Using superposition with MNA method

Utilities

Instead simplfiy

$SN(x\#) := \text{str2num}(\text{eval}(\text{num2str}(x\#)))$

Complex Signum

$$csign(z) := \begin{cases} \text{sign}(\text{Re}(z)) & \text{if } \text{Re}(z) \neq 0 \\ \text{sign}(\text{Im}(z)) & \text{otherwise} \end{cases}$$

Circuit and admittance matrix, by hand or using MNA function from the other worksheet.

V1	1	0
R1	1	2
C1	2	0
RT	2	3
CT	2	3
LK	3	4
RK	4	5
CK	5	0
RO	5	0
CO	5	0

$$A := \begin{bmatrix} \frac{1}{R1} & -\frac{1}{R1} & 0 & 0 & 0 & 1 \\ -\frac{1}{R1} & \frac{R1\cdot(1+s\cdot CT\cdot RT+s\cdot C1\cdot RT)+RT}{R1\cdot RT} & -\frac{1+s\cdot CT\cdot RT}{RT} & 0 & 0 & 0 \\ 0 & -\frac{1+s\cdot CT\cdot RT}{RT} & \frac{RT\cdot(1+s^2\cdot CT\cdot LK)+s\cdot LK}{s\cdot LK\cdot RT} & -\frac{1}{s\cdot LK} & 0 & 0 \\ 0 & 0 & -\frac{1}{s\cdot LK} & \frac{RK+s\cdot LK}{s\cdot LK\cdot RK} & -\frac{1}{RK} & 0 \\ 0 & 0 & 0 & -\frac{1}{RK} & \frac{RK\cdot(1+s\cdot CO\cdot RO+s\cdot CK\cdot RO)+RO}{RK\cdot RO} & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\chi := \text{"}\chi\text{"}$

Solve the Linear System

$B := [\,0\ 0\ 0\ 0\ 0\ V1s\,]^T$

$V := SN\left(A^{-1}\cdot B\right)$

Complex Freq

$H(s, C1) := SN\left(\frac{V_2}{V_1}\right)$

or $H(s, C1, R1, XT, RT, \ldots)$

Symbolic Soltuions

$v_{12}(t) := v_1(t, f) - v_2(t)$

As time function only, faster.

$$v_2(t) := \sum_{\kappa=1}^{\Sigma max} H_n(2\cdot\kappa-1)\cdot V_1(t, \mathrm{i}\cdot 2\cdot\pi\cdot f, 2\cdot\kappa-1)$$

$v_{12}(t, f, C) := v_1(t, f) - v_2(t, f, C)$

As function of several variables

$$v_2(t, f, C) := \sum_{\kappa=1}^{\Sigma max} H(\mathrm{i}\cdot(2\cdot\kappa-1)\cdot 2\cdot\pi\cdot f, C)\cdot V_1(t, \mathrm{i}\cdot 2\cdot\pi\cdot f, 2\cdot\kappa-1)$$

Square Wave

$v_1(t, f) := Vo\cdot csign\left(-\mathrm{i}\cdot\mathbf{e}^{\mathrm{i}\cdot 2\cdot\pi\cdot f\cdot t}\right)$

$V_1(t, s, n) := -\mathrm{i}\cdot\frac{4\cdot Vo}{n\cdot\pi}\cdot\mathbf{e}^{n\cdot s\cdot t}$

Values

$R1 := 327.1\ \text{k}\Omega$ $\quad C1 := 200\ \text{pF}$ $\quad Vo := 0.8\ \text{V}$

$RT := 9\ \text{M}\Omega$ $\quad CT := 12.78\ \text{pF}$ $\quad LK := 250\ \text{nH}$

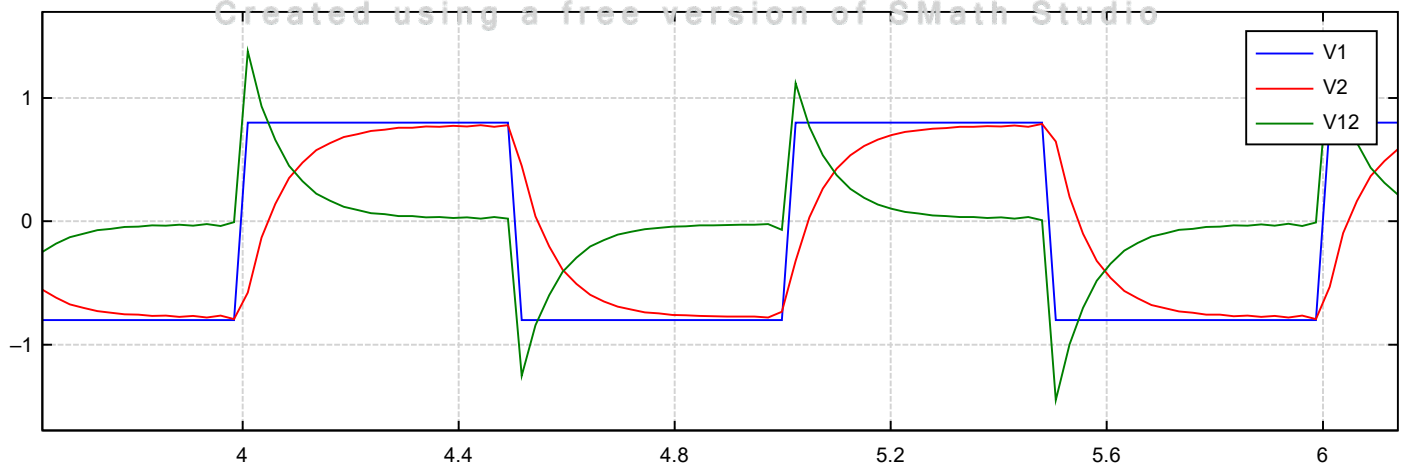
$RK := 1.5\ \Omega$ $\quad CK := 100\ \text{pF}$ $\quad \Sigma max := 10$

