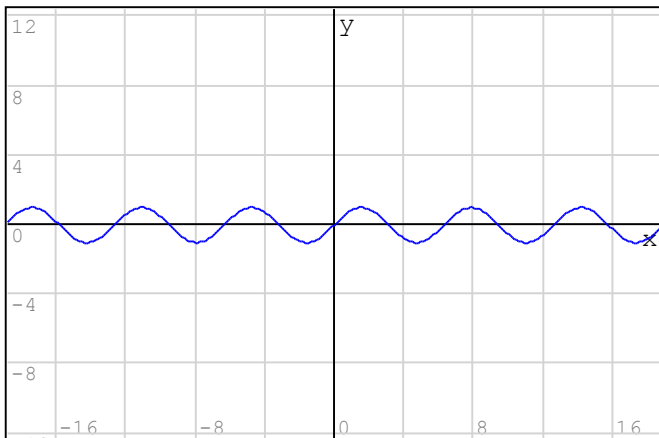


Examples of two- and three-dimensional graphics in Smath Studio

By Gilberto E. Urroz, Ph.D., P.E.
December 2009

Plotting a single function of x :

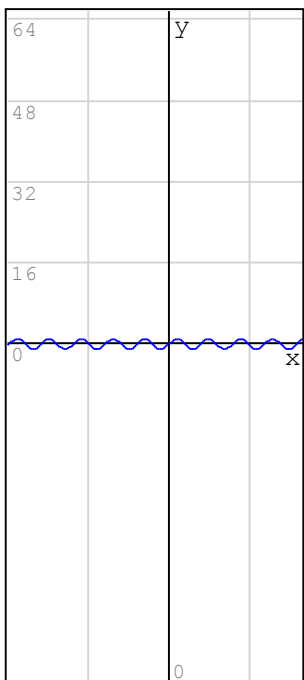


- 1 - Click on point in your worksheet where upper left corner of graph will go
- 2 - Click on the "2D" option in the "Functions" palette or use the "Insert > Graph > 2D" menu option
- 3 - Type the function name in the placeholder below the graph

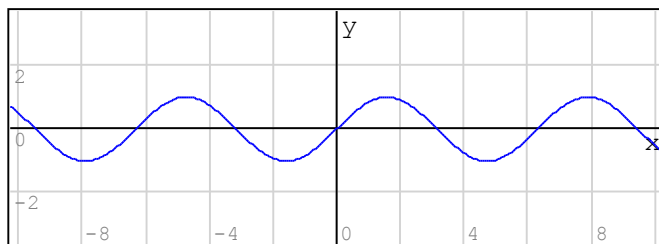
In this example we plot the function: $f(x) = \sin(x)$

Changing the size of the graph window:

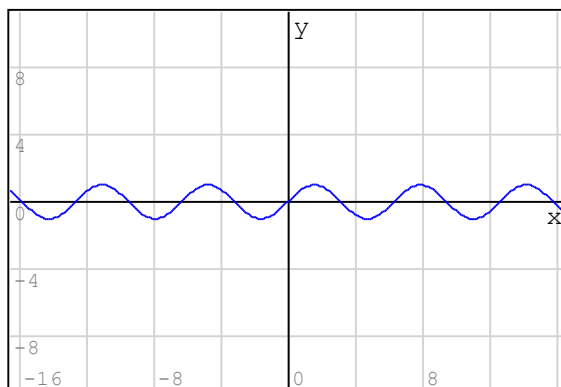
Click on the graph window, then drag one of the three black handlers in the graph window to adjust its size



$\sin(x)$

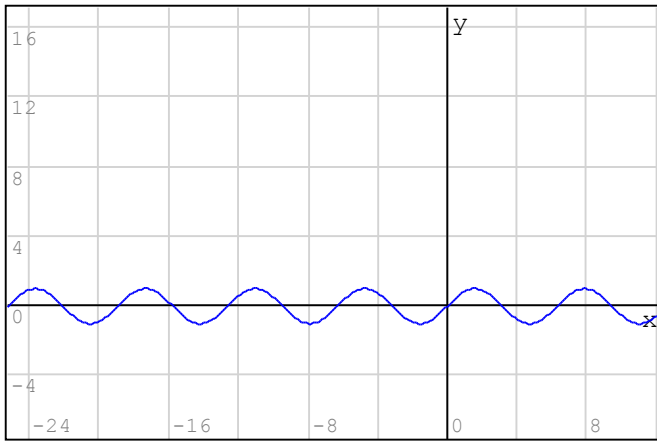


$\sin(x)$

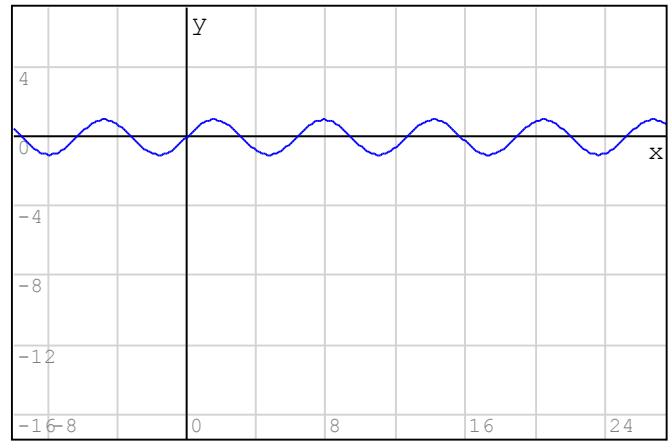


$\sin(x)$

Moving the axes about the graph window:



