{r0}^{r} 1\, dr = \int_0^t v\, dt$ --> $\displaystyle r - r0 = \int_0^t v\, dt$ -->

$\displaystyle r = r0 + \int_0^t v\, dt$. For this problem, $r0 := \begin{pmatrix} 2 \\ 4 \\ 5 \end{pmatrix}$, t = 4s, and $v = \begin{pmatrix} -5 \cdot t^2 \\ 2 \cdot t - 3 \cdot \sqrt{t} \\ \dfrac{1}{1+t^2} \end{pmatrix}$

Therefore, $\displaystyle r = r0 + \int_0^t v\, dt$ gets calculated as:

$$r := r0 + \int_0^4 \begin{pmatrix} -5 \cdot t^2 \\ 2 \cdot t - 3 \cdot \sqrt{t} \\ \dfrac{1}{1+t^2} \end{pmatrix} dt$$, i.e., $r = \begin{pmatrix} -104.6667 \\ 4.0007 \\ 6.3258 \end{pmatrix}$ m

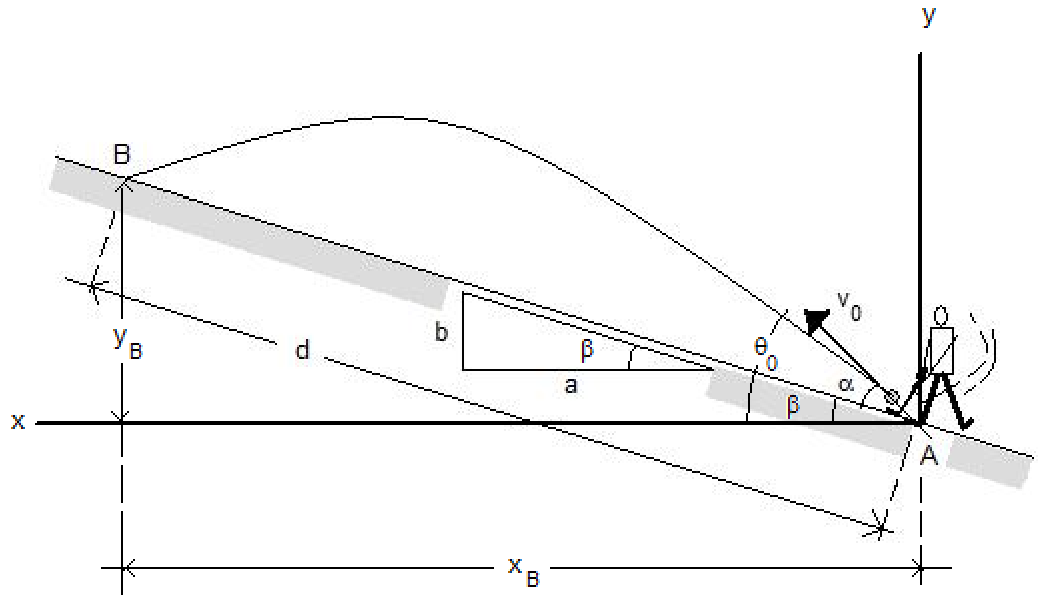
The magnitude of r is: $r\ mag := \sqrt{{r_1}^2 + {r_2}^2 + {r_3}^2}$ or, $r\ mag = 104.9339$ m

Alternatively, use function "norme" which provides the so-called Euclidean norm (i.e., the magnitude, or length) of the vector r:

$r\ mag\ alt := \text{norme}(r)$ --> $r\ mag\ alt = 104.9339$ m

6. EX01-Q06-Projectile Motion (Points: 10)

The golfer hits the ball while on a slope of 7H:1V (i.e., a = 7, b = 1) as shown in the figure. The ball leaves the ground at an angle α (alpha) = 5° measured from the slope, and hits the ground at a distance *d* = 41 ft along the slope. Determine the initial velocity of the ball, v_0.



Solution: Note: I've added the x and y axes (as shown), the coordinates of
--------- point B, and angles β and θ0 to the original figure.

Given: $a := 7$ $b := 1$, the angle β is calculated as: $\beta := \mathrm{atan}\left(\frac{b}{a}\right)$ or $\beta = 0.1419$ rad

Note: in most software, trigonometric functions and their inverses use radians.

The value of β in degrees would be: $\beta := \frac{180}{\pi}\cdot\beta$, or $\beta = 8.1301$ deg

With $\alpha := 5$ deg, then the initial velocity angle is: $\theta 0 := \alpha + \beta$, or $\theta 0 = 13.1301$ deg.

The coordinates of point B, with $d := 41$ ft can be calculated as follows (using similar triangles):

$$xB := d\cdot\frac{a}{\sqrt{a^2+b^2}} \quad \text{and} \quad yB := d\cdot\frac{b}{\sqrt{a^2+b^2}}$$

i.e., $xB = 40.5879$ and $yB = 5.7983$ ft

The equations for the positions x and y for the ball are given by:

$$x = x0 + v0\cdot\cos(\theta 0)\cdot t \quad , \text{ and } \quad y = y0 + v0\cdot\sin(\theta 0)\cdot t - \frac{1}{2}\cdot g\cdot t^2$$

Isolating t from the first equation we have: $t = \frac{x - x0}{v0\cdot\cos(\theta 0)}$

Inserting this result into the second equation provides an equation for the (parabolic) trajectory of the ball:

$$y = y0 + v0\cdot\sin(\theta 0)\cdot\left(\frac{x - x0}{v0\cdot\cos(\theta 0)}\right) - \frac{1}{2}\cdot g\cdot\left(\frac{x - x0}{v0\cdot\cos(\theta 0)}\right)^2$$

which simplifies to:

$$y = y0 + \tan(\theta 0)\cdot(x - x0) - \frac{g\cdot(x - x0)^2}{2\cdot v0^2\cdot\cos(\theta 0)^2}$$

For the coordinate system shown above: $x0 := 0$ and $y0 := 0$

Also, $g := 32.2\,\frac{ft}{s^2}$. To proceed with the solution we need to

convert θ0 to radians: $\theta 0 := \theta 0\cdot\frac{\pi}{180}$, i.e., $\theta 0 = 0.2292$ rad

The solution, v0, is found by taking x = xB and y = yB in the equation for the trajectory, i.e.,

$$yB = y0 + \tan(\theta 0)\cdot(xB - x0) - \frac{g\cdot(xB - x0)^2}{2\cdot v0^2\cdot\cos(\theta 0)^2}$$

Solving for v0: $\frac{g\cdot(xB - x0)^2}{2\cdot v0^2\cdot\cos(\theta 0)^2} = y0 + \tan(\theta 0)\cdot(xB - x0) - yB$

$$\frac{2\cdot v0^2\cdot\cos(\theta 0)^2}{g\cdot(xB - x0)^2} = \frac{1}{y0 + \tan(\theta 0)\cdot(xB - x0) - yB}$$

$$v0 := \sqrt{\frac{g\cdot(xB - x0)^2}{2\cdot\cos(\theta 0)^2\cdot(y0 + \tan(\theta 0)\cdot(xB - x0) - yB)}}$$ --> $v0 = 87.3016\,\frac{ft}{s}$

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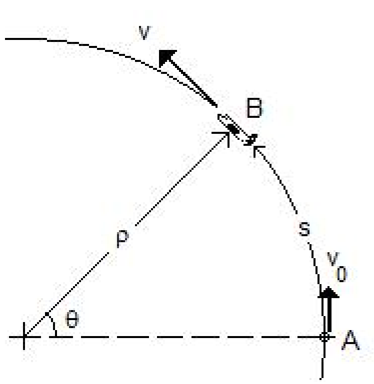
7. EX01-Q07-Normal-tangential components (Points: 10)

An airplane is describing a vertical circular path of radius ρ (rho) = 1,009 m as shown, such that its tangential acceleration is constant a_t = 14 m/s^2. If the plane's velocity at point A is v_0 = 149 m/s, determine the magnitude of the plane's acceleration at point B located such that the angle θ (theta) is 35°.



Solution:

For constant tangential acceleration we can use:

$$v^2 = v0^2 + 2\cdot at\cdot(s - s0)$$

taking s0 = 0, the velocity at position s is:

$$v = \sqrt{v0^2 + 2\cdot at\cdot s}$$

For this problem: $v0 := 149\,\frac{m}{s}$, $at := 14\,\frac{m}{s^2}$,and

the position s can be calculated using the angle θ, in radians, i.e.,

$\theta := 35\cdot\frac{\pi}{180}$, or $\theta = 0.6109$ rad . With the radius of the circular path being $\rho := 1009$ m , the arc length is $s := \rho\cdot\theta$, or $s = 616.363$ m

The velocity at that position is: $v := \sqrt{v0^2 + 2\cdot at\cdot s}$, or $v = 198.6433 \frac{m}{s}$

The corresponding normal acceleration is: $an := \frac{v^2}{\rho}$, or $an = 39.1072 \frac{m}{s^2}$

The magnitude of the plane's acceleration is. then, calculated as:

$$a := \sqrt{at^2 + an^2} \quad \text{, i.e.,} \quad a = 41.5376 \frac{m}{s^2}$$

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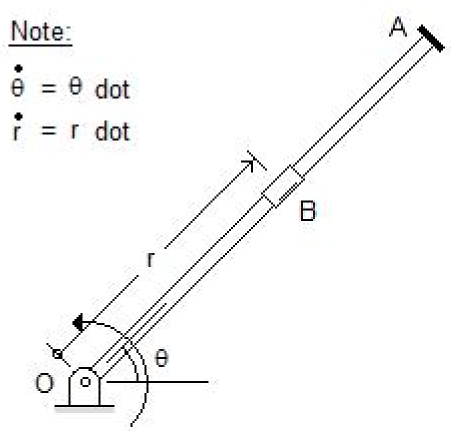
8. EX01-Q08-Cylindrical (polar) components (Points: 10)

Rod OA rotates counterclockwise with an angular velocity *theta-dot* = ($0.2t^2$) rad/s. Through mechanical means collar *B* moves along the rod with a speed of *r-dot* = ($1.0t^{1/2}$) ft/s. If *theta* = 0 and *r* = 0.3 ft when *t* = 0, determine the magnitude of the collar's acceleration at *t* = 0.7 s.



Solution:

From the definition of r-dot:

$$rdot = \frac{dr}{dt} = 1.0\cdot\sqrt{t}$$

and with initial conditions

$r = 0.3\cdot ft$ when $t = 0$ we can find r

at t = 0.7 s as:

$$\int_{0.3}^{r} 1\, dr = \int_{0}^{0.7} 1.0\cdot\sqrt{t}\, dt$$

With $r - 0.3 = \int_{0}^{0.7} 1.0\cdot\sqrt{t}\, dt$, then $r := 0.3 + \int_{0}^{0.7} 1.0\cdot\sqrt{t}\, dt$

or, $r = 0.6904$ ft

To find $rddot = \frac{d}{dt}(rdot) = \frac{d^2\cdot r}{dt^2}$ use $\frac{d}{dt}\left(1.0\cdot\sqrt{t}\right) \rightarrow \frac{1}{2\cdot\sqrt{t}}$

To find $\theta ddot = \frac{d}{dt}(\theta dot) = \frac{d^2\cdot\theta}{dt^2}$ use $\frac{d}{dt}\left(0.2\cdot t^2\right) \rightarrow \frac{2\cdot t}{5}$

At $t := 0.7$ s $rdot := 1.0\cdot\sqrt{t}$ --> $rdot = 0.8367 \frac{ft}{s}$

$rddot := \frac{1}{2\cdot\sqrt{t}}$ --> $rddot = 0.5976 \frac{ft}{s^2}$

$$\dot\theta := 0.2 \cdot t^2 \quad \text{-->} \quad \dot\theta = 0.098\ \frac{rad}{s}$$

$$\ddot\theta := \frac{2 \cdot t}{5} \quad \text{-->} \quad \ddot\theta = 0.28\ \frac{rad}{s^2}$$

The components of the acceleration are:

$$a_r := \ddot r - r \cdot \dot\theta^2 \quad \text{-->} \quad a_r = 0.591\ \frac{ft}{s^2}$$

$$a_\theta := r \cdot \ddot\theta + 2 \cdot \dot r \cdot \dot\theta \quad \text{-->} \quad a_\theta = 0.3573\ \frac{ft}{s^2}$$

The magnitude of the acceleration is:

$$a := \sqrt{a_r^2 + a_\theta^2} \quad \text{-->} \quad a = 0.6906\ \frac{ft}{s^2}$$

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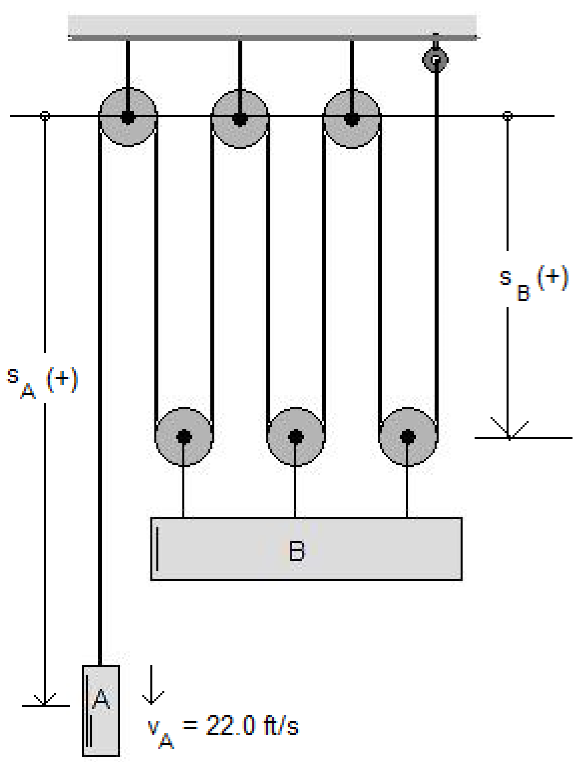
9. EX01-Q09-Absolute dependent motion (Points: 10)

Determine the velocity of block B if block A is moving downwards at a speed of 22.0 ft/s.



Solution: The length of the cord is calculated as:

$$s_A + 6 \cdot s_B = L = \text{constant}$$

Taking derivatives with respect to time:

$$v_A + 6 \cdot v_B = 0$$

Therefore, $v_B = -\frac{v_A}{6}$

Since sA, sB are positive downwards, and vA is downwards, then we take:

$$v_A := +22\ \frac{ft}{s}$$

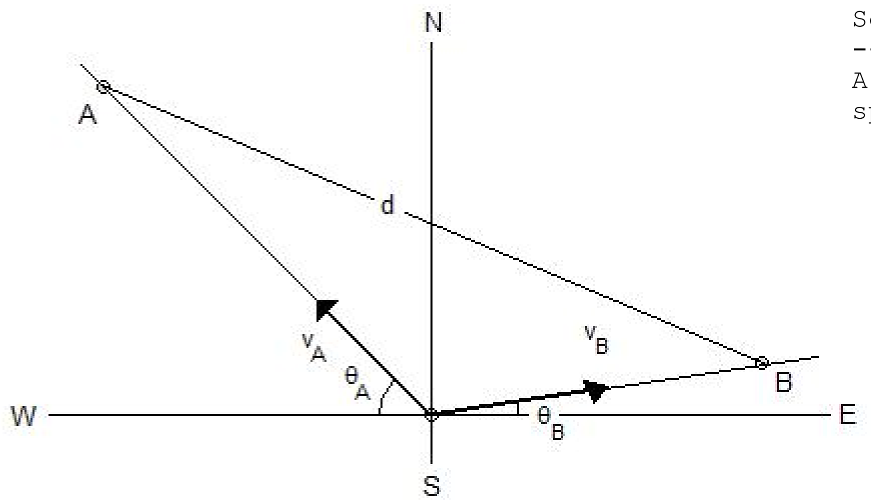
and calculate vB as: $v_B := -\frac{v_A}{6}$, or

$v_B = -3.6667$. This is to say that

$$v_B = 3.6667\ \frac{ft}{s}\ \text{upwards}$$

10. EX01-Q10-Relative Motion of Particles (Points: 10)

1Two boats, A and B, leave the same point in the shoreline of a lake (the E-W line) moving with constant velocities in the directions shown, i.e., with v_A = 17 ft/s, v_B = 5 ft/s, angle *theta-A* = 36°, and angle *theta-B* = 38°. Determine the distance *d* separating the two boats after *t* = 8 s.



Solution: The distances
--------- traveled by
A and B, at constant speeds:

$vA := 17\ \frac{ft}{s}$

and

$vB := 5\ \frac{ft}{s}$

for a time period of:

$t := 8\ s$

are:

$sA := vA \cdot t$, or $sA = 136\ ft$, and $sB := vB \cdot t$, or $sB = 40\ ft$

The angle between the two directions of motion is (in degrees):

$\theta A := 36$, $\theta B := 38$, and $\theta := 180 - (\theta A + \theta B)$, or $\theta = 106$

Next, apply the law of cosines for triangle solutions to the triangle above, with sA = distance from origin to A, and sB = distance from origin to B, and d being opposite to the angle θ (herein converted to radians):

$d := \sqrt{sA^2 + sB^2 - 2 \cdot sA \cdot sB \cdot \cos\left(\theta \cdot \frac{\pi}{180}\right)}$, then $d = 151.9702\ ft$

Alternative solution: calculate the relative velocity vector vBA = vBv-vAv, find its magnitude, vBA_mag, and calculate d = vBA_mag*t: (Here, vBv means the velocity vB as a vector, etc.):

$vBv := \begin{pmatrix} vB \cdot \cos\left(\theta B \cdot \frac{\pi}{180}\right) \\ vB \cdot \sin\left(\theta B \cdot \frac{\pi}{180}\right) \end{pmatrix}$ $vAv := \begin{pmatrix} -vA \cdot \cos\left(\theta A \cdot \frac{\pi}{180}\right) \\ vA \cdot \sin\left(\theta A \cdot \frac{\pi}{180}\right) \end{pmatrix}$

$vBv = \begin{pmatrix} 3.9401 \\ 3.0783 \end{pmatrix} \frac{ft}{s}$ $vAv = \begin{pmatrix} -13.7533 \\ 9.9923 \end{pmatrix} \frac{ft}{s}$

The relative velocity vBA is: $vBA := vBv - vAv$ --> $vBA = \begin{pmatrix} 17.6933 \\ -6.914 \end{pmatrix} \frac{ft}{s}$

Its magnitude is: $vBA\ mag := norme(vBA)$ --> $vBA\ mag = 18.9963\ \frac{ft}{s}$

and $d\ alt := vBA\ mag \cdot t$ --> $d\ alt = 151.9702\ ft$