$RO := 1\ \text{M}\Omega$ $\quad CO := 15\ \text{pF}$

$$\Pi := \begin{cases} \text{Re}(v_1(x\cdot\tau, f)) \\ \text{Re}(v_2(x\cdot\tau)) \\ \text{Re}(v_{12}(x\cdot\tau)) \end{cases}$$

Frequency

$f := 1000\ \text{Hz}$ $\quad H_n(n) := SN(H(\mathrm{i}\cdot n\cdot 2\cdot\pi\cdot f, C1))$ $\quad \tau := \frac{1}{f}$



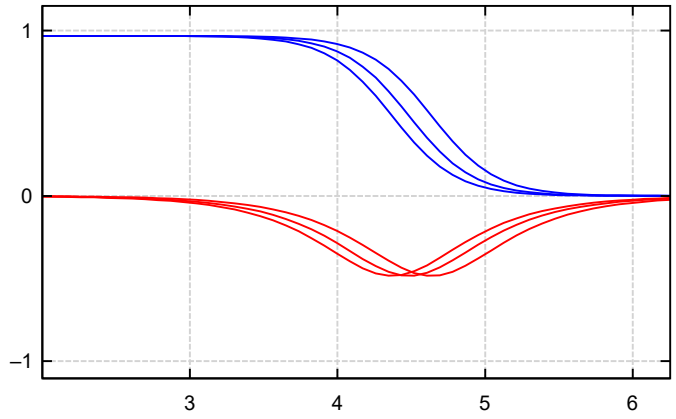
Transfer function $Co := \text{stack}(0.1, 5, 10)\ \text{pF}$ $k := [1..3]$ $sx := i \cdot 2 \cdot \pi \cdot 10^{x}\ \text{Hz}$

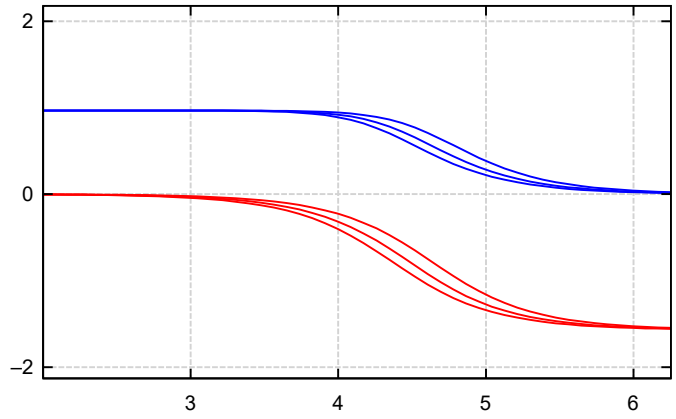
$\delta_k := \text{str2num}(\text{strrep}("PH'Traces\#@'LineStyle'LineColor:"blue"", "@", \text{num2str}(k-1)))$ δ = *dummy*

$\delta_k := \text{str2num}(\text{strrep}("PH'Traces\#@'LineStyle'LineColor:"red"", "@", \text{num2str}(k-1+3)))$

$$\Pi 1 := \begin{cases} \text{mat2sys}_1(\delta_k := \text{Re}(H(sx, Co_k))) \\ \text{mat2sys}_1(\delta_k := \text{Im}(H(sx, Co_k))) \end{cases}$$

$$\Pi 2 := \begin{cases} \text{mat2sys}_1(\delta_k := |H(sx, Co_k)|) \\ \text{mat2sys}_1(\delta_k := \arg(H(sx, Co_k))) \end{cases}$$





Utility

$$NR(y, a) := \left| \begin{array}{l} \text{for } iter \in [1..1] \\ \quad \left| \begin{array}{l} [ya\ y\delta] := \left[y\big|_{t=a}\ y\big|_{t=a+\Delta t} \right] \\ a := \text{eval}\left(a - \frac{ya \cdot \Delta t}{y\delta - ya} \right) \end{array} \right. \\ a \end{array} \right.$$

$\Delta t := 0.00001\ \text{s}$

Freqs $\text{Clear}(f) = 1$

$nf := [1..10]$ $X := \frac{nf-1}{10-1} \cdot 3 + \log10(50)$ $F := \overrightarrow{10^{X}}\ \text{Hz}$ $\tau := \overrightarrow{\frac{1}{F}}$

Caps $Co := \text{stack}(0.1, 5)\ \text{pF}$ $nc := [1..\text{length}(Co)]$

Measures $VPP1_{nf\,nc} := v_1(0.5 \cdot \tau_{nf}, F_{nf}) - v_1(0, F_{nf})$

$V2PP_{nf\,nc} := \text{Re}\left(v_2(0.5 \cdot \tau_{nf}, F_{nf}, Co_{nc}) - v_2(0, F_{nf}, Co_{nc}) \right)$

$V12PP_{nf\,nc} := \mathrm{Re}\left(v_{12}\left(0.5\cdot\tau_{nf}, F_{nf}, Co_{nc}\right) - v_{12}\left(0, F_{nf}, Co_{nc}\right)\right)$

$t1_{nf\,nc} := 0$

$t2_{nf\,nc} := \mathrm{mod}\left(NR\left(\mathrm{Re}\left(v_2\left(t, F_{nf}, Co_{nc}\right)\right), 0\right), \tau_{nf}\right)\ s$

$t12_{nf\,nc} := \mathrm{mod}\left(NR\left(\mathrm{Re}\left(v_{12}\left(t, F_{nf}, Co_{nc}\right)\right), 0\right), \tau_{nf}\right)\ s$

$\Delta t2_{nf\,nc} := t1_{nf\,nc} - t2_{nf\,nc}$

$\Delta t12_{nf\,nc} := t1_{nf\,nc} - t12_{nf\,nc}$

$Pha2_{nf\,nc} := 2\cdot\pi\cdot F_{nf}\cdot\Delta t2_{nf\,nc}$

$Pha12_{nf\,nc} := 2\cdot\pi\cdot F_{nf}\cdot\Delta t12_{nf\,nc}$

$tbl := \mathrm{augment}(F, \mathrm{col}(VPP1, 1), \mathrm{col}(V2PP, 1), \mathrm{col}(V12PP, 1), \mathrm{col}(Pha2, 1), \mathrm{col}(Pha12, 1))$

$tbl := \mathrm{augment}(tbl, \mathrm{col}(VPP1, 2), \mathrm{col}(V2PP, 2), \mathrm{col}(V12PP, 2), \mathrm{col}(Pha2, 2), \mathrm{col}(Pha12, 2))$

$tblH := \begin{bmatrix} \text{"S:VPP1"} & \text{"S:VPP2"} & \text{"S:VPP12"} & \text{"S:Pha2"} & \text{"S:Pha12"} \\ \text{"S:'V"} & \text{"S:'V"} & \text{"S:'V"} & \text{"S:'deg"} & \text{"S:'deg"} \end{bmatrix}$

$tblH := \mathrm{eval}\left(\mathrm{augment}\left(\begin{bmatrix} \text{"S:Freq"} \\ \text{"S:'Hz"} \end{bmatrix}, tblH, tblH\right)\right)$ $\quad tblU := \mathrm{eval}\left(\mathrm{row}\left(\overrightarrow{\mathrm{str2num}(tblH)}, 2\right)\right)$

< - - - - - - - - Co1 - - - - - - - - - - - >< - - - - - - - Co2 - - - - - - - - - - - >

Freq	VPP1	VPP2	VPP12	Pha2	Pha12	VPP1	VPP2	VPP12	Pha2	Pha12
Hz	V	V	V	deg	deg	V	V	V	deg	deg
50.00	1.60	0.02	1.57	-0.06	-0.09	1.60	0.03	1.56	-0.09	-0.08
107.70	1.60	0.04	1.55	-0.14	-0.19	1.60	0.06	1.53	-0.20	-0.19
232.10	1.60	0.10	1.49	-0.30	-0.42	1.60	0.14	1.45	-0.43	-0.41
500.00	1.60	0.22	1.37	-0.66	-0.95	1.60	0.31	1.28	-0.92	-0.89
1077.00	1.60	0.45	1.14	-1.41	-2.27	1.60	0.60	0.99	-1.94	-1.93
2321.00	1.60	0.80	0.79	-3.12	-6.36	1.60	0.97	0.62	-3.99	-4.49
5000.00	1.60	1.14	0.45	-8.30	-11.40	1.60	1.25	0.34	-9.63	-7.23
10770.00	1.60	1.35	0.24	-19.55	-18.57	1.60	1.41	0.18	-21.94	-10.40
23210.00	1.60	1.44	0.15	-43.25	-31.47	1.60	1.43	0.16	-47.26	-20.76
50000.00	1.60	1.31	0.28	-90.00	-90.00	1.60	1.12	0.47	-90.00	-90.00

tbl

Alvaro $\quad \mathrm{appVersion}(4) = \text{"1.74.9654.0"}$