Numeric integration method
(Simpson's rule with Richardson extrapolation)

Input data:

$f(x) := \sin(x)$ integrand

$a := 1$ inferior limit

$b := 5$ superior limit

$maxerr := 10^{-7}$ accuracy

$$simp(a, b, h, n) := \frac{1}{3}\cdot\left(\frac{h}{2}\cdot\left(f(a)+f(b)\right)+h\cdot\sum_{k=1}^{n-1} f(a+k\cdot h)\right)+\frac{2}{3}\cdot h\cdot\sum_{k=1}^{n} f\left(a-\frac{h}{2}+k\cdot h\right)$$

Calculation:

$n := 2$

$h := \frac{b-a}{n}$

$dintp := simp(a, b, h, n)$

$\varepsilon := dintp$

while $|\varepsilon| > maxerr$

- $n := 2\cdot n$
- $h := \frac{b-a}{n}$
- $dint := simp(a, b, h, n)$
- $\varepsilon := \frac{dint - dintp}{15}$
- $dintp := dint$

$n = 32$

$\varepsilon = -2.1798215\cdot 10^{-8}$

$dint := dint + \varepsilon$

Result:

$dint = 0.2566401$

Control:

$$\int_a^b f(x)\,dx = 0.2566401$$

Analytical solution:

$-\cos(b)+\cos(a) = 0.2566401$