CEE3030 - Test 02 - Fall 2009 - Solutions:

[1]. Binomial. The probability of a forest fire on a given summer day at a state park is estimated to be 0.2. What would be the probability of having exactly 3 fires in a 10-day period that summer?

Solution: X = number of forest fires, X ~ Bin(n=10, p=0.2). We want: P(X=3) = ?

For the binomial distribution the pmf is:

$$P(X = x) = f(x) = \frac{n!}{x! \cdot (n-x)!} \cdot p^{x} \cdot (1-p)^{n-x} \quad , \text{ for } x = 0,1,\ldots,n$$

Thus, $P(X=3) = f(3) = \frac{10!}{(3!) \cdot (7!)} \cdot 0.2^{3} \cdot 0.8^{7} = 0.2013$

[2]. Poisson. A service engineer at a UDOT service station takes care of servicing 3 cars per day, on the average. What is the probability that, on a given day, he'll have to take care of servicing 4 or more vehicles?

Solution: X = number of vehicles serviced on a day, then X ~ Poisson(λ = 3). We want:

$$P(X \geq 4) = 1 - P(X \leq 3) = 1 - F(3) = 1 - \sum_{x=0}^{3} f(x)$$

For the Poisson distribution the pmf is:

$$P(X = x) = f(x) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!} \quad , \text{ for } x = 0, 1, \ldots, \infty$$

Thus, we want:

$$P(X \geq 4) = 1 - \sum_{x=0}^{3} \left(\frac{e^{-3} \cdot 3^{x}}{x!} \right) = 0.3528$$

[3]. Negative Binomial. Suppose you are testing a well field for a certain contaminant. The probability that a given well is contaminated is estimated to be 0.1. A clean-up of the field will be required if you find 3 contaminated wells. What is the probability that you will find 3 contaminated wells after testing only 5 wells?

Solution: X = number of wells tested before finding 3 contaminated wells. X ~ NB(r=3, p=0.1), and we want to find P(X=5) = ?

For the negative binomial distribution, the pmf is:

$$P(X=x) = f(x) = \binom{x-1}{r-1} \times p^{r} \cdot (1-p)^{x-r} \quad , \text{ for } x = r, r+1, \ldots, \infty$$

Thus,

$$P(X=5)=f(5)=\binom{4}{2}\times 0.1^3\cdot 0.9^2 = \frac{4!}{2!\cdot 2!}\cdot 0.1^3\cdot 0.9^2 = 0.0049$$

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[4]. Geometric. The probability that a manufacturing machine requires repairs on a given day is 0.05. What is the probability that the first time that the machine require repairs occurs in the 5-th day of operation?

Solution: X = number of days for machine repairs, X ~ geom(p=0.05). Thus, we need to find P(X=5) = f(5) = ?

For the geometric distribution, the pmf is:

$$P(X=x)=f(x)=p\cdot(1-p)^{x-1}, \quad x = 1,2,...,\infty$$

The solution is: $P(X=5)=f(5)=0.05\cdot 0.95^4 = 0.0407$

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[5]. Hypergeometric. You receive a sample of 20 net book computers to provide to your engineering field crew. After you receive the shipment of computers, and send a crew of 10 people with computers into the field, the manufacturer calls you and tells you that 6 of the computers have a defective WiFi unit. What is the probability that exactly 2 of the computers in the field have a defective WiFi unit?

Solution: X = number of defective computers out of 10 sent to the field, knowing that in a population of 20 computers, 6 are defective. X ~ H(N=20,R=6,n=10). Find P(X=2) = ?

For the hypergeometric distribution the pmf is:

$$P(X=x)=f(x)=\frac{\binom{R}{x}\cdot\binom{N-R}{n-x}}{\binom{N}{n}} =$$

Then,

$$P(X=2)=f(2)=\frac{\binom{6}{2}\cdot\binom{14}{8}}{\binom{20}{10}} = \frac{\frac{6!}{2!\cdot 4!}\cdot\frac{14!}{8!\cdot 6!}}{\frac{20!}{10!\cdot 10!}} = 0.2438$$

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[6]. The tensile strength of nylon strings used for a test in the lab follows the normal distribution with a mean of 200 N/mm^2 and a standard deviation of 10 N/mm^2. What percentage of nylon strings would have a tensile strength between 190 and 210 N/mm^2?

Solution: X = tensile strength of nylon strings in N/mm^2, X~N(μ = 200, σ = 10). We want:

$$P(190<X<210) = P\left(\frac{190-200}{10} < \frac{X-\mu}{\sigma} < \frac{210-200}{10}\right) = P(-1<Z<1) = \Phi(1) - \Phi(-1),$$

where $\Phi(z)$ is the CDF of the standard normal variable $Z = \frac{X-\mu}{\sigma}$.

Values of $\Phi(z)$ can be found in tables:

$$\Phi(1) = 0.8413 \quad \text{and} \quad \Phi(-1) = 0.1587$$

Thus, $P(190<X<210) = 0.8413 - 0.1587 = 0.6826$

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[7]. The hydraulic conductivity (m/s) of soil samples from a given aquifer is found to follow the lognormal distribution with parameters $\mu = 0.2$ and $\sigma = 0.05$. What is the mean value of the permeability of the aquifer?

Solution. X = hydraulic conductivity (m/s) of soil samples, $X \sim \text{lognormal}(\mu=0.2, \sigma=0.05)$. The mean value of X is:

$$\mu_X = \exp\left(\mu + \frac{\sigma^2}{2}\right) = \exp\left(0.2 + \frac{0.05^2}{2}\right) = 1.2229 \text{ m/s}$$

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[8]. The time to failure, in days, of a field soil moisture probe follows the exponential distribution. The mean failure time is reported to be 3 days. What is the probability that a given soil moisture probe will last less than 2 days?

Solution: X = time to failure (days) of a probe, $X \sim \exp(\lambda)$

with: $\mu_X = \frac{1}{\lambda}$. Since $\mu_X = 3$, then $\lambda = \frac{1}{\mu_X} = \frac{1}{3}$.

We are seeking $P(X<2) = F(2)$. For the exponential distribution

$P(X<x) = F(x) = 1 - e^{-\lambda \cdot x}$, $x>0$. Thus,

$$P(X<2) = F(2) = 1 - e^{-\left(\frac{1}{3}\right)\cdot 2} = 1 - e^{-\frac{2}{3}} = 0.4866$$

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[9]. The maximum daily discharge in a small stream, in cubic feet per second (cfs), follows the Weibull distribution with parameters $\alpha = 2$ and $\beta = 3$. Determine the probability that, on a given day, the maximum daily discharge will be between 0.5 and 5.5 cfs.

Solution: X = max. daily discharge (cfs) in a small stream, $X \sim \text{Weibull}(\alpha=2, \beta=3)$. We seek $P(0.5<X<5.5) = ?$

For the Weibull distribution, $F(x) = 1 - e^{-(\beta x)^{\alpha}}$, for $x>0$, thus:

$$P(0.5 < X < 5.5) = F(5.5) - F(0.5) = \left(1 - e^{-(3\cdot 5.5)^2}\right) - \left(1 - e^{-(3\cdot 0.5)^2}\right)$$

$$P(1.5 < X < 2.5) = e^{-1.5^2} - e^{-16.5^2} = 0.1054$$

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[10]. A cylindrical Uranium rod emits particles radially so that the angle of emission is uniformly distributed in the range a = -π and b = +π. If a Geiger counter is located in front of the cylinder and it detects particles emitted in the range -π/4 to +π/4 only, what is the probability that a given particle emitted from the rod will be detected by the Geiger counter?

Solution: X = angle along which radioactive particles are emitted from a rod, X~uniform(a=-π,b=+π), we want to find P(-π/4<X<π/4) = ?

For the uniform distribution, the CDF is given by:

$$F(x) = \frac{x-a}{b-a} = \frac{x-(-\pi)}{\pi-(-\pi)} = \frac{x+\pi}{2\cdot\pi}$$

Therefore,

$$P(-\pi/4<X<\pi/4) = F\left(\frac{\pi}{4}\right) - F\left(-\frac{\pi}{4}\right) = \frac{\frac{\pi}{4}+\pi}{2\cdot\pi} - \frac{-\frac{\pi}{4}+\pi}{2\cdot\pi} = 0.25$$