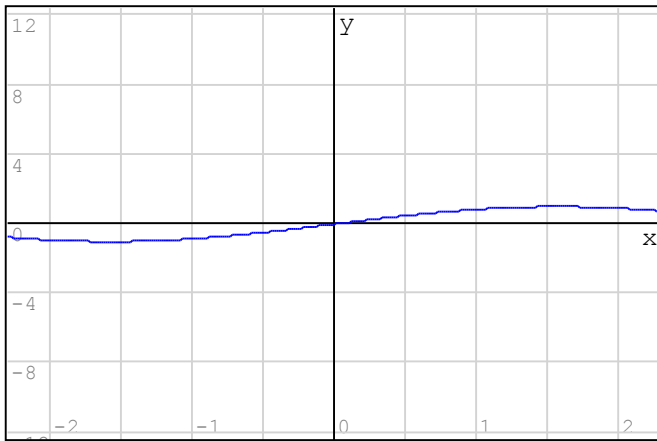
$\sin(x)$



$\sin(x)$

- 1 - Click on the "Move" option in the "Plot" palette
- 2 - Click on the graph window and drag the mouse in the direction where you want to move the axes.

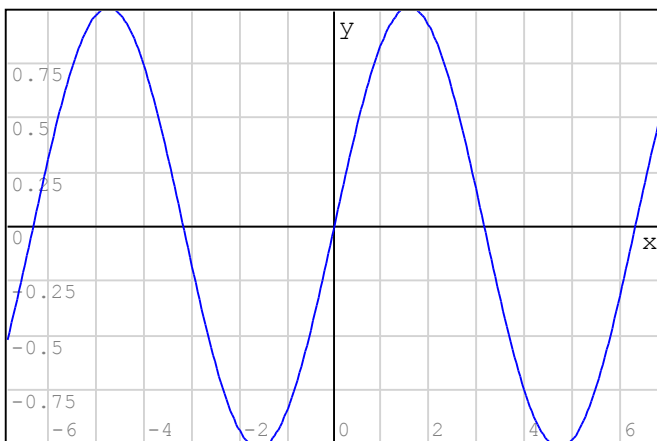
Scaling (zooming) the graph:



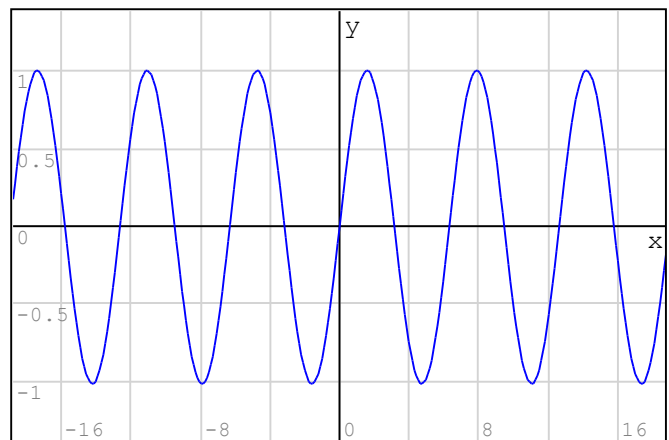
$\sin(x)$

- To zoom the x-axis only: ----->
- 1 - Click on "Scale" in the "Plot" palette
 - 2 - Click on the graph window
 - 3 - Hold down the "Control" key
 - 4 - Roll the mouse wheel up or down

- <--- To zoom the x-axis only:
- 1 - Click on "Scale" in the "Plot" palette
 - 2 - Click on the graph window
 - 3 - Hold down the "Shift" key
 - 4 - Roll the mouse wheel up or down



$\sin(x)$

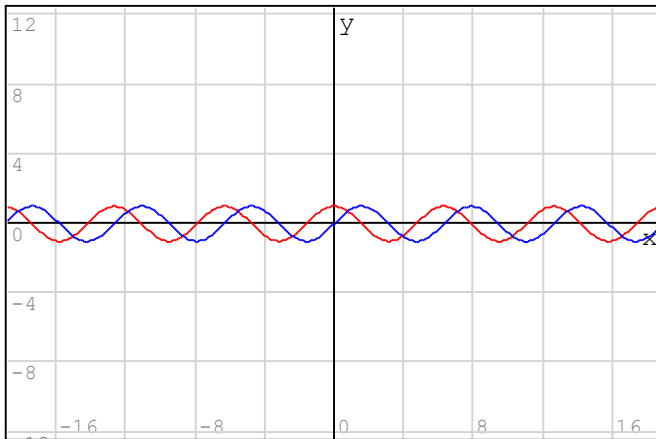


$\sin(x)$

- <--- You can zoom both axes by zooming one axis at a time. In this case, I zoomed the x axis first, and then the y axis.

Note: Use the "Refresh" option in the "Plot" menu to recover the original version of any plot.

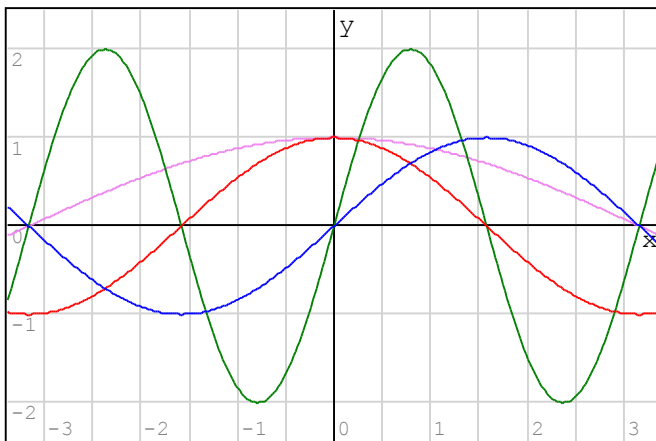
Plotting various functions simultaneously:



$\left\{ \begin{array}{l} \sin(x) \\ \cos(x) \end{array} \right.$

- 1 - create a 2D graph
- 2 - click on the function placeholder (lower left corner) to select it
- 3 - click on the "Equation System" option in the "Functions" palette to produce a minimum of two function entries
- 4 - Type the two functions to be plotted

Note: the first function listed is plotted using a blue line, the second one uses a red line



$\left\{ \begin{array}{l} \sin(x) \\ \cos(x) \\ 2 \cdot \sin(2 \cdot x) \\ \cos\left(\frac{x}{2}\right) \end{array} \right.$

- 1 - To add more than two functions to plot, use the "Equation System" option in the "Functions" palette as above
- 2 - Click on the "Equation System" cell, and drag down corner of cell adding as many placeholders as you want.
- 3 - Type the functions to be plotted, one at a time, clicking outside of the graph after you enter each one of them (this will allow you to see each function plot as they are added to the graph)

Notes:

- 1 - As you add functions to plot, SMATH Studio uses the following colors for the plots:

1 - blue	2 - red
3 - black	4 - magenta, etc.

- 2 - In this example the graph has been zoomed using the approach described above

Plotting a function using vectors:

$x := -\pi, -\pi + \frac{\pi}{20} \dots \pi$

$n := \text{length}(x)$
 $n = 41$

for $k \in 1 \dots n$

$y_k := \sin(x_k)^2 + \sin(2 \cdot x_k)$

Vectors of x and y data are created using ranges, example:

Create x vector as follows

Type: $x : \text{range} - p \text{ cntl-G}, p \text{ cntl-G}, - p \text{ cntl-G} + p \text{ cntl-G} / 20$

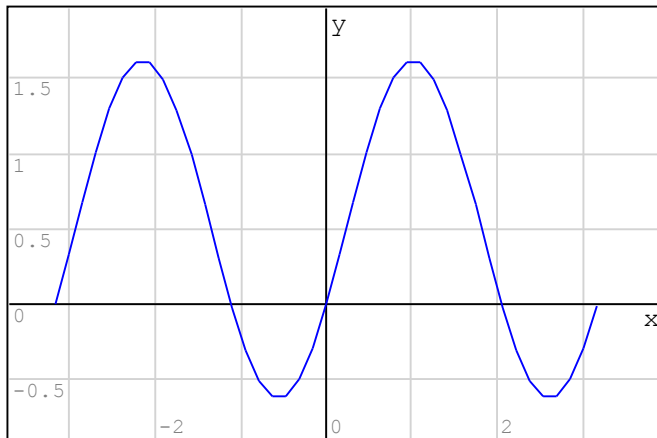
Calculate the length of vector = n

Fill out y vector using a for loop. Click "for" in the "Programming" palette, then use: $\text{range } 1, n$

Use sub-indices, e.g., $y [k \dots \text{etc.}]$

```
M:= augment(x, y)
```

Form augmented matrix M with vectors x and y,
place M in graph as a function name:

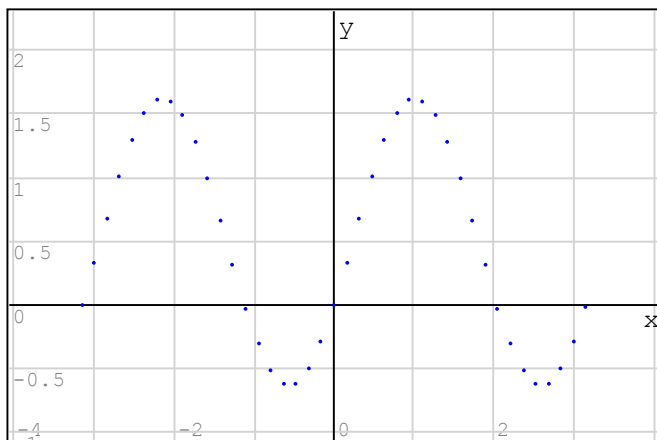


M

< --- The graph was zoomed in and the axes moved by using the following procedures:

- 1 - To zoom x-axis only: click on "Scale" in the "Plot" palette, hold the "Control" key, and use the mouse wheel
- 2 - To zoom y-axis only: click on "Scale" in the "Plot" palette, hold the "Shift" key and use the mouse wheel
- 3 - To move axes, drag mouse across graph window

Using points or lines for a plot:



M

Using the sparse data in matrix M we reproduce the graph above, but then we selected the "Graph by points" option in the "Plot" palette to produce the graph shown to the left.

You can click the option "Graph by lines" option in the "Plot" palette to return to the default graph format of continuous lines.

Matrices can be used for plotting parametric plots:

```
t:= -π, -π + π/50 .. π
```

```
n:= length(t)
```

```
for k ∈ 1 .. n
```

```
  | xk := sin(3 · tk)  
  | yk := 2 · cos(2 · tk)
```

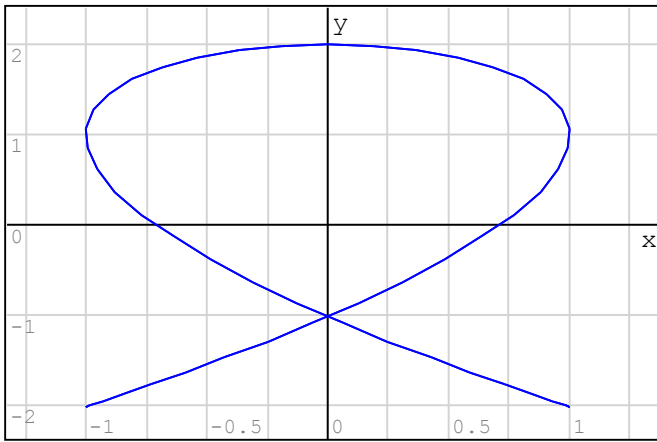
```
M:= augment(x, y)
```

Define the vector of the parameter t

Determine length of vector t = n

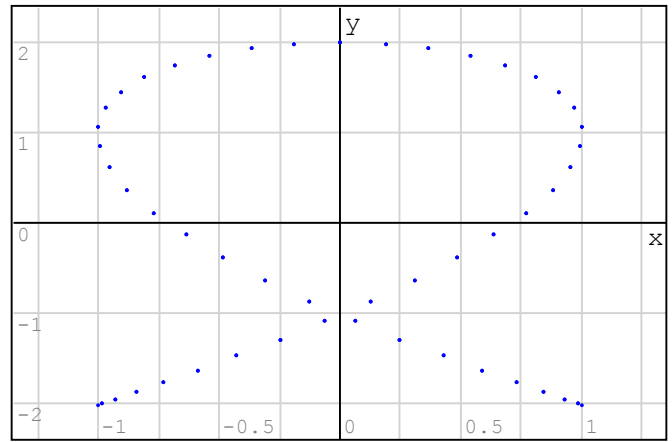
Calculate vectors of $x = x(t)$ and $y = y(t)$

Produce matrix of (x,y) and plot it



M

1 - Using "Plot by Lines" option in the "Plot" palette



M

2 - Using "Plot by Lines" option in the "Plot" palette

Polar plots can be produced using vectors and matrices:

$$\theta := 0, \frac{\pi}{50} \dots 2\pi$$

$$n := \text{length}(\theta)$$

for $k \in 1 \dots n$

$$r_k := 2 \cdot (1 + 2 \cdot \sin(\theta_k))$$

for $k \in 1 \dots n$

$$\begin{cases} xx_k := r_k \cdot \cos(\theta_k) \\ yy_k := r_k \cdot \sin(\theta_k) \end{cases}$$

$$P := \text{augment}(xx, yy)$$

Generate vector of θ between 0 and 2π

Determine length of vector θ

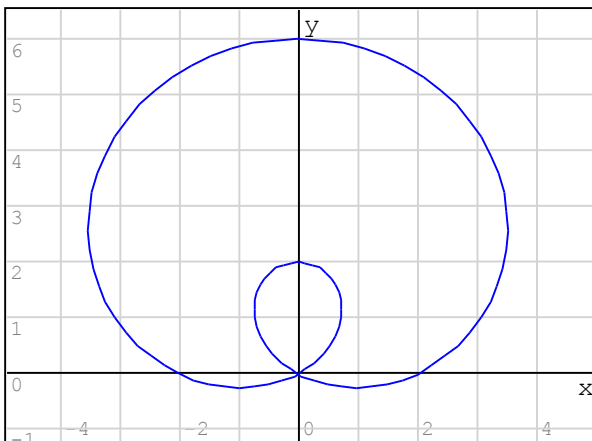
Generate values of $r = f(\theta)$

Generate coordinates:

$$x = r \cos(\theta)$$

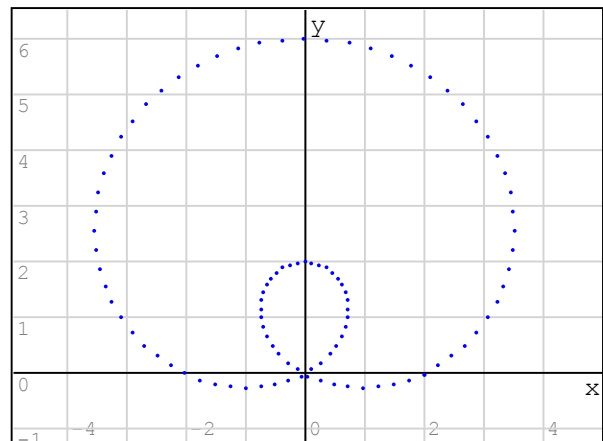
$$y = r \sin(\theta)$$

Produce matrix of (x, y) and plot it



P

1 - Using "Plot by Lines" option in the "Plot" palette



P

2 - Using "Plot by Lines" option in the "Plot" palette

Using graphs in solving equations:

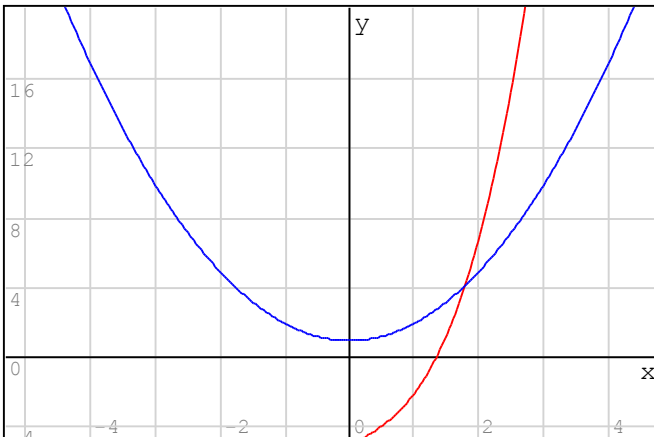
In this example we seek the solution(s) for the equation:

$$x^2 + 1 = x^3 + 2 \cdot x - 5$$

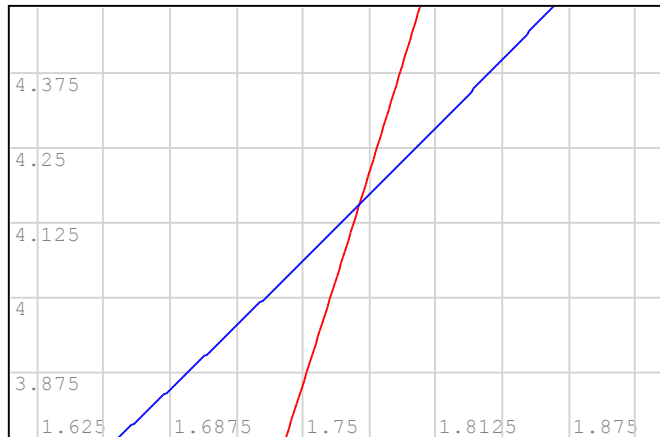
A solution can be found by determining the intersection of the functions:

$$f(x) := x^2 + 1 \quad g(x) := x^3 + 2 \cdot x - 5$$

Using graphics and zooming the intersection we estimate the solution to be close to $x = 1.80$



$\left\{ \begin{array}{l} f(x) \\ g(x) \end{array} \right.$

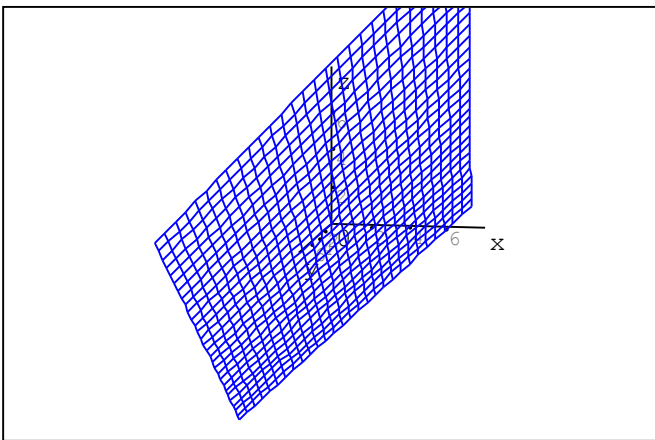


$\left\{ \begin{array}{l} f(x) \\ g(x) \end{array} \right.$

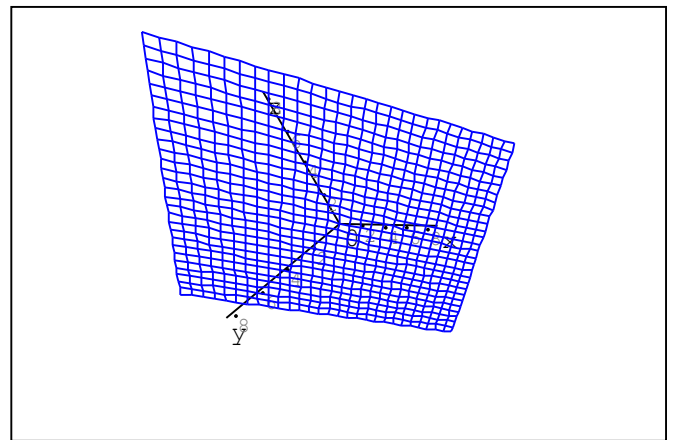
The exact solution can be found using:

$$\text{solve}(x^2 + 1 = x^3 + 2 \cdot x - 5, x) = 1.776$$

Three-dimensional graphs - surfaces:



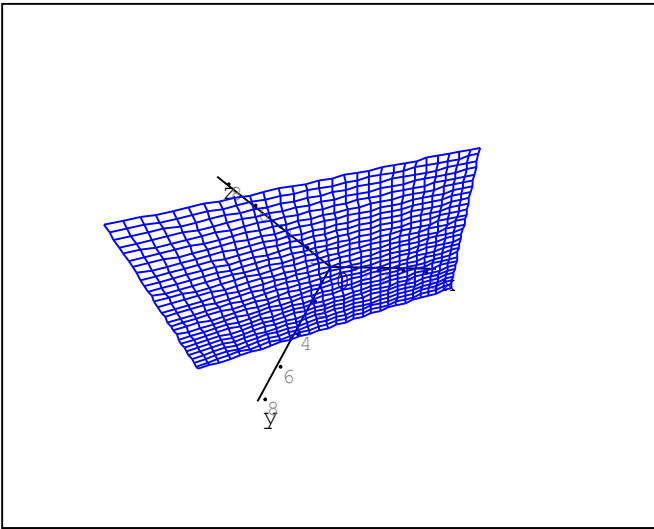
$x + y$



$x + y$

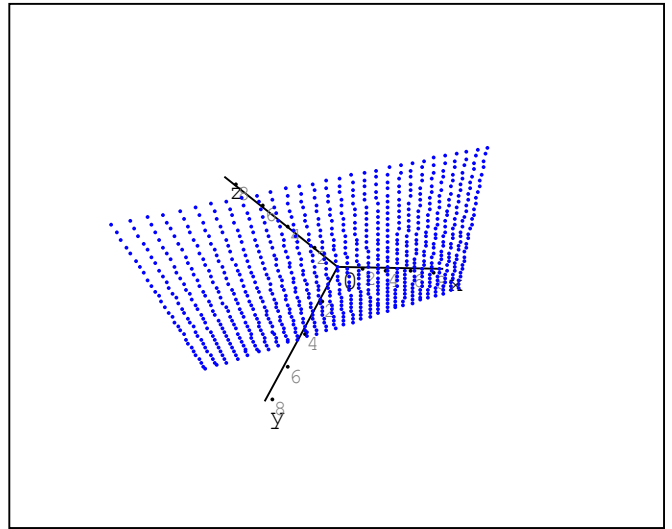
Use the option "3D" in the "Functions" palette, and enter the function $f(x, y)$ in the placeholder. The result is a 3D surface, in this case, a plane. The original plot is shown above.

Use the "Rotate" option in the "Plot" palette to change the surface view.



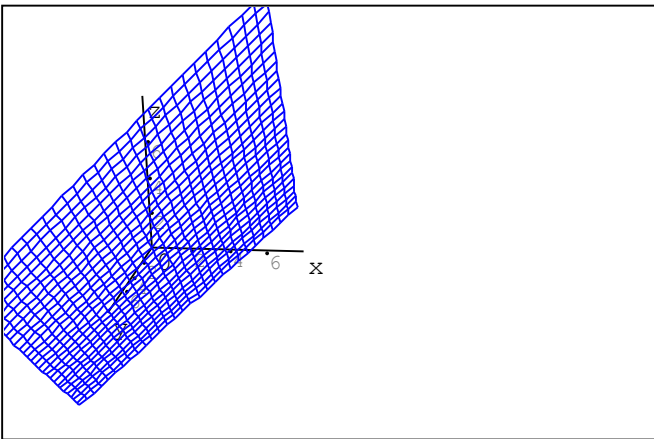
$x + y$

This figure uses the option "Graph by Lines" in the "Plot" palette (default).



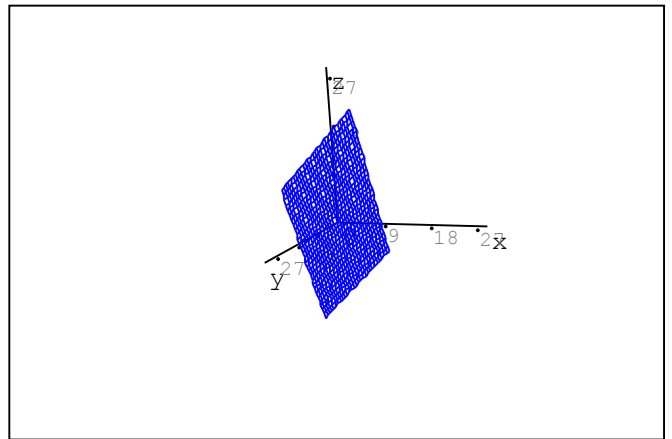
$x + y$

This figure uses the option "Graph by Points" in the "Plot" palette.



$x + y$

Use the "Move" option in the "Plot" palette to move the location of the origin in the graphics window.

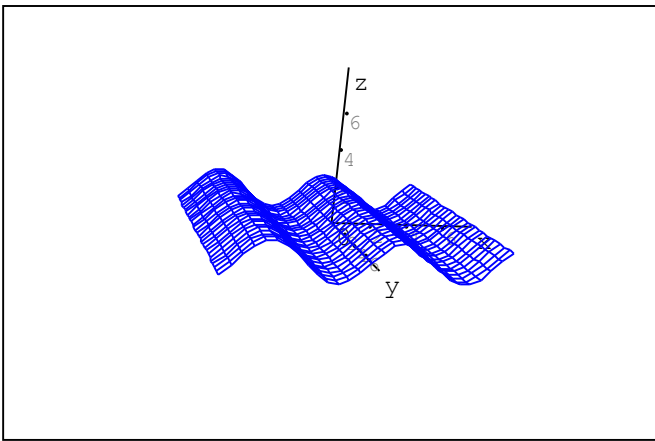


$x + 2 \cdot y$

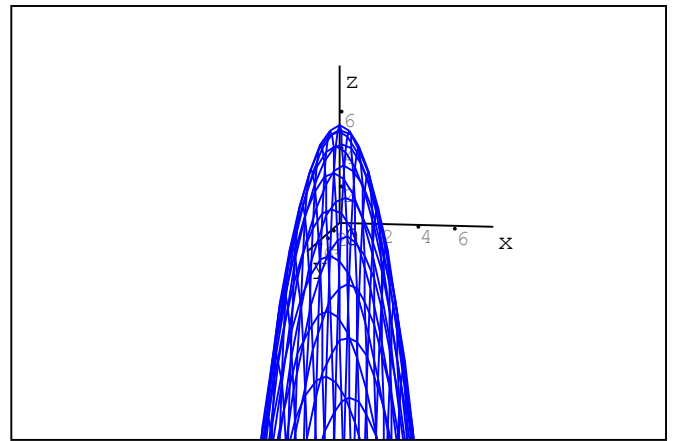
Use the "Scale" option in the "Plot" palette, click on the graph, and drag the mouse over it to zoom in or out.

Note: Use the "Refresh" option in the "Plot" menu to recover the original version of any plot.

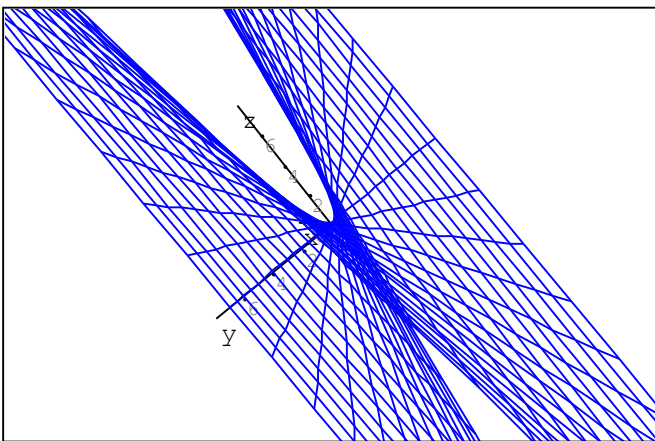
The following examples use more complex surfaces:



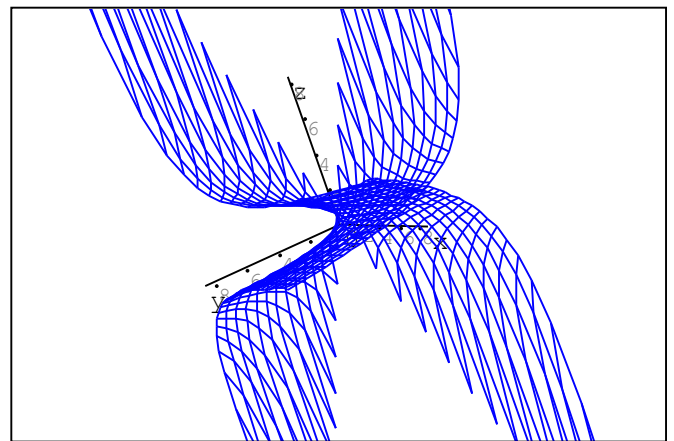
$$\sin(x)$$



$$5 - (x^2 + y^2)$$



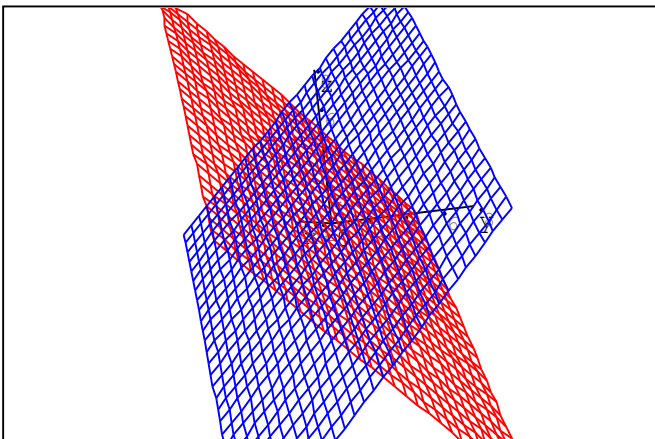
$$x \cdot y$$



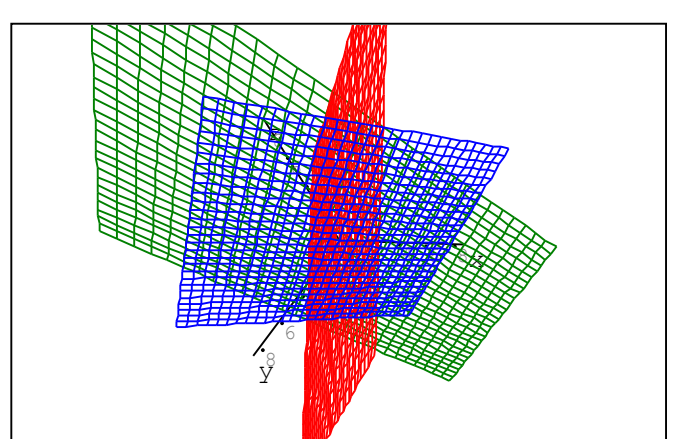
$$\frac{x - y}{x + y}$$

The type of 3D graphs of surfaces produced by SMath Studio are referred to as wireframe plots.

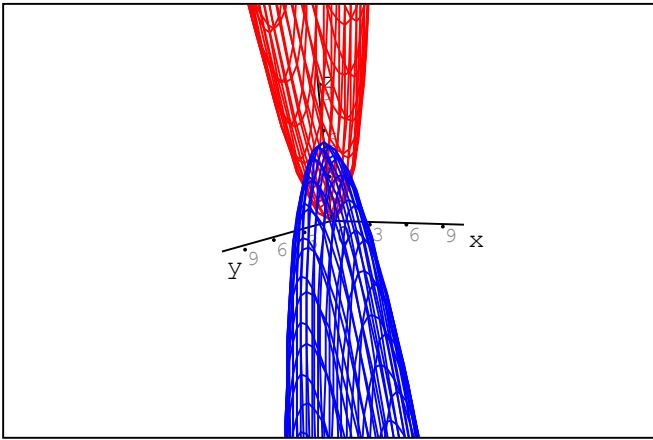
The following examples show more than one surface plot together in 3D:



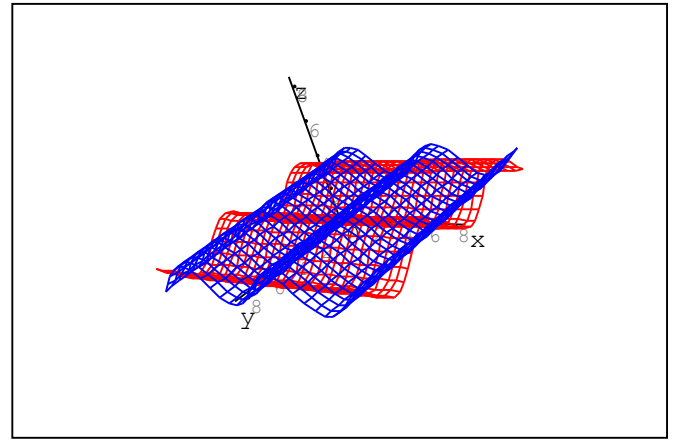
$$\begin{cases} x + y \\ x - y \end{cases}$$



$$\begin{cases} x + y \\ x - y \\ x + 2 \cdot y \end{cases}$$



$$\begin{cases} 5 - (x^2 + y^2) \\ x^2 + y^2 \end{cases}$$



$$\begin{cases} \sin(x) \\ \sin(y) \end{cases}$$

Drawing a space curve in 3D:

t:= 0 , 0.1 .. 10

n:= length(t)

n= 101

for k ∈ 1 .. n

$$\begin{cases} x_k := \sin(t_k) \\ y_k := \cos(t_k) \\ z_k := \frac{t_k}{2} \end{cases}$$

M:= augment(x, y, z)

A space curve is defined by a matrix of three columns corresponding to coordinates x, y, and z of the curve.

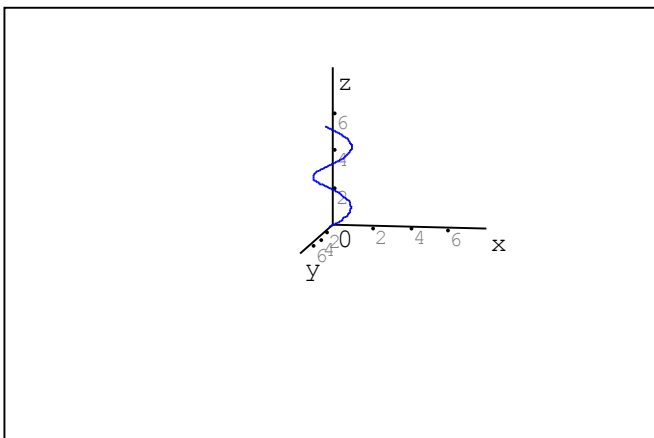
Create a vector t with values of the parameter that will produce $x = x(t)$, $y = y(t)$, and $z = z(t)$.

Determine the length of vector t

Generate vectors x, y, and z using a "for" loop

Build matrix M with coordinates (x,y,z)

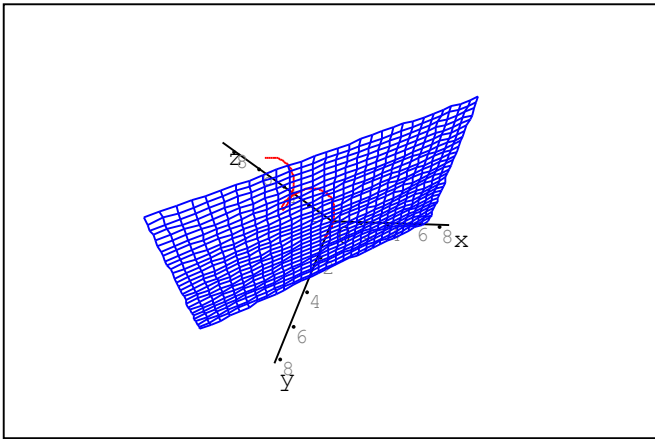
Plot matrix M in a 3D plot



M

Drawing a surface and a space curve:

Use the "Equation System" option in the "Plot" palette, and enter the equation of the surface (e.g., $x+y$) and the matrix that represents the space curve (e.g., M)



$$\begin{cases} x+y \\ M \end{cases}$$

Plotting 2 space curves:

In this example two straight lines in 3D are produced by using linear parametric equations.

$t := -2, -1.9 \dots 2$

$n := \text{length}(t)$

$n = 40$

Vector t is the parameter, and the coordinates of the two curves are given by (x_1, y_1, z_1) and (x_2, y_2, z_2) . These lines are represented by the matrices P and Q , respectively.

for $k \in 1 \dots n$

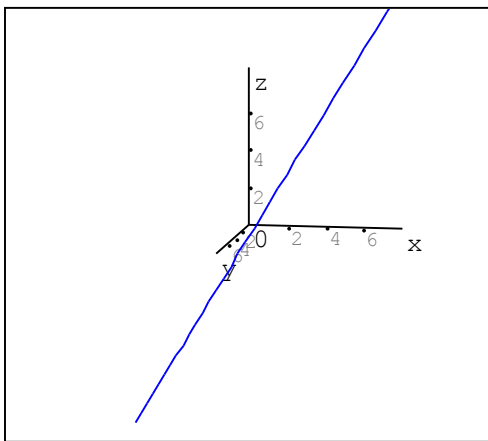
$$\begin{cases} x1_k := 1 + 5 \cdot t_k \\ y1_k := -1 + 4 \cdot t_k \\ z1_k := 1 + 8 \cdot t_k \end{cases}$$

for $k \in 1 \dots n$

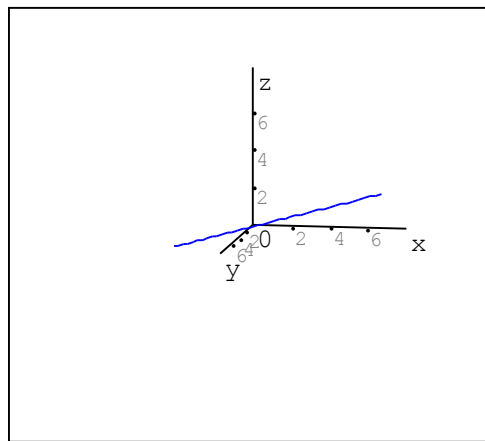
$$\begin{cases} x2_k := 1 + 3 \cdot t_k \\ y2_k := -1 + t_k \\ z2_k := t_k \end{cases}$$

$P := \text{augment}(x1, y1, z1)$

$Q := \text{augment}(x2, y2, z2)$

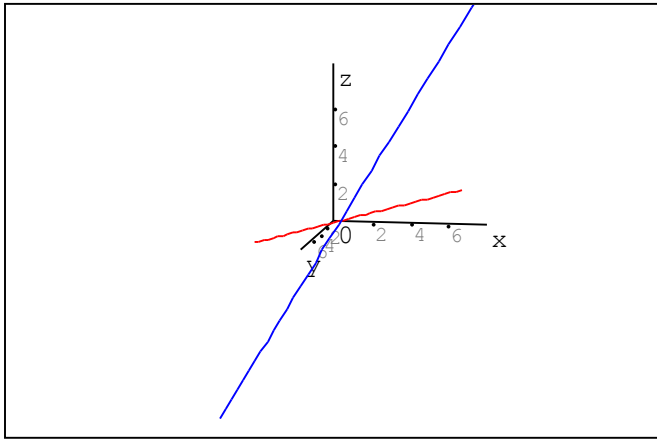


P



Q

Individual plots of curves given by matrices P and Q .



<--- Join plot of
 lines given
 by matrices
 P and Q

{ P
 { Q